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1. (a) $\boxed{45\ \text{mW}}$

(b) $\boxed{2\ \text{nJ}}$

(c) $\boxed{100\ \text{ps}}$

(d) $\boxed{39.212\ \text{fs}}$

(e) $\boxed{3\ \Omega}$

(f) $\boxed{18\ \text{km}}$

(g) $\boxed{2.5\ \text{Tb}}$

(h) $\boxed{100\ \text{exaatoms/m}^3}$

2. (a) $\boxed{1.23 \text{ ps}}$

(b) $\boxed{1\ \mu\text{m}}$

(c) $\boxed{1.4 \text{ K}}$

(d) $\boxed{32 \text{ nm}}$

(e) $\boxed{13.56 \text{ MHz}}$

(f) $\boxed{2.021 \text{ millimoles}}$

(g) $\boxed{130 \text{ ml}}$

(h) $\boxed{100 \text{ m}}$

3. $\boxed{\text{(a) 1.212 V}}$

$\boxed{\text{(b) 100 mA}}$

$\boxed{\text{(c) 1 zs}}$

$\boxed{\text{(d) 33.9997 zs}}$

$\boxed{\text{(e) 13.1 fs}}$

$\boxed{\text{(f) 10 Ms}}$

$\boxed{\text{(g) 10 }\mu\text{s}}$

$\boxed{\text{(h) 1 s}}$

4. (a) $\boxed{10^{21}\ \text{m}}$

(b) $\boxed{10^{18}\ \text{m}}$

(c) $\boxed{10^{15}\ \text{m}}$

(d) $\boxed{10^{12}\ \text{m}}$

(e) $\boxed{10^{9}\ \text{m}}$

(f) $\boxed{10^{6}\ \text{m}}$

5. (a) $\boxed{373.15\text{ K}}$

(b) $\boxed{255.37\text{ K}}$

(c) $\boxed{0\text{ K}}$

(d) $\boxed{149.1\text{ kW}}$

(e) $\boxed{914.4\text{ mm}}$

(f) $\boxed{1.609\text{ km}}$

6. (a) $\boxed{373.15\text{ K}}$

(b) $\boxed{273.15\text{ K}}$

(c) $\boxed{4.2\text{ K}}$

(d) $\boxed{112\text{ kW}}$

(e) $\boxed{528\text{ kJ}}$

(f) $\boxed{100\text{ W}}$ (100 J/s is also acceptable)

7. (a) $P = 550 \text{ mJ} / 15 \text{ ns} = \boxed{36.67 \text{ MW}}$

(b) $P_{avg} = (550 \text{ mJ/pulse})(100 \text{ pulses/s}) = 55 \text{ J/s} = \boxed{55 \text{ W}}$

8. (a) 500×10^{-6} J/50×10^{-15} s = 10 GJ/s = $\boxed{\text{10 GW}}$

(b) (500×10^{-6} J/pulse)(80×10^{6} pulses/s) = 40 kJ/s = $\boxed{\text{40 kW}}$

9. Energy = (40 hp)(1 W/ 1/745.7 hp)(3 h)(60 min/h)(60 s/ min) = $\boxed{322.1 \text{ MJ}}$

10. $(20\ \text{hp})(745.7\ \text{W/hp})/[(500\ \text{W/m}^2)(0.1)] = \boxed{298\ \text{m}^2}$

11. (a) (100 pW/device)(N devices) = 1 W. Solving, N = $\boxed{10^{10} \text{ devices}}$

(b) Total area = (1 μm^2/ 5 devices)(10^{10} devices) = $\boxed{2000 \text{ mm}^2}$

(roughly 45 mm on a side, or less than two inches by two inches, so yes).

12. (a) 20×10^3 Wh/ 100 W = 200 h

So, in one day we remain at the \$0.05/kWh rate.

(0.100 kW)(N 100 W bulbs)(\$0.05/kWh)(7 days)(24 h/day) = \$10

Solving, N = 11.9

Fractional bulbs are not realistic so rounding down, $\boxed{\text{11 bulbs maximum.}}$

(b) Daily cost = (1980)(\$0.10/kWh)(24 h) + (20 kW)(\$0.05/kWh)(24 h) = $\boxed{\$4776}$

13. Between 9 pm and 6 am corresponds to 9 hrs at \$0.033 per kWh.
Thus, the daily cost is (0.033)(2.5)(9) + (0.057)(2.5)(24 – 9) = \$2.88

Consequently, 30 days will cost $\boxed{\$86.40}$

14. $$\frac{(9\times10^{9}\ \text{person})(100\ \text{W/person})}{(800\ \text{W/m}^{2})(0.1)}=\boxed{11.25\times10^{9}\ \text{m}^{2}}$$

15. $q(t) = 5e^{-t/2}$ C

$dq/dt = -(5/2)\,e^{-t/2}$ C/s $=$ $\boxed{-2.5e^{-t/2}\text{ A}}$

16. $q = i \cdot t = (10^{-9}\text{ A})(60\text{ s}) = \boxed{60\text{ nC}}$

17. (a) # electrons = -10^{13} C/(-1.602×10^{-19} C/electron) = $\boxed{6.242\times10^{31}\text{ electrons}}$

(b) $$\left[\frac{6.242\times10^{31}\text{ electrons}}{\pi\left(\frac{1\text{ cm}}{2}\right)^2}\right]\left(\frac{100\text{ cm}}{1\text{ m}}\right)^2 = \boxed{7.948\times10^{35}\text{ electrons/m}^2}$$

(c) current = (10^6 electons/s)(-1.602×10^{-19} C/electron) = $\boxed{160.2\text{ fA}}$

18. $q(t) = 9 - 10t$ C

(a) $q(0) = \boxed{9\text{ C}}$

(b) $q(1) = \boxed{-1\text{ C}}$

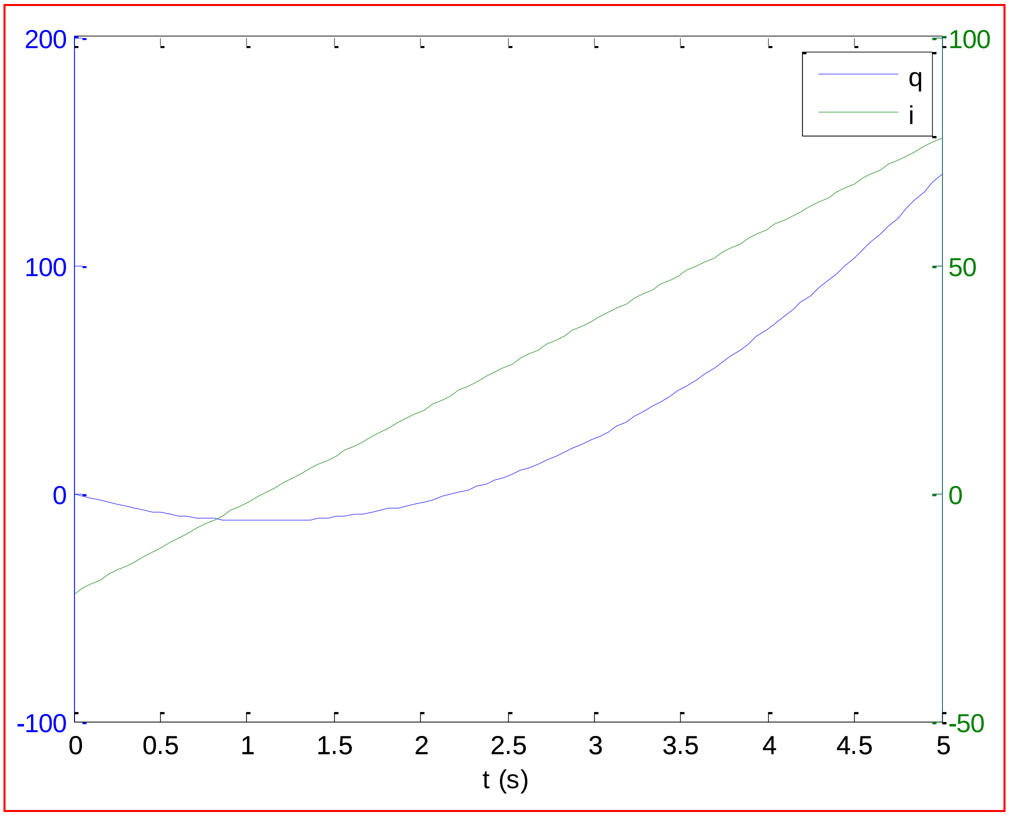
(c) $i(t) = dq/dt = \boxed{-10\text{ A, regardless of value of } t}$

19. (a) $q = 10t^2 - 22t$

$i = dq/dt = 20t - 22 = 0$

Solving, $t = \boxed{1.1\text{ s}}$

(b)



20. i(t) = 114sin 100πt A

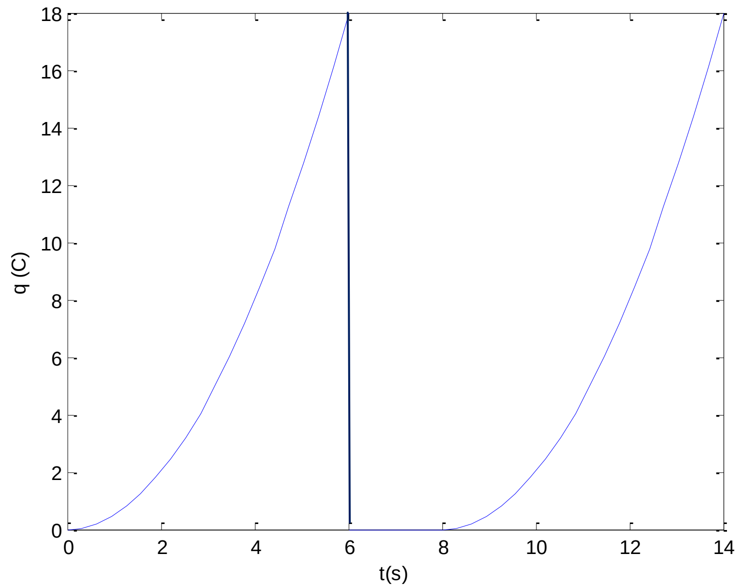
(a) This function is zero whenever 100πt = πn, n = 1, 2, …
or when t = 0.01n.
Therefore, the current drops to zero $\boxed{\text{201 times}}$ (t = 0, t = 0.01, … t = 2) in the interval.

(b) $q = \int_0^1 i dt = 114\int_0^1 \sin 100\pi t = -\frac{114}{100\pi}\cos 100\pi t\Big|_0^1 = \boxed{0 \text{ C net}}$

21. (a) Define $i_{avg} = \frac{1}{T}\int_0^T i(t)dt = \frac{1}{8}\int_0^8 tdt = \boxed{2.25 \text{ A}}$

(b) $q(t) = \int_0^t i(t')dt' = \int_0^t t'dt' =$

$$\boxed{\begin{array}{ll} 500t^2 \text{ mC}, & 0 \le t < 6 \\ 0, & 6 \le t < 8 \\ 500(t-8)^2 \text{ mC}, & 8 \le t < 14 \end{array}}$$



22. (a) $i_{avg} = \frac{(3)(1)+(-1)(1)+(1)(1)+(0)(1)}{4} = \boxed{750 \text{ mA}}$

(b) $i_{avg} = \frac{(3)(1)+(-1)(1)+(1)(1)}{3} = \boxed{1 \text{ A}}$

(c) $q(t) = \int i(t)dt = \boxed{\begin{cases} 3t+q(0) = 3t+1 \text{ C}, & 0 \le t \le 1 \text{ s} \\ -t+q(1) = -t+4 \text{ C}, & 1 \le t \le 2 \text{ s} \\ t+q(2) = t+2 \text{ C}, & 2 \le t \le 3 \text{ s} \\ q(3) = 5 \text{ C}, & 3 \le t \le 4 \text{ s} \end{cases}}$

23. A to C = 5 pJ. B to C = 3 pJ. Thus, A to B = 2 pJ.
A to D = 8 pJ so C to D = 3 pJ

(a) $V_{CB} = 3\times10^{-12}/–1.602\times10^{-19} = \boxed{–18.73 \text{ MV}}$

(b) $V_{DB} = (3 + 3) \times10^{-12}/ –1.602\times10^{-19} = \boxed{–37.45 \text{ MV}}$

(c) $V_{BA} = 2\times10^{-12}/–1.602\times10^{-19} = \boxed{–12.48 \text{ MV}}$

24. $v_x = 10^{-3}$ J/ -1.602×10^{-19} C $= \boxed{-6.24\times10^{15} \text{ V}}$

$v_y = -v_x = \boxed{+6.24\times10^{15} \text{ V}}$

25. (a) Voltage is defined as the potential difference between two points, hence two wires are needed (one to each 'point').

(b) The reading will be the negative of the value displayed previously.

26. (a) $P_{abs} = (+6)(+10^{-12}) = \boxed{6 \text{ pW}}$

(b) $P_{abs} = (+1)(+10\times10^{-3}) = \boxed{10 \text{ mW}}$

(c) $P_{abs} = (+10)(-2) = \boxed{-20 \text{ W}}$

27. (a) $P_{abs} = (2)(-1) = \boxed{-2\text{ W}}$

(b) $P_{abs} = (-16e^{-t})(0.008e^{-t}) = \boxed{-47.09\text{ mW}}$

(c) $P_{abs} = (2)(-10^{-3})(0.1) = \boxed{-200\ \mu\text{W}}$

28. $P_{abs} = v_p(1)$

(a) $(+1)(1) = \boxed{1\text{ W}}$

(b) $(-1)(1) = \boxed{-1\text{ W}}$

(c) $(2 + 5\cos 5t)(1) = (2 + 5\cos 5)(1) = \boxed{3.418\text{ W}}$

(d) $(4e^{-2t})(1) = (4e^{-2})(1) = \boxed{541.3\text{ mW}}$

(e) A negative value for absorbed power indicates the element is actually supplying power to whatever it is connected to.

29. $P_{supplied} = (2)(2) = \boxed{4\text{ W}}$

30. (a) Short circuit corresponds to zero voltage, hence i_{sc} = $\boxed{3.0\ \text{A}}$.

(b) Open circuit corresponds to zero current, hence v_{oc} = $\boxed{500\ \text{mV.}}$

(c) $P_{max} \approx (0.375)(2.5) = \boxed{938\ \text{mW}}$ (near the knee of curve)

31. Looking at sources left to right,

$P_{supplied}$ =

(2)(2)	= 4 W;
(8)(2)	= 16 W;
(10(-4)	= -40 W;
(10)(5)	= 50 W;
(10)(-3)	= -30 W

Note these sum to zero, as expected.

32. The remaining power is leaving the laser as heat, due to losses in the system. Conservation of energy requires that the total output energy, regardless of form(s), equal the total input energy.

33. (a) $V_R = 10$ V, $V_x = 2$ V

$P_{abs} =$

$(2)(-10) = -20$ W;
$(10)(10) = 100$ W;
$(8)(-10) = -80$ W

(b) element A is a passive element, as it is absorbing positive power

34. (a) $V_R = 10\ 0$ V, $V_x = 92$ V

$P_{Vx(supplied)} = (92)(5V_x) = (92)(5)(92)$ $= 42.32$ kW

$P_{VR(supplied)} = (100)(-5V_x) = -100(5)(92)$ $= -46.00$ kW

$P_{5Vx(supplied)} = (8)(5V_x) = (8)(5)(92)$ $= 3.68$ W

(b) $\boxed{42.32 - 46 + 3.68 = 0}$

35. $i_2 = -3v_1$ therefore $v_1 = -100/3$ mV = $\boxed{-33.33 \text{ mV}}$

36. First, it cannot dissipate more than 100 W and hence imax = 100/12 = 8.33 A

It must also allow at least 12 W or imin = 12/12 = 1 A

10 A is too large; 1 A is just on the board and likely to blow at minimum power operation, so 4 A is the optimum choice among the values available.

37. $(-2i_x)(-i_x) = 1$

Solving, $\boxed{i_x = 707 \text{ mA}}$

38. (a) $10^{-3}/4.7\times10^{3}$ = $\boxed{210\text{ nA}}$

(b) $10/4.7\times10^{3}$ = $\boxed{2.1\text{ mA}}$

(c) $4e^{-t}/4.7\times10^{3}$ = $\boxed{850e^{-t}\ \mu\text{A}}$

(d) $100\cos5t / 4.7\times10^{3}$ = $\boxed{21\cos5t\ \text{mA}}$

(e) $\left|\dfrac{-7}{4.7\times10^{3}}\right|$ = $\boxed{1.5\text{ mA}}$

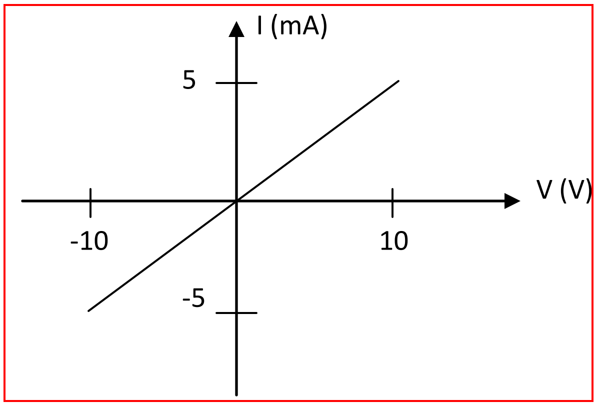
39. (a) $(1980)(0.001) = \boxed{1.98 \text{ V}}$;
$(2420)(0.001) = \boxed{2.42 \text{ V}}$

(b) $(1980)(4\times10^{-3}\sin 44t) = \boxed{7.92 \sin 44t \text{ V}}$;
$(2420)(4\times10^{-3}\sin 44t) = \boxed{9.68 \sin 44t \text{ V}}$

40. $I = V/R$; $I_{min} = V_{min}/R = -10/2000 = -5$ mA

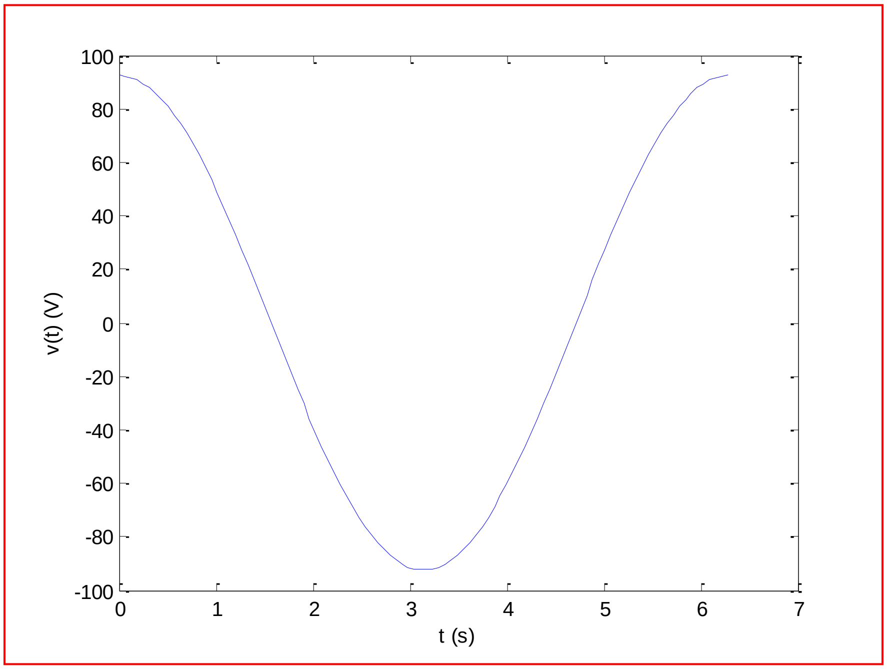
$I_{max} = V_{max}/R = +10/2000 = 5$ mA

(a)



(b) slope = dI/dV = $5\times10^{-3}/10$ = $\boxed{500\ \mu s}$

41. We expect the voltage to be 33 times larger than the current, or 92.4 cos t V.



42. (a) $R = 5/0.05\times10^{-3} = \boxed{100\ \text{k}\Omega}$

(b) $R = \boxed{\infty}$

(c) $R = \boxed{0}$

43. (a) $\boxed{\infty}$

(b) $\boxed{10 \text{ ns}}$

(c) $\boxed{5 \text{ s}}$

44. $G = 10$ mS; $R = 1/G = 100\ \Omega$

(a) $i = 2\times10^{-3}/100 = \boxed{20\ \mu\text{A}}$

(b) $i = 1/100 = \boxed{10\ \text{mA}}$

(c) $i = 100e^{-2t}/100 = \boxed{e^{-2t}\ \text{A}}$

(d) $i = 0.01(5)\sin 5t = \boxed{50 \sin 5t\ \text{mA}}$

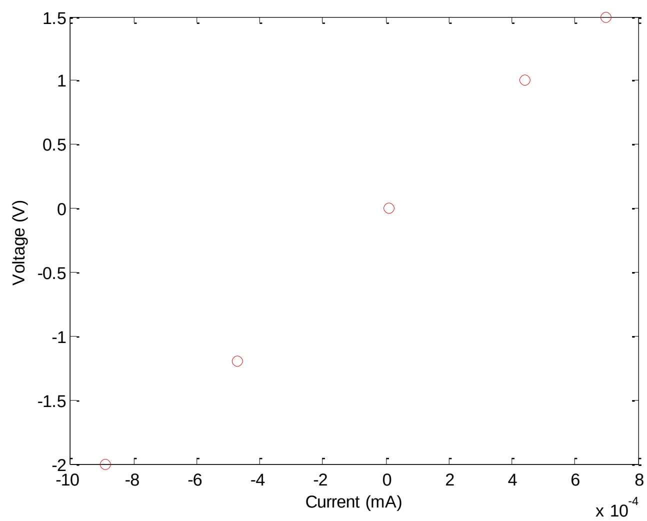
(e) $i = \boxed{0}$

45. (a) $i_{lo} = 9/1010 = \boxed{8.91 \text{ mA}}$;
$i_{hi} = 9/990 = \boxed{9.09 \text{ mA}}$

(b) $P_{lo} = 9^2/1010 = \boxed{80.20 \text{ mW}}$;
$P_{hi} = 9^2/990 = \boxed{81.82 \text{ mW}}$

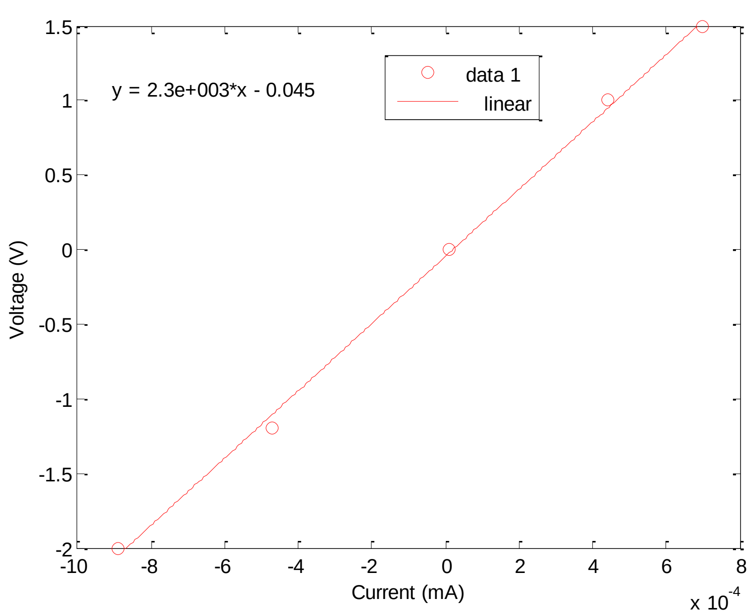
(c) $9/1100 = \boxed{8.18 \text{ mA}}$;
$9/900 = \boxed{10 \text{ mA}}$;
$9^2/1100 = \boxed{73.6 \text{ mW}}$;
$9^2/900 = \boxed{90.0 \text{ mW}}$

46. (a) Plotting the data in the table provided,



(b) A best fit (using MATLAB fitting tool in plot window) yields a slope of $\boxed{2.3\ \text{k}\Omega}$.

However, this is only approximate as the best fit does not intersect zero current at zero voltage.



47. Define I flowing clockwise. Then

P_{Vs}(supplied) = $V_s I$

P_{R1}(absorbed) = $I^2 R_1$

P_{R2}(absorbed) = $(V_{R2})^2/R_2$

Equating, $V_s I = I^2 R_1 + (V_{R2})^2/R_2$ [1]

Further, $V_s I = I^2 R_1 + I^2 R_2$

or $I = V_s/(R_1 + R_2)$ [2]

We substitute Eq. [2] into Eq. [1] and solve for $(V_{R2})^2$:

$$(V_{R2})^2 = V_s^2\left[\frac{R_2^2}{(R_1+R_2)^2}\right] \text{ hence } V_{R2} = V_s\left[\frac{R_2}{R_1+R_2}\right]. \boxed{\text{QED.}}$$

48. TOP LEFT:

$I = 5/10\times10^3 = 500\ \mu A$

$P_{abs} = 5^2/10\times10^3 = 2.5\ mW$

TOP RIGHT:

$I = -5/10\times10^3 = -500\ \mu A$

$P_{abs} = (-5)^2/10\times10^3 = 2.5\ mW$

BOTTOM LEFT:

$I = -5/10\times10^3 = -500\ \mu A$

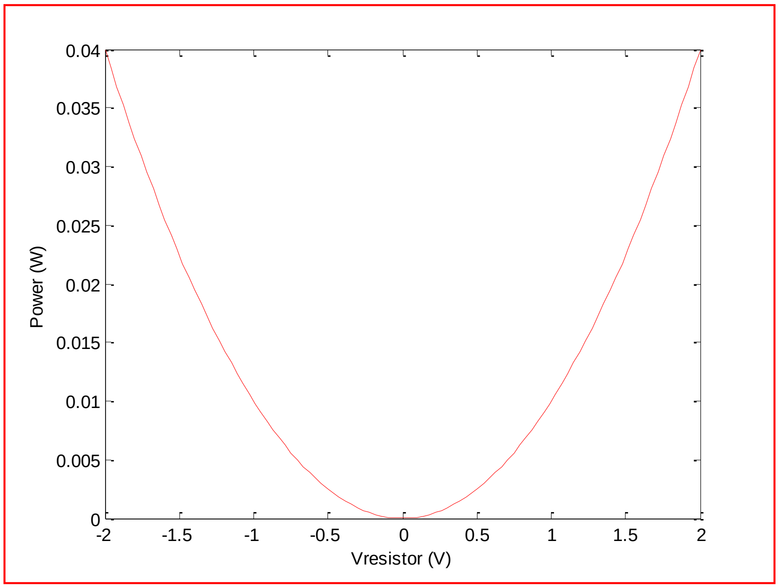
$P_{abs} = (-5)^2/10\times10^3 = 2.5\ mW$

BOTTOM RIGHT:

$I = -(-5)/10\times10^3 = 500\ \mu A$

$P_{abs} = (-5)^2/10\times10^3 = 2.5\ mW$

49. Power = V^2/R so



50. <DESIGN>

One possible solution:

$$R = \rho \frac{L}{A} = \frac{L}{A(qN_D \mu_n)} = 10 .$$

Select $N_D = 10^{14}$ atoms/cm^2, from the graph, $\mu_n = 2000$ cm^2/Vs.

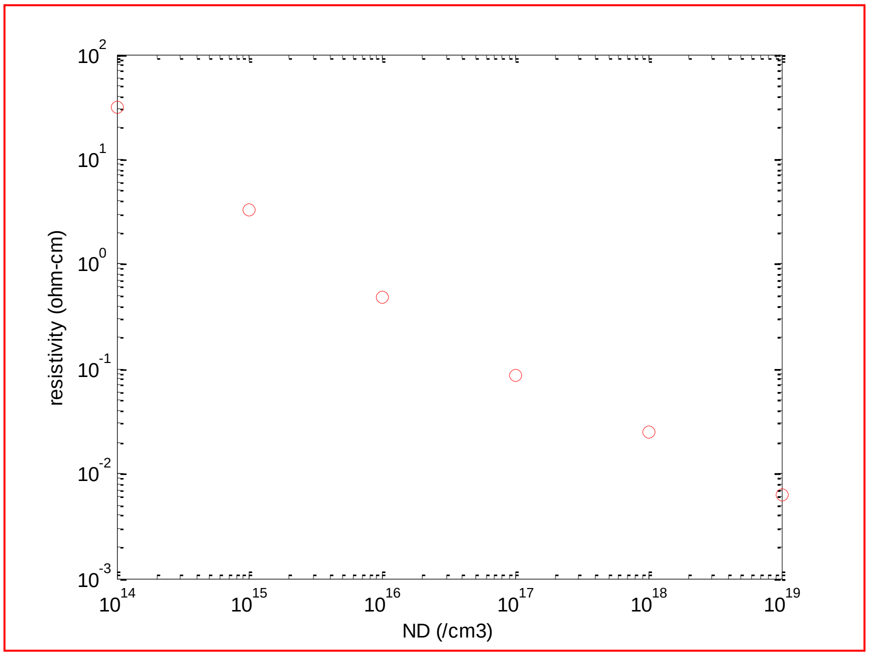
Hence, $qN_D\mu_n = 0.032$, so that L/A = 0.32 μm^{-1}.

Using the wafer thickness as one dimension of our cross sectional area A, A = 300x μm^2 and y is the other direction on the surface of the wafer, so L = y.

Thus, L/A = y/300x = 0.32. Choosing y = 6000 μm, x = 62.5 μm.

Summary: Select wafer with phosphorus concentration of 10^{14} atoms/cm^3, cut surface into a rectangle measuring 62.5 μm wide by 6000 μm long. Contact along narrow sides of the 6000 μm long strip.

51. $\rho = (qN_D\mu_n)^{-1}$. Estimating μ_n from the graph, keeping in mind "half-way" on a log scale corresponds to 3, not 5,



52. <DESIGN> One possible solution:

We note 28 AWG wire has a resistance of 65.3 ohms per 1000 ft length (at 20^{o}C).

There, 1531 ft is approximately 100 ohms and 7657 ft is approximately 500 ohms.

These are huge lengths, which reincorces the fact that copper wire is a very good conductor.

*Wrap the 7657 ft of 28 AWG wire around a (long) nonconducting rod.

*Connect to the left end.

*The next connection slides along the (uninsulated) coil. When connection is approximately 20% of the length as measured from the left, R = 100 ohms. When it is at the far (right) end, R = 500 ohms.

53. 14 AWG = 2.52 ohms per 1000 ft.
R = (500 ft)(2.52 ohms/1000 ft) = 1.26 ohms

$$P = I^2R = (25)^2(1.26) = \boxed{787.5 \text{ W}}$$

54. (a) 28 AWG = 65.3 ohms per 1000 ft therefore length = 1000(50)/65.3 = $\boxed{766 \text{ ft}}$

(b) 110.5°F = 43.61°C

Thus, $R_2 = (234.5 + 43.61)(50)/(234.5 + 20) = 54.64$ ohms

We therefore need to reduce the length to 50(766)/54.64 = $\boxed{701 \text{ ft}}$

55. <Design> One possible solution:

Choose 28 AWG wire. Require (10)(100)/(65.3) = 153 feet (rounding error within 1% of target value). Wrap around 1 cm diameter 47 cm long wooden rod.

56. B415 used instead of B33; gauge unchanged.
The resistivity is therefore 8.4805/1.7654 = 4.804 times larger

(a) With constant voltage, the current will be (100/4.804) = $\boxed{\text{20.8\% of expected value}}$

(b) $\boxed{\text{No additional power wil be wasted}}$ since the error leads to lower current: P = V/R where V = unchanged and R is larger (0%)

57. $R = \rho L/A = (8.4805\times10^{-6})(100)(2.3/7.854\times10^{-7}) = 2483\ \Omega$

$P = i^2R$ so

$\boxed{\text{B415: } 2.48\text{ mW}; \quad \text{B75: } 504\ \mu\text{W}}$

58. $\beta = 100$, $I_B = 100\ \mu A$

(a) $I_C = \beta\ I_B = \boxed{10\ \text{mA}}$

(b) $P_{BE} = (0.7)(100\times10^{-6}) = \boxed{70\ \mu\text{W}}$

59. Take the maximum efficiency of a tungsten lightbulb as 10%. Then only ~10 W (or 10 J/s) of optical (visible) power is expected. The remainder is emitted as heat and invisible light.

60. Assuming the batteries are built the same way, each has the same energy density in terms of energy storage.

The AA, being larger, therefore stores more energy.

Consequently, for the same voltage, we would anticipate the larger battery can supply the same current for longer, or a larger maximum current, before discharging completely.

1. (a) $\boxed{\text{5 nodes}}$

(b) $\boxed{\text{7 elements}}$

(c) $\boxed{\text{7 branches}}$

2. (a) $\boxed{\text{4 nodes;}}$

(b) $\boxed{\text{7 elements}}$;

(c) $\boxed{\text{6 branches}}$ (we omit the 2 Ω resistor as it is not associated with two distinct nodes)

3. (a) $\boxed{\text{4 nodes}}$

(b) $\boxed{\text{path, yes; loop, no}}$

(c) $\boxed{\text{path, yes; loop, no}}$

4. (a) $\boxed{\text{6 elements}}$;

(b) $\boxed{\text{path, yes; loop, no.}}$

(c) $\boxed{\text{path, yes; loop, no.}}$

5. (a) $\boxed{\text{4 nodes}}$

(b) $\boxed{\text{5 elements}}$

(c) $\boxed{\text{5 branches}}$

(d) i) $\boxed{\text{neither}}$ (only one node) ; ii) $\boxed{\text{path only;}}$

iii) $\boxed{\text{both path and loop;}}$ iv) $\boxed{\text{neither}}$ ('c' enountered twice).

6. The parallel-connected option would allow most of the sign to still light, even if one or more bulbs burn out. For that reason, it would be more useful to the owner.

7. $iA + iB = iC + iD + iE$

(a) $iB = iC + iD + iE - iA = 3 - 2 + 0 - 1 = \boxed{0 \text{ A}}$

(b) $iE = iA + iB - iC - iD = -1 - 1 + 1 + 1 = \boxed{0 \text{ A}}$

8. (a) By KCL, $I = 7 - 6 = \boxed{1\ A}$

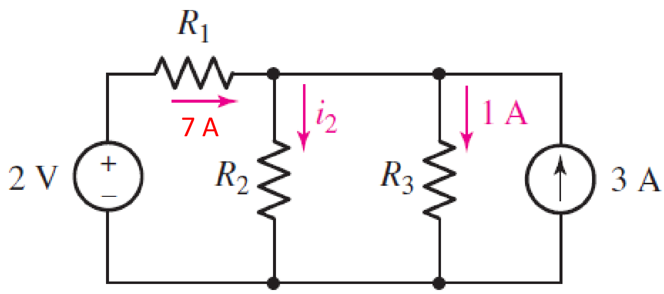
(b) There is a typographical error in the 1st printing. The source should be labelled 9 A. Then,

By KCL, $9 = 3 + I + 3$ so $I = \boxed{3\ A.}$

(c) No net current can flow through the this resistor or KCL would be violated.

Hence, $\boxed{I = 0.}$

9. We note that KCL requires that if 7 A flows out of the "+" terminal of the 2 V source, it flows left to right through R1. Equating the currents into the top node of R2 with the currents flowing out of the same node, we may write



7 + 3 = i2 + 1

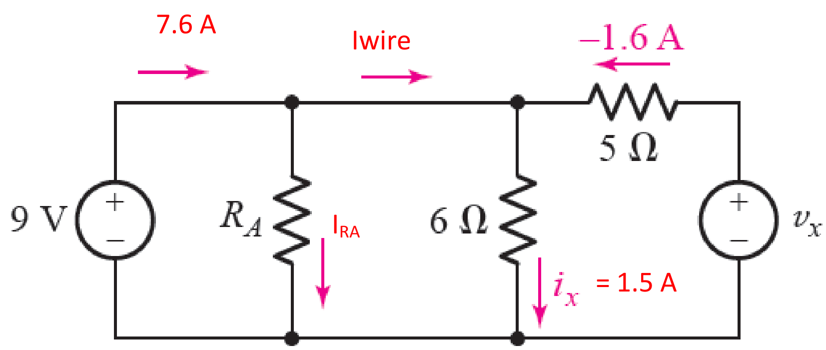
or

i2 = 10 – 1 = $\boxed{9 \text{ A}}$

10. By KCL, $1 = i2 - 3 + 7$

Hence, $i2 = \boxed{-3\text{ A}}$

11. We can determine RA from Ohm's law if either the voltage across, or the current through the element is known. The problem statement allows us to add labels to the circuit diagram:



Applying KCL to the common connection at the top of the 6 Ω resistor,
Iwire = 1.5 – (-1.6) = 3.1 A

Applying KCL to the top of RA then results in

I_{RA} = 7.6 – Iwire = 7.6 – 3.1 = 4.5 A.

Since the voltage across RA = 9 V, we find that RA = 9/4.5 = $\boxed{2\ \Omega}$

12. Applying KCL,

$$IE = IB + IC = IB + 150IB = 151IB = 151(100\times10^{-6}) = \boxed{15.1\text{ mA}}$$

$$\text{And,}\quad IC = \beta IB = \boxed{15.0\text{ mA}}$$

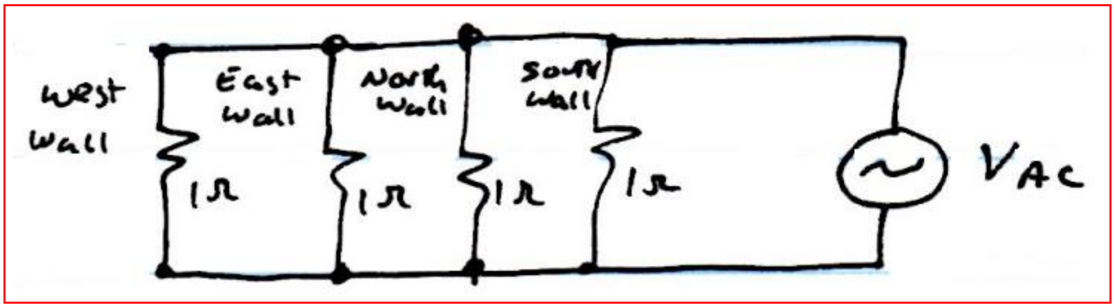
13. I3 = –5Vx

$Vx = (2\times10^{-3})(4.7\times10^{3}) = 9$ V

Therefore I3 = –47 A

14. With finite values of R1 some value of current I will flow out of the source and through the left-most resistor. If we allow some small fraction of that current kI (k < 1) to flow through the resistor connected by a single node, to wher does the current continue? With nowhere for the current to go, KCL is violated. Thus at best we must imagine an equal current flowing the opposite direction, yielding a net zero current. Consequently, Vx must be zero.

15. The order of the resistors in the schematic below is unimportant.



16. (a) -v1 + v2 – v3 = 0

Hence, v1 = v2 – v3 = 0 + 17 = $\boxed{17\text{ V}}$

(d) v1 = v2 – v3 = -2 – 2 = $\boxed{-4\text{ V}}$

(e) v2 = v1 + v3 = 7 + 9 = $\boxed{16\text{ V}}$

(f) v3 = v1 – v2 = -2.33 + 1.70 = $\boxed{-0.63\text{ V}}$

17. By KVL, $+9 + 4 + vx = 0$

Therefore $vx = \boxed{-13\text{ V}}$

From Ohm's law, $ix = vx/7 = -13/7 = \boxed{-1.86\text{ A}}$

18. (a) $-1 + 2 + 2i - 5 + 10i = 0$

Hence, $12i = 4$ so $i = \boxed{333 \text{ mA}}$

(b) $10 + 2i - 1.5 + 2i + 2i + 2 - 1 + 2i = 0$

Hence, $8i = 8$ so $i = \boxed{1 \text{ A}}$

19. From KVL, $+4 - 23 + v_R = 0$ so $v_R = 23 - 4 = 19$ V

Also, $-v_R + 12 + 1.5 - v_2 - v_3 + v_1 = 0$
Or $-19 + 12 + 1.5 - v_2 - 1.5 + 3 = 0$

Solving, $v_2 = \boxed{-4 \text{ V}}$

20. We note that vx does not appear across a simple element, and there is more than one loop that may be considered for KVL.

$+4 – 23 + 12 + v3 + vx = 0$

Or

$+4 – 23 +12 – 1.5 + vx = 0$

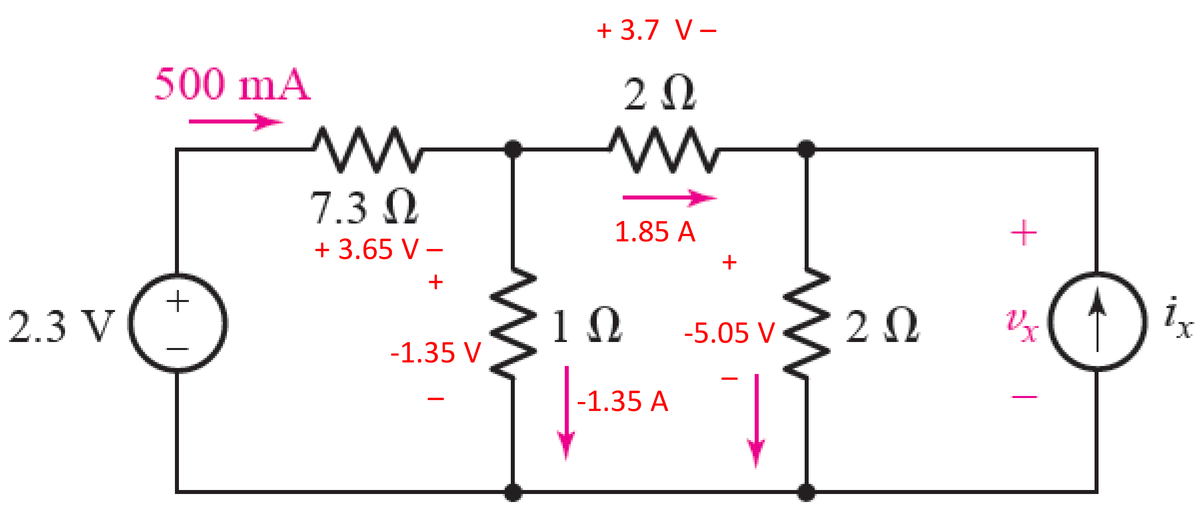
Solving, $vx = \boxed{8.5 \text{ V}}$

21. We apply KCL/KVL and Ohm's law alternately, beginning with the far left. Knowing that 500 mA flows through the 7.3 Ω resistor, we calculate 3.65 V as labelled below. Then application of KVL yields 2.3 – 7.3 = –1.35 V across the 1 Ω resistor.

This tells us that -1.35/1 = -1.35 A flows downward through the 1 Ω resistor. KCL now tells us that 0.5 – (–1.35) = 1.85 flows through the top 2 Ω resistor. Ohm's law dictates a voltage of 3.7 V across this resistor in this case.

Application of KVL determines that –(–1.35) + 3.7 + v2 = 0 or -5.05 V appears across the right-most 2 Ω resistor, as labelled below. Since this voltage also appears across the current source, we know that

$$\boxed{Vx = -5.05 \text{ V}}$$



22. By KVL, $-v_s + v_1 + v_2 = 0$ [1]
Define a clockwise current I. Then $v_1 = IR_1$ and $v_2 = IR_2$.

Hence, Eq. [1] becomes

$$-v_s + IR_1 + IR_2 = 0$$

Thus, $v_s = (R_1 + R_2)I$ and $I = v_1/R_1$

so $v_s = (R_1 + R_2)v_1/R_1$ or $v_1 = v_sR_1/(R_1 + R_2)$. QED

Similarly, $v_2 = IR_2$ so we may also write,

$v_s = (R_1 + R_2)v_2/R_1$ or $v_2 = v_sR_2/(R_1 + R_2)$. QED

$\boxed{\text{(PROOF)}}$

23. $\boxed{v_1 = 2\ \text{V};}$ $\boxed{v_2 = 2\ \text{V}};$ i2 = 2/6 = $\boxed{333\ \text{mA}}$

i3 = 5v1 = $\boxed{10\ \text{A}}$

i1 = i2 + i3 = $\boxed{10.33\ \text{A}}$

v4 = v5 = 5i2 = 5(1/3) = $\boxed{5/3\ \text{V}}$

i5 = (5/3)/5 = $\boxed{333\ \text{mA}}$

i3 = i4 + i5 therefore i3 – i5 = 10 – 1/3 = $\boxed{9.67\ \text{A}}$

-v2 + v3 + v4 = 0 thefore v3 = v4 + v2 = 6/3 = $\boxed{2\ \text{V}}$

24. Define the current I flowing out of the "+" reference terminal of the 5 V source. This current also flows through the 100 Ω and 470 Ω resistors since no current can flow into the input terminals of the op amp.

Then, $-5 + 100I + 470I + v_{out} = 0$ [1]

Further, since Vd = 0, we may also write

$-5 + 100I = 0$ [2]

Solving, $V_{out} = 5 - 570I = 5 - 570(5/100) = \boxed{-23.5 \text{ V}}$

25. With a clockwise current i, KVL yields

–(–8) + (1)i + 16 +4.7i = 0

Therefore, i = –(16 – 8)/5.7 = $\boxed{–4.21 \text{ A}}$

Absorbed power:
Source vs1: +(–8)(+4.2) = $\boxed{–33.60 \text{ W}}$

Source vs2: (16)(–4.21) = $\boxed{–67.36 \text{ W}}$

R1: $(4.21)^2(1)$ = $\boxed{17.21 \text{ W}}$

R2: $(4.21)^2(4.7)$ = $\boxed{83.30 \text{ W}}$

26. Define a clockwise current I. Then, KVL yields

$-4.5 + 2I + 8vA + 5I = 0$ [1]

also, $vA = -5I$ so Eq. [1] can be written as

$-4.5 + 2I - 40I + 5I = 0$

Solving, $I = 4.5/(-33) = -136.0$ mA

ABSORBED POWER

4.5 V source:	$(4.5)(+0.136)$	= 612 mW
2 W:	$(2)(-0.136)^2$	= 37.0 mW
Dep. source:	$(8vA)(-0.136) = (8)(-5)(-0.136)^2$	= –740 mW
5 W:	$(5)(-0.136)^2$	= 92.5 mW

27. Define a clockwise current I

Then invoking KVL and Ohm's law we may write

$500I - 2 + 1000I + 3vx + 2200I = 0$

Since vx = -500I, the above equation can be recast as

$500I - 2 + 1000I - 1500I + 2200I = 0$

Solving, $I = 2/2200 = 909.1\ \mu A$

ABSORBED POWER

500 Ω:	$500(I^2)$	= 413.2 μW
2 V source:	(2)(-I)	= -1.818 mW
1 kΩ:	$(1000)(I^2)$	= 826.5 μW
Dep source:	[(3)(-500I)](I)	= -1.240 mW
2.2 kΩ:	$(2200)(I^2)$	= 1.818 mW

28. (a) By KVL, -12 + 27ix + 33ix + 13ix + 2 + 19ix = 0
Hence, ix = 10/92 = 108.7 mA

Element	$P_{absorbed}$	(W)
12 V	(12)(-0.1087) =	-1.304
27 Ω	$(27)(0.1087)^2$ =	0.3190
33 Ω	$(33)(0.1087)^2$ =	0.3899
13 Ω	$(13)(0.1087)^2$ =	0.1536
19 Ω	$(19)(0.1087)^2$ =	0.2245
2 V	(2)(0.1087) =	0.2174

(b) By KVL, -12 + 27ix + 33ix + 4v1 + 2 + 19ix = 0
and v1 = 33ix. Solving together, ix = 10/211 = 47.39 mA

Element	$P_{absorbed}$ (W)
12 V	-0.5687
27 Ω	0.06064
33 Ω	0.07411
Dep source	0.2964
19 Ω	0.04267
2 V	0.09478

(c) By KVL, -12 + 27ix + 33ix + 4ix + 2 + 19ix + 2= 0
Solving, ix = 10/83 = 120.5 mA

Element	$P_{absorbed}$ (W)
12 V	-1.446
27 Ω	0.3920
33 Ω	0.4792
Dep source	0.05808
19 Ω	0.2759
2 V	0.2410

29. A simple KVL equation for this circuit is

$-3 + 100I_D + V_D = 0$

Substituting in the equation which related the diode current and voltage,

$$-3 + (100)(29\times10^{-12})\left(e^{V_D/27\times10^{-3}} - 1\right)$$

Solving either by trial and error or using a scientific calculator's equation solver,

$V_D = \boxed{560 \text{ mV}}$

30. (a) KCL yields $3 - 7 = i1 + i2$

Or, $-4 = v/4 + v/2$

Solving, $v = -5.333$ V

Thus, $i1 = v/4 = \boxed{-1.333 \text{ A}}$ and $i2 = v/2 = \boxed{-2.667 \text{ A}}$

(b)

Element	**Absorbed Power**		
3 A	(-5.333)(-3)	=	16.00 W
7 A	(-5.333)(7)	=	-37.33 W
4 Ω	$(4)(-1.333)^2$	=	7.108 W
2 Ω	$(2)(-2.667)^2$	=	14.23 W

31. Consider the currents flowing INTO the top node. KCL requires

$-2 - i1 - 3 - i2 = 0$
Or $i1 + i2 = -5$ [1]

Also, $i1 = v/10$ and $i2 = v/6$ so Eq. [1] becomes

$v/10 + v/6 = -5$

Solving, $v = \boxed{-18.75 \text{ V}}$

Thus the power supplied by the –2 A source is $(-2)(-18.75) = \boxed{37.5 \text{ W}}$
and the power supplied by the 3 A source is $-(3)(-18.75) = \boxed{56.25 \text{ W}}$

32. Combining KCL and Ohm's law in a single step results in

$1 + 2 = v/5 + 5 + v/5$

Solving, $v = \boxed{-5\text{ V}}$

33. Summing the currents flowing into the top node, KCL yields

$-v/1 + 3i_x + 2 - v/3 = 0$ [1]

Also, $i_x = -v/3$. Substituting this into Eq. [1] results in

$-v - v + 2 - v/3 = 0$

Solving, $\boxed{v = 6/7 \text{ V} \quad (857 \text{ mV})}$

The dependent source supplies power $= (3i_x)(v)$
$= (3)(-v/3)(v) = \boxed{-36/49 \text{ W} \quad (-735 \text{ mW})}$

The 2 A source supplies power $= (2)(v) = \boxed{12/7 \text{ W} = (1.714 \text{ W})}$

34. Define the center node as +v; the other node is then the reference terminal.

KCL yields $3\times10^{-3} - 5\times10^{-3} = \dfrac{v}{1000} + \dfrac{v}{4700} + \dfrac{v}{2800}$

Solving, v = –1.274 V

(a)

R	$\mathbf{P_{absorbed}}$
1 kΩ	1.623 mW
4.7 kΩ	345.3 μW
2.8 kΩ	579.7 μW

(b)

Source	$\mathbf{P_{absorbed}}$	
3 mA	$(v)(3\times10^{-3})$	= –3.833 mW
5 mA	$(v)(-5\times10^{-3})$	= +6.370 mW

(c) $\sum P_{absorbed} = 2.548$ mW

$\sum P_{\text{supplied}} = 2.548$ mW

Thus, $\sum P_{\text{supplied}} = \sum P_{absorbed}$.

35. By KVL, $v_{eq} = v_1 + v_2 - v_3$

(a) $v_{eq} = 0 - 3 - 3 = \boxed{-6 \text{ V;}}$

(b) $v_{eq} = 1 + 1 - 1 = \boxed{1 \text{ V;}}$

(c) $v_{eq} = -9 + 4.5 - 1 = \boxed{-5.5 \text{ V}}$

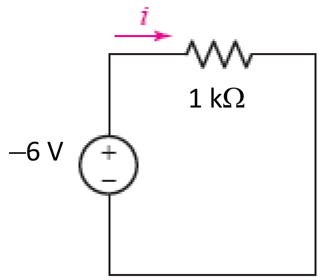
36. KCL requires that ieq = i1 – i2 + i3

(a) ieq = 0 + 3 + 3 = $\boxed{6\text{A}}$

(b) ieq = 1 – 1 + 1 = $\boxed{1\text{ A}}$

(c) ieq = –9 – 4.5 + 1 = $\boxed{-12.5\text{ A}}$

37. The voltage sources are in series, hence they may be replaced with veq = –2 + 2 – 12 + 6 = –6 V. The result is the circuit shown below:



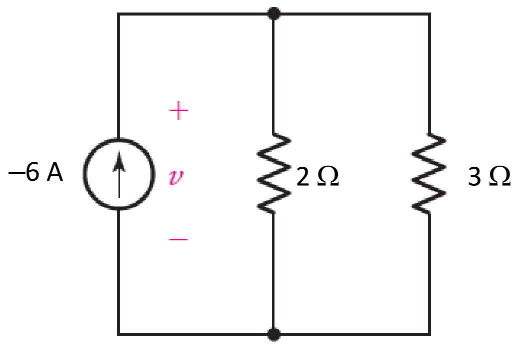
Analyzing the simplifed circuit, i = –6/1000 = $\boxed{-6 \text{ mA}}$

38. We may first reduce the series connected voltage sources, or simply write a KVL equation around the loop as it is shown:

$+2 + 4 + 7i + v1 + 7i + 1 = 0$

Setting i = 0, v1 = $\boxed{–7 \text{ V}}$

39. The current sources are combined using KCL to obtain ieq = 7 – 5 – 8 = –6 A. The resulting circuit is shown below.



(a) KCL stipulates that $-6 = \frac{v}{2} + \frac{v}{3}$ v.

Solving, $\boxed{\text{v = –36/5 V}}$

(b) $P_{supplied}$ by equivalent source = (v)(-6) = $\boxed{\text{43.2 W}}$

Source	$\mathbf{P_{supplied}}$ **(W)**	
7 A source	(7)(-36/5)	= –50.4 W
5 A source	(-5)(-36/5)	= 36 W
8 A source	(-8)(-36/5)	= 57.6 W
		$\boxed{\text{43.2 W so confirmed.}}$

40. We apply KCL to the top node to write

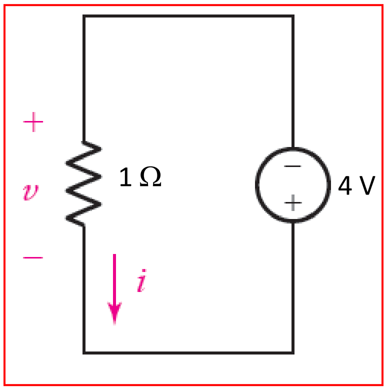
$1.28 - 2.57 = v/1 + Is + vs/1$

Setting v = 0, we may solve our equation to obtain $\boxed{Is = -1.29 \text{ A}}$

41. (a) Employing KCL, by inspection $\boxed{I_x = 3\ \text{A};\ V_y = 3\ \text{V}}$

(b) Yes. Current sources in series must carry the same current. Voltage sources in parallel must have precisely the same voltage.

(c)



(all other sources are irrelevant to determining i and v).

42. Left-hand network: $1 + 2 \parallel 2 = 1 + 1 = \boxed{2\ \Omega}$

Right-hand network: $4 + 1\parallel 2 + 3 = 4 + 0.6667 + 3 = \boxed{7.667\ \Omega}$

43. (a) $R_{eq} = 1 + 2\|4 = 1 + 8/6 = \boxed{2.333\ \Omega}$

(b) $R_{eq} = 1\|4\|3$

$1/R_{eq} = 1 + 1/4 + 1/3 = 19/12\ \Omega^{-1}$

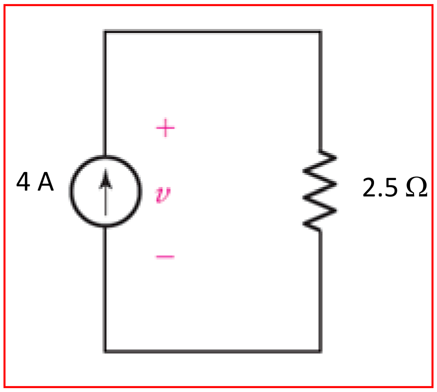
So, $R_{eq} = \boxed{632\ \text{m}\Omega}$

44. The 2 sources may be replaced with a 2 V source located such that i flows out of the "+" reference terminal.

The 4 resistors may be replaced with a single resistor having value $2 + 7 + 5 + 1 = 15\ \Omega$.

45. KCL yields ieq = -2 + 5 + 1 = 4 A
And Req = 5 || 5 = 2.5 Ω

(a)



(b) Ohm's law yields v = (4)(2.5) = $\boxed{10 \text{ V}}$

(c) Power *supplied* by 2 A source = (–2)(10) = $\boxed{–20 \text{ W}}$

46. Looking at the far right of the circuit, we note the following resistor combination is possible: $3 \| 9 + 3 + 5 + 3 \| 6 = 2.25 + 8 + 2 = 12.25\ \Omega$

We now have three resistors in parallel: $3 \| 5 \| 12.25 = 1.626\ \Omega$

Invoking Ohm's law, vx = (1)(1.626) = 1.626 V.

Since this voltage appears across the current source and each of the three resistance (3 Ω, 5 Ω, 12.25 Ω), Ohm's law again applies: i3 = vx/3 = $\boxed{542\text{ mA}}$

Finally, the source supplies $(1.626)^2/3 + (1.626)^2/5 + (1.626)^2/12.25 = \boxed{1.626\text{ W}}$

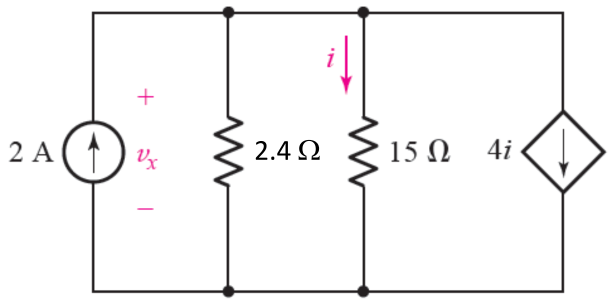
47. At the far right we have the resistor combination 9 + 6 || 6 = 9 + 3 = 12 Ω.
After this, we have three resistors in parallel but should not involve the 15 Ω resistor as it controls the dependent source. Thus, R_{eq}^{-1} = 1/3 + 1/12 and R_{eq} = 2.4 Ω.
The simplified circuit is shown below.
Summing the currents flowing into the top node,

$$2 - \frac{v_x}{2.4} - \frac{v_x}{15} - 4i = 0 \quad [1]$$

Since i = v_x/15, Eq. [1] becomes

$$2 - \frac{v_x}{2.4} - \frac{v_x}{15} - \frac{4}{15}v_x = 0$$

Solving, $\boxed{v_x = 2.667 \text{ V}}$



48. We combine the left-hand set of resistors: 6 + 3 || 15 = 8.5 Ω
The independent sources may be combined into a single 4 + 3 – 9 = -2 A source (arrow pointing up). We leave the 6 Ω resistors; at least one has to remain as it controls the dependent source. A voltage v is defined across the simplified circuit, with the + terminal at the top node.

Applying KCL to the top node,

$$-2 - 2i = v/8.5 + v/6 + v/6 \qquad [1]$$

where i = v/6. Thus, Eq. [1] becomes -2 – 2v/6 = v/8.5 + 2v/6 or v = -19 V.

We have lost the 15 Ω resistor temporarily, however. Fortunately, the voltage we just found appears across the original resistor combination we replaced. Hence, a current -19/8.5 = -2.235 A flows downward through the combination.

Hence, the voltage acrossw the 3 Ω || 15 Ω combination is

$$v - 6(-2.235) = -5.59 \text{ V}$$

Thus, $P_{15\Omega} = (-5.59)2/15 = \boxed{2.083 \text{ W}}$

49. $R_{11} = 3\ \Omega$, so $R_1 = 11R_{11} = 33\ \Omega$, $R_2 = 33/2\ \Omega$ etc.

Starting from the far right, we define $R_A = R_7 \parallel [R_8 + R_{10} \parallel R_{11} + R_9]$
$= (33/7) \parallel [33/8 + 33/9 + 1.571] = 3.1355\ \Omega$

Next, $R_4 \parallel [R_5 + R_A + R_6] = 33/4 \parallel [33/5 + 3.1355 + 33/6] = 5.352\ \Omega$

Finally, $R_{eq} = R_1 \parallel [R_2 + 5.352 + 33/3] = \boxed{16.46\ \Omega}$

50. Four 100 Ω resistors may be combined as:

(a) 25 Ω = $\boxed{100\ \Omega \parallel 100\ \Omega \parallel 100\ \Omega \parallel 100\ \Omega}$

(b) 60 Ω = $\boxed{[(100\ \Omega \parallel 100\ \Omega) + 100\ \Omega] \parallel 100\ \Omega}$

(c) 40 Ω = $\boxed{(100\ \Omega + 100\ \Omega) \parallel 100\ \Omega \parallel 100\ \Omega}$

51. (a) $v2 = v1 - v2 = 9.2 - 3 = \boxed{6.2\ \text{V}}$

(b) $v1 = v - v2 = 2 - 1 = \boxed{1\ \text{V}}$

(c) $v = v1 + v2 = 3 + 6 = \boxed{9\ \text{V}}$

(d) $v1 = vR1/(R1 + R2) = v2R2/(R1 + R2)$.

Thus, setting $v1 = v2$ and $R1 = R2$, $R1/R2 = \boxed{1}$

(e) $v2 = vR2/(R1 + R2) = vR2/(2R2 + R2) = v/3 = \boxed{1.167\ \text{V}}$

(f) $v1 = vR1/(R1 + R2) = (1.8)(1)/(1 + 4.7) = \boxed{315.8\ \text{mV}}$

52. (a) $i1 = i - i2 = 8 - 1 = \boxed{7A}$

(b) $v = i(R1 \parallel R2) = i(50\times10^3) = \boxed{50\text{ V}}$

(c) $i2 = iR1/(R1 + R2) = (20\times10^{-3})(1/5) = \boxed{4\text{ mA}}$

(d) $i1 = iR2/(R1 + R2) = (10)(9)/18 = \boxed{5A}$

(e) $i2 = iR1/(R1 + R2) = (10)(10\times10^6)/(10\times10^6 + 1) = \boxed{10\text{ A}}$

53. **One** possible solution: $\boxed{\text{Choose } v = 1\text{ V}}$. Then

$$\boxed{\begin{array}{l} R1 = v/i1 = 1\ \Omega \\ R2 = v/i2 = 833.3\ \text{m}\Omega \\ R3 = v/i3 = 125\ \text{m}\Omega \\ R4 = v/i4 = 322.6\ \text{m}\Omega \end{array}}$$

54. First, replace the 2 Ω || 10 Ω combination with 1.66 Ω. Then

$$v_x = 3\frac{2}{2+3+1.667} = \boxed{900 \text{ mV}}$$

55. Employing voltage division,

$V_{3\Omega} = (9)(3)/(1 + 3 + 5 + 7 + 9) = \boxed{1.08\text{ V}}$

$V_{7\Omega} = (9)(7)/(1 + 3 + 5 + 7 + 9) = \boxed{2.52\text{ V}}$

56. We begin by simplifying the circuit to determine i1 and i2.
We note the resistor combination (4 + 4) || 4 + 5 = 8 || 4 + 5 = 7.667 Ω.
This appears in parallel with the 1 Ω and 2 Ω resistors, and experiences current i2.

Define the voltage v across the 25 A source with the '+' reference on top. Then,

$$25 = \frac{v}{1} + \frac{v}{2} + \frac{v}{7.667}$$

Solving, v = 15.33 V. Thus, i1 = 25 – v/1 = 9.67 A
i2= i1 – v/2 = 2.005 A

Now, from current division we know i2 is split between the 4 Ω and 4 Ω + 4 Ω = 8 Ω branches, so we may write

v3 = 4[4i2/(4 + 8)] = $\boxed{\text{2.673 V}}$

57. We may do a little resistor combining:

2 kΩ || 4 kΩ = 8/6 kΩ

3 kΩ || 7 kΩ = 21/10 kΩ

The 4 kΩ resistor is in parallel with the series combination of 8/6 + 3 + 21/10 = 6.433 kΩ. That parallel combination is equivalent to 2.466 kΩ.

Thus, voltage division may be applied to yield

$v_{4k\Omega} = (3)(2.466)/(1 + 2.466) = 2.134$ V

and $v_x = (2.134)(21/10)/(8/6 + 3 + 21/10) = \boxed{697.0 \text{ mV}}$

58. We *could* use voltage division for determining those voltages if $i_1 = 0$. Since it is *nonzero* (presumably – circuit analysis will verify whether this is the case), we do not have equal currents through the two resistors, hence voltage division is not valid.

59. v_π = (12 cos 1000t) (15000)/(15030) mV = 11.98 cos 1000t mV

v_{out} appears across 10 kΩ || 1 kΩ = 909.1 Ω

Thus, $v_{out} = (-g_m v_\pi)(909.1) = \boxed{-13.1\cos 10^3 t \text{ mV}}$

60. We can apply voltage division to obtain v_π by first combining the 15 kΩ and 3 kΩ resistors: 15 kΩ || 3 kΩ = 2.5 kΩ.
Then by voltage division,

$$v_\pi = 6\times10^{-6}\cos 2300t\left(\frac{2.5}{1+2.5}\right)\text{ V} = 4.28\cos 2300t\ \mu\text{V}$$

By Ohm's law, $v_{out} = -3.3\times103(g_m\, v_p)$

$= -3300(322\times10^{-3})(4.28\times10^{-6})\cos 2300t$

$= \boxed{-4.55\cos 2300t \text{ mV}}$

61. (a) Looking to the right of the 10 Ω resistors, we see

$40 + 50 \parallel (20 + 4) = 56.22\ \Omega$

Further, the 10 Ω || 10 Ω can be replaced with a 5 Ω resistor without losing the desired voltage. Hence, v appears across $5 \parallel 56.22 = 4.592\ \Omega$

So $v = 2 (4.592)/(20 + 4.592) = \boxed{371.0 \text{ mV}}$

(b) The 4 Ω resistor has been "lost" but we can return to the original circuit and note the voltage determined in the previous part. By voltage division, the voltage across the 50 Ω resistor is $v(50 \parallel 24)/(40 + 50 \parallel 24) = 108.5$ mV.

$v_{4\Omega} = v_{50\Omega}(4)/(20 + 4) = 18.08$ mV

Hence, the power dissipated by the 4 W resistor is $(v_{4\Omega})^2/4 = \boxed{81.75\ \mu\text{W}}$

62. First, note the equivalent resistance 40 + 50 ||(20 + 4) = 56.22 Ω.

This is in parallel with 10 Ω, yielding 8.49 Ω in series with 20 Ω.

Thus, I = 2/28.49 = 70.2 mA flows through the left-most 2 Ω resistor.

This is split between the 10 Ω and 56.22 Ω combination.

(a) I_{40} = 0.0702 (10)/(10 + 56.22) = $\boxed{10.6 \text{ mA}}$

(b) The source supplies (2)(0.0702) = $\boxed{140.4 \text{ mW}}$

(c) I_{40} is split between the 50 Ω and (20 + 4) = 24 Ω branches.

Thus, $I_{4\Omega}$ = (0.0106)(50)/(50 + 24) = 7.16 mA

Hence, $P_{4\Omega}$ = (4)(i4Ω)2 = $\boxed{205.2\ \mu\text{W}}$

63. (a) $\boxed{\text{5 nodes; 6 loops; 7 branches;}}$

(b) Define all currents as flowing clockwise.
Note the combination $R_{eq} = 2/3 + 2 + 5 = 7.667\ \Omega$.

By inspection, $\boxed{I_{2\Omega left} = -2\ \text{A};}$

$I5_{\Omega(left)} = (-2)(7.667)/(5 + 7.667) = \boxed{-1.21\ \text{A};}$

$I_{2\Omega right} = I_{5\Omega} = (-2)(5)/(5 + 7.667) = \boxed{-790\ \text{mA};}$

The remaining currents are found by current division:

$I_{1\Omega} = -0.7895(2)/3 = \boxed{-526\ \text{mA};}$
$I_{2\Omega middle} = -0.7895(1)/3 = \boxed{-263\ \text{mA}}$

1. (a) $$\begin{bmatrix} -4 & 2 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$$

Solving, $\boxed{v_1 = -2.056}$ and $\boxed{v_2 = 0.389}$

(b) $$\begin{bmatrix} -1 & 0 & 2 \\ 2 & 1 & -5 \\ 4 & 5 & 8 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$$

Solving, $\boxed{v_1 = -8.667}$ $\boxed{v_2 = 8.667}$ $\boxed{v_3 = -0.3333}$

2. $$\begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix} = (2)(3) - (1)(-4) = 6 + 4 = \boxed{10}$$

$$\begin{vmatrix} 0 & 2 & 11 \\ 6 & 4 & 1 \\ 3 & -1 & 5 \end{vmatrix} = 0 - 6[2(5) - (11)(-1) + 3[(2)(1) - (11)(4)] = \boxed{-252}$$

3. (a) By Cramer's Rule,

$$\begin{vmatrix} -4 & 2 \\ 1 & -5 \end{vmatrix} = (-4)(-5) - (2)(1) = 18$$

$$v_1 = \frac{\begin{vmatrix} 9 & 2 \\ -4 & -5 \end{vmatrix}}{\begin{vmatrix} -4 & 2 \\ 1 & -5 \end{vmatrix}} = \frac{-45+8}{18} = \boxed{-2.056}$$

$$v_2 = \frac{\begin{vmatrix} -4 & 9 \\ 1 & -4 \end{vmatrix}}{\begin{vmatrix} -4 & 2 \\ 1 & -5 \end{vmatrix}} = \frac{16-9}{18} = \boxed{0.3889}$$

(b)

$$\begin{vmatrix} -1 & 0 & 2 \\ 2 & 1 & -5 \\ 4 & 5 & 8 \end{vmatrix} = -1[(1)(8) - (-5)(5)] - 0 + 2[(2)(5) - (1)(4)] = -21$$

$$v_1 = \frac{\begin{vmatrix} 8 & 0 & 2 \\ -7 & 1 & -5 \\ 6 & 5 & 8 \end{vmatrix}}{-21} = \frac{182}{-21} = \boxed{-8.667}$$

$$v_2 = \frac{\begin{vmatrix} -1 & 8 & 2 \\ 2 & -7 & -5 \\ 4 & 6 & 8 \end{vmatrix}}{-21} = \frac{-182}{-21} = \boxed{8.667}$$

$$v_3 = \frac{\begin{vmatrix} -1 & 0 & 8 \\ 2 & 1 & -7 \\ 4 & 5 & 6 \end{vmatrix}}{-21} = \frac{7}{-21} = \boxed{-0.3333}$$

4. (a) Grouping terms,

$$990 = (66 + 15 + 100)v_1 - 15v_2 - 110v_3$$
$$308 = -14v_1 + 36v_2 - 22v_3$$
$$0 = -140v_1 - 30v_2 + 212v_3$$

Solving, $v_1 = 13.90$ V
$v_2 = 21.42$ V
$v_3 = 12.21$ V

(b)

```
>> e1 = '990 = (66 + 15 + 110)*v1 - 15*v2 - 110*v3';
>> e2 = '308 = -14*v1 + 36*v2 - 22*v3';
>> e3 = '0 = -140*v1 - 30*v2 + 212*v3';
>> a = solve(e1,e2,e3,'v1','v2','v3');
>> a.v1
ans =
1070/77
>> a.v2
ans =
4948/231
>> a.v3
ans =
940/77
```

5. (a) Grouping terms,

$$1596 = (114 + 19 + 12)v_1 - 19v_2 - 12v_3$$

$$180 = -v_1 + (1 + 6)v_2 - 6v_3$$

$$1064 = -14v_1 - 133v_2 + (38 + 14 + 133)v_3$$

Solving, $\boxed{v_1 = 29.98; \quad v_2 = 96.07; \qquad v_3 = 77.09}$

(b) MATLAB code:

```
>> e1 = '7 = v1/2 - (v2 - v1)/12 + (v1 - v3)/19';
>> e2 = '15 = (v2 - v1)/12 + (v2 - v3)/2';
>> e3 = '4 = v3/7 + (v3 - v1)/19 + (v3 - v2)/2';
>> a = solve(e1,e2,e3,'v1','v2','v3');
>> a.v1
ans =
16876/563
>> a.v2
ans =
54088/563
>> a.v3
ans =
43400/563
```

6. The corrected code is as follows (note there were no errors in the e2 equation):

```
>> e1 = '3 = v1/7 - (v2 - v1)/2 + (v1 - v3)/3';
>> e2 = '2 = (v2 - v1)/2 + (v2 - v3)/14';
>> e3 = '0 = v3/10 + (v3 - v1)/3 + (v3 - v2)/14';
>> a = solve(e1,e2,e3,'v1','v2','v3');
>> a.v1
ans =
1178/53
>> a.v2
ans =
9360/371
>> a.v3
ans =
6770/371
```

7. All the errors are in Eq. [1], which should read,

$$\boxed{7 = \frac{v_1}{4} - \frac{v_2 - v_1}{2} + \frac{v_1 - v_3}{19}}$$

8. Our nodal equations are:

$$5 = \frac{v_1}{1} + \frac{v_1 - v_2}{5} \qquad [1]$$

$$-4 = \frac{v_2}{2} + \frac{v_2 - v_1}{5} \qquad [2]$$

Solving, v_1 = 3.375 V and v_2 = -4.75 V.
Hence,

$$i = (v_1 - v_2)/5 = \boxed{1.625 \text{ A}}$$

9. Define nodal voltages v_1 and v_2 on the top left and top right nodes, respectively; the bottom node is our reference node. Our nodal equations are then,

$$-3 = \frac{v_1}{3} + \frac{v_1 - v_2}{2} \Rightarrow (2+3)v_1 - 3v_2 = -18$$

$$2 = v_2 + \frac{v_2 - v_1}{2} \Rightarrow -v_1 + (2+1)v_2 = 4$$

Solving the set where terms have been grouped together,

v_1 = -3.5 V and v_2 = 166.7 mV

$P_{1\Omega} = (v_2)^2/1 = \boxed{27.79 \text{ mW}}$

10. Our two nodal equations are:

$$2 = \frac{v_1}{9} + \frac{v_1 - v_2}{1} \Rightarrow 10v_1 - 9v_2 = 18$$
$$15 = \frac{v_2}{2} + \frac{v_2 - v_1}{1} \Rightarrow -2v_1 + 3v_2 = 30$$

Solving, v_1 = 27 V and v_2 = 28 V. Thus, $\boxed{v_1 - v_2 = -1 \text{ V}}$

11. We note that he two 6 Ω resistors are in parallel and so can be replaced by a 3 Ω resistor. $\boxed{\text{By inspection, } i_1 = 0.}$

Our nodal equations are therefore

$$2 = \frac{v_A}{3} + \frac{v_A - v_B}{1} \Rightarrow (1+3)v_A - 3v_B = 6 \qquad [1]$$

$$-4 = \frac{v_B}{3} + \frac{v_B - v_A}{1} \Rightarrow -3v_A + (1+3)v_B = -12 \qquad [2]$$

Solving,

v_A = -1.714 V and v_B = -4.28 V. Hence, $\boxed{v_1 = v_A - v_B = 2.572 \text{ V}}$

12. Define v_1 across the 10 A source, '+' reference at the top.
Define v_2 across the 2.5 A source, '+' reference at the top.
Define v_3 acros the 200 Ω resistor, '+' reference at the top.

Our nodal equations are then

$$10 = \frac{v_1}{20} + \frac{v_1 - v_p}{40} \qquad [1]$$

$$0 = \frac{v_p - v_1}{40} + \frac{v_p - v_2}{50} + \frac{v_p}{100} \qquad [2]$$

$$2 - 2.5 = \frac{v_2 - v_p}{50} + \frac{v_2 - v_3}{10} \qquad [3]$$

$$5 - 2 = \frac{v_3 - v_2}{10} + \frac{v_3}{200} \qquad [4]$$

Solving, $\boxed{v_p = 171.6 \text{ V}}$

13. Choose the bottom node as the reference node. Then, moving left to right, designate the following nodal voltages along the top nodes: v_1, v_2, and v_3, respectively.

Our nodal equations are then

$$8+4=\frac{v_1-v_3}{3}+\frac{v_1-v_2}{3} \qquad [1]$$

$$-4=\frac{v_2}{5}+\frac{v_2-v_1}{3}+\frac{v_2-v_3}{1} \qquad [2]$$

$$-5=\frac{v_3-v_1}{3}+\frac{v_3-v_2}{1}+\frac{v_3}{7} \qquad [3]$$

Solving, v_1 = 26.73 V, v_2 = 8.833 V, v_3 = 8.633 V

$v_{5\Omega} = v_2 = \boxed{8.833 \text{ V}}$

Thus, $P_{7\Omega} = (v_3)^2/7 = \boxed{10.65 \text{ W}}$

14. Assign the following nodal voltages: v_1 at top node; v_2 between the 1 Ω and 2 Ω resistors; v_3 between the 3 Ω and 5 Ω resistors, v_4 between the 4 Ω and 6 Ω resistors. The bottom node is the reference node.

Then, the nodal equations are:

$$2 = \frac{v_1 - v_2}{1} + \frac{v_2 - v_3}{3} + \frac{v_2 - v_3}{4} \qquad [1]$$

$$-3 = \frac{v_2}{2} + \frac{v_2 - v_1}{1} \qquad [2]$$

$$-3 = \frac{v_3}{5} + \frac{v_3 - v_1}{3} + \frac{v_3 - v_4}{7} \qquad [3]$$

$$0 = \frac{v_4}{6} + \frac{v_4 - v_1}{4} + \frac{v_4 - v_3}{7} \qquad [4]$$

Solving, v_3 = 11.42 V and so $i_5 = v_3/5 = \boxed{2.284 \text{ A}}$

15. First, we note that it is possible to separate this circuit into two parts, connected by a single wire (hence, the two sections cannot affect one another).

For the left-hand section, our nodal equations are:

$$2 = \frac{v_1}{2} + \frac{v_1 - v_3}{6} \qquad [1]$$

$$-2 = \frac{v_2}{5} + \frac{v_2 - v_3}{2} + \frac{v_2 - v_4}{10} \qquad [2]$$

$$1 = \frac{v_3 - v_1}{6} + \frac{v_3 - v_2}{2} + \frac{v_3 - v_4}{5} \qquad [3]$$

$$0 = \frac{v_4 - v_2}{10} + \frac{v_4 - v_3}{5} + \frac{v_4}{5} \qquad [4]$$

Solving,

$$\boxed{\begin{aligned} v_1 &= 3.078 \text{ V} \\ v_2 &= -2.349 \text{ V} \\ v_3 &= 0.3109 \text{ V} \\ v_4 &= -0.3454 \text{ V} \end{aligned}}$$

For the right-hand section, our nodal equations are:

$$-2 = \frac{v_5}{1} + \frac{v_5 - v_7}{4} \qquad [1]$$

$$2 = \frac{v_6}{4} + \frac{v_6 - v_8}{4} + \frac{v_6 - v_7}{2} \qquad [2]$$

$$6 = \frac{v_7 - v_5}{4} + \frac{v_7 - v_6}{2} + \frac{v_7 - v_8}{10} \qquad [3]$$

$$0 = \frac{v_8}{1} + \frac{v_8 - v_6}{4} + \frac{v_8 - v_7}{10} \qquad [4]$$

Solving,

$$\boxed{\begin{aligned} v_5 &= 1.019 \text{ V} \\ v_6 &= 9.217 \text{ V} \\ v_7 &= 13.10 \text{ V} \\ v_8 &= 2.677 \text{ V} \end{aligned}}$$

16. We note that the far-right element should be a 7 Ω resistor, not a dependent current source.
The bottom node is designated as the reference node. Naming our nodal voltages from left to right along the top nodes then: v_A, v_B, and v_C, respectively.

Our nodal equations are then:

$$0.02v_1 = \frac{v_A - v_C}{5} + \frac{v_A - v_B}{3} \quad [1]$$

$$10 = \frac{v_B - v_A}{3} + \frac{v_B - v_C}{2} \quad [2]$$

$$0 = \frac{v_C - v_B}{2} + \frac{v_C - v_A}{5} \quad [3]$$

However, we only have three equations but there are four unknowns (due to the presence of the dependent source). We note that $v_1 = v_C - v_B$. Substituting this into Eq. [1] and solving yields:

v_A = 85.09 V $\quad$ v_B = 90.28 V $\quad$ v_C = 73.75 V

Finally, $i_2 = (v_C - v_A)/5 = \boxed{-2.268 \text{ A}}$

17. Select the bottom node as the reference node. The top node is designated as v_1, and the center node at the top of the dependent source is designated as v_2.

Our nodal equations are:

$$1 = \frac{v_1 - v_2}{5} + \frac{v_1}{2} \qquad [1]$$

$$v_x = \frac{v_2}{3} + \frac{v_2 - v_1}{5} \qquad [2]$$

We have two equations in three unknowns, due to the presence of the dependent source. However, $v_x = -v_2$, which can be substituted into Eq. [2]. Solving,

$v_1 = 1.484$ V and $v_2 = 0.1936$ V

Thus, $i_1 = v_1/2 = \boxed{742 \text{ mA}}$

18. We first create a supernode from nodes 2 and 3. Then our nodal equations are:

$$3-5=\frac{v_1-v_3}{1}+\frac{v_1-v_2}{5} \qquad [1]$$

$$5-8=\frac{v_2}{3}+\frac{v_2-v_1}{5}+\frac{v_3}{2}+\frac{v_3-v_1}{1} \qquad [2]$$

We also require a KVL equation that relates the two nodes involved in the supernode:

$$v_2-v_3=4 \qquad [3]$$

Solving, $\boxed{v_1 = -8.6 \text{ V}, \quad v_2 = -36 \text{ V and } v_3 = -7.6 \text{ V}}$

19. We name the one remaining node v_2. We may then form a supernode from nodes 1 and 2, resulting in a single KCL equation:

$$3-5=\frac{v_1}{5}+\frac{v_2}{9}$$

and the requisite KVL equation relating the two nodes is $v_1 - v_2 = 9$

Solving these two equations yields $\boxed{v_1 = -3.214 \text{ V}}$

20. We define v_1 at the top left node; v_2 at the top right node; v_3 the top of the 1 Ω resistor; and v_4 at the top of the 2 Ω resistor. The remaining node is the reference node.

We may now form a supernode from nodes 1 and 3. The nodal equations are:

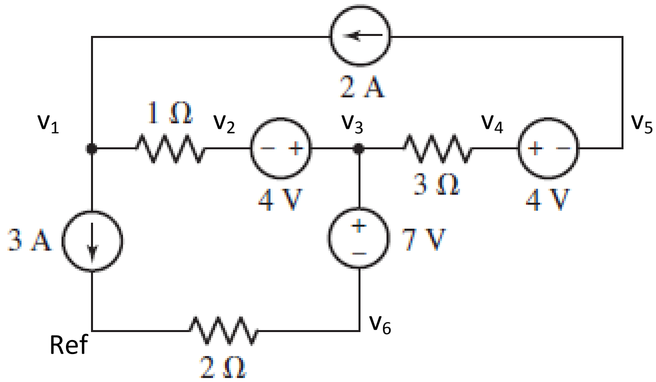
$$-2 = \frac{v_3}{1} + \frac{v_1 - v_2}{10} \qquad [1]$$

$$2 = \frac{v_4}{2} + \frac{v_4 - v_2}{4} \qquad [2]$$

By inspection, $v_2 = 5$ V and our necessary KVL equation for the supernode is $v_1 - v_3 = 6$. Solving,

$$\boxed{\begin{array}{l} v_1 = 4.019\text{ V} \\ v_2 = 5\text{ V} \\ v_3 = -1.909\text{ V} \\ v_4 = 4.333\text{ V} \end{array}}$$

21. We first select a reference node then assign nodal voltages as follows:



There are two supernodes we can consider: the first is formed by combining nodes 2, 3 and 6. The second supernode is formed by combining nodes 4 and 5. However, since we are asked to only find the power dissipated by the 1 Ω resistor, we do not need to perform a complete analysis of this circuit.

At node 1, $-3 + 2 = (v_1 - v_2)/1$ or $v_1 - v_2 = -1$ V

Since this is the voltage across the resistor of interest, $P_{1\Omega} = (-1)^2/1 = \boxed{1 \text{ W}}$

22. We begin by selecting the bottom center node as the reference node. Then, since 4 A flows through the bottom 2 Ω resistor, 4 V appears across that resistor.
Naming the remaining nodes (left to right) v_1, v_2, v_3, v_4, v_5, and v_6, respectively, we see two supernodes: combine nodes 2 and 3, and then nodes 5 and 6.

Our nodal equations are then

$$4-6=\frac{v_1-v_2}{14} \qquad [1]$$

$$0=\frac{v_2-v_1}{14}+\frac{v_3-v_4}{7} \qquad [2]$$

$$0=\frac{v_4-v_3}{7}+\frac{v_4-1}{2}+\frac{v_4-v_5}{7} \qquad [3]$$

$$6=\frac{v_6}{3}+\frac{v_5-v_4}{7} \qquad [4]$$

with

$$v_2-v_3=4 \qquad [5]$$

$$v_6-v_5=3 \qquad [6]$$

Solving, $v_4 = 0$. Thus, the current flowing out of the 1 V source is (1 – 0)/2 = 500 mA

and so the 1 V source supplies (1)(0.5) = $\boxed{500 \text{ mW}}$

23. We select the central node as the reference node. We name the left-most node v_1; the top node v_2, the far-right node v_3 and the bottom node v_4.

By inspection, $v_1 = 5$ V

We form a supernode from nodes 3 and 4 then proceed to write appropriate KCL equations:

$$-1 = \frac{v_2}{2} + \frac{v_2 - v_3}{10} \qquad [1]$$

$$5 = \frac{v_3 - v_2}{10} + \frac{v_3}{20} + \frac{v_4 - v_1}{12} \qquad [2]$$

Also, we need the KVL equation relating nodes 3 and 4, $v_4 - v_3 = 10$

Solving, $v_2 = v = \boxed{1.731 \text{ V}}$

24. A strong choice for the reference node is the bottom node, as this makes one of the quantities of interest (v_x) a nodal voltage. Naming the far left node v_1 and the far right node v_3, we are ready to write the nodal equations after making a supernode from nodes 1 and 3:

$$1+8=\frac{v_1-v_x}{8}+\frac{v_3}{2} \qquad [1]$$

$$-8=\frac{v_x-v_1}{8}+\frac{v_x}{5} \qquad [2]$$

Finally, our supernode's KVL equation: $v_3 - v_1 = 2v_x$

Solving, $v_1 = 31.76$ V and $\boxed{v_x = -12.4 \text{ V}}$

Finally, $P_{supplied|1\text{ A}} = (v_1)(1) = \boxed{31.76 \text{ W}}$

25. We select the bottom center node as the reference. We next name the top left node v_1, the top middle node v_2, the top right node v_3, and the bottom left node v_4.

A supernode can be formed from nodes 1, 2 and 4. $v_3 = 4$ V by inspection.

Our nodal equation is then

$$-2 = \frac{v_4}{4} + \frac{v_2 - v_3}{2} \qquad [1]$$

Then the KVL equation is $v_2 - v_4 = 0.5i_1 + 3$ where $i_1 = (v_2 - v_3)/2$.

Solving, $v_2 = 727.3$ mV and hence $\boxed{i_1 = -1.136 \text{ A}}$

26. Our nodal equations may be written directly, noting that two nodal voltages are available by inspection:

$$0 = \frac{v_x + 2}{1} + \frac{v_x}{1} + \frac{v_x - v_y}{4} \qquad [1]$$

$$-1 = \frac{v_y - v_x}{4} + \frac{v_y - kv_y}{3} \qquad [2]$$

Setting $v_x = 0$, Eq. [1] becomes $0 = 2 - v_y/4$ or $v_y = 8$ V.

Consequently, Eq. [2] becomes $-1 = 8/4 + (8 - 8k)/3$ or $\boxed{k = 2.125 \text{ (dimensionless)}}$

27. If we select the bottom node as our reference, and name the top three nodes (left to right) v_A, v_B and v_C, we may write the following nodal equations (noting that $v_B = 4v_1$):

$$2 = \frac{v_A - 4v_1}{2} + \frac{v_A - v_C}{3} \qquad [1]$$

$$v_1 = \frac{v_C - 4v_1}{5} + \frac{v_C - v_A}{3} \qquad [2]$$

And $v_1 = v_A - v_C$

Solving, $\boxed{v_1 = 480 \text{ mV}}$

28. With the selected reference node, v1 = 1 V by inspection. Proceeding with nodal analysis,

$$3 = \frac{v_2 - v_1}{1} + \frac{v_2 - v_3}{2} \qquad [1]$$

$$-2v_x = \frac{v_3 - v_2}{2} + \frac{v_3 - v_4}{1} \qquad [2]$$

$$0 = \frac{v_4}{3} + \frac{v_4 - v_3}{1} + \frac{v_4 - v_1}{4} \qquad [3]$$

And to account for the additional variable introduced through the dependent source,

$v_x = v_3 - v_4$

Solving, $\boxed{v_1 = 1\text{ V},\ v_2 = 3.085\text{ V},\ v_3 = 1.256\text{ V and } v_4 = 951.2\text{ mV}}$

29. We define two clockwise flowing mesh currents i_1 and i_2 in the lefthand and righthand meshes, respectively. Our mesh equations are then

$$-1 = 5i_1 - i_2 \quad [1]$$
$$-2 = -i_1 + 6i_2 \quad [2]$$

Solving,

$\boxed{i_1 = -275.9 \text{ mA} \quad \text{and } i_2 = -379.3 \text{ mA}}$

30. Our two mesh equations are:

$$5 + 7i_2 + 14(i_2 - i_1) = 0 \qquad [1]$$
$$14(i_1 - i_2) + 3i_1 - 12 = 0 \qquad [2]$$

Solving,

$\boxed{i_1 = 1.130 \text{ A} \quad \text{and } i_2 = 515.5 \text{ mA}}$

31. Our two mesh equations are:

$$-15-11=10i_1-i_2 \qquad [1]$$
$$-21+11=-i_1+10i_2 \qquad [2]$$

Solving, $\boxed{i_1 = -2.727 \text{ A} \quad \text{and} \quad i_2 = -1.273 \text{ A}}$

32. We need to construct three mesh equations:

$$-2+(1)(i_1-i_2)-3+5(i_1-i_3)=0 \qquad [1]$$
$$(1)(i_2-i_1)+6i_2+9(i_2-i_3)=0 \quad [2]$$
$$5(i_3-i_1)+3+9(i_3-i_2)+7i_3=0 \qquad [3]$$

Solving, $\boxed{i_1 = 989.2 \text{ mA}, \; i_2 = 150.1 \text{ mA and } i_3 = 157.0 \text{ mA}}$

33. We require three mesh equations:

$$-2-3=6i_1-i_2-5i_3 \quad [1]$$
$$0=-i_1+16i_2-9i_3 \quad [2]$$
$$3=-5i_1-9i_2+21i_3 \quad [3]$$

Solving, i_1 = -989.2 mA, i_2 = -150.2 mA, and i_3 = -157.0 mA

Thus,

$P_{1\Omega} = (i_2 - i_1)^2(1)$	=	703.9 mW
$P_{6\Omega} = (i_2)^2(6)$	=	135.4 mW
$P_{9\Omega} = (i_2 - i_3)^2(9)$	=	41.62 mA
$P_{7\Omega} = (i_3)^2(7)$	=	172.5 mW
$P_{5\Omega} = (i_3 - i_1)^2(5)$	=	3.463 W

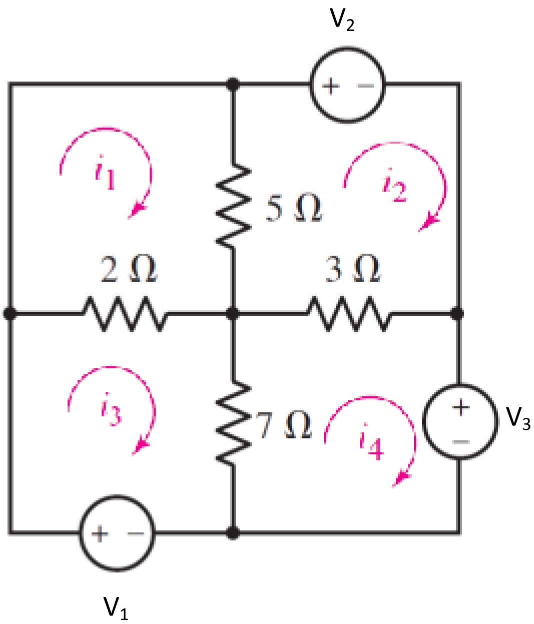
34. The 220 Ω resistor carries no current and hence can be ignored in this analysis. Defining three clockwise mesh currents i_1, i_2, i_y left to right, respectively,

$$-5 + 2200i_1 + 4700(i_1 - i_2) = 0 \qquad [1]$$
$$-4700(i_2 - i_1) + (4700 + 1000 + 5700)i_2 - 5700i_y = 0 \qquad [2]$$
$$-5700i_2 + (5700 + 4700 + 1000)i_y = 0 \qquad [3]$$

(a) Solving Eqs. [1-3], i_y = $\boxed{318.4\ \mu\text{A}}$

(b) Since no current flows through the 220 Ω resistor, it dissipates $\boxed{\text{zero power.}}$

35. We name the sources as shown and define clockwise mesh currents i_1, i_2, i_3 and i_4:



To obtain $i_1 - i_3 = 0$ [1], $i_1 - i_2 = 0$ [2], $i_3 - i_4 = 0$ [3], $i_2 - i_4$ [4], we begin with our mesh equations:

$-V_1 = 9i_3 - 2i_1 - 7i_4$ [5]

$0 = 7i_1 - 2i_3$ [6]

$V_2 = 8i_2 - 5i_1 - 3i_4$ [7]

$V_3 = 10i_4 - 3i_2 - 7i_3$ [9]

Solving Eqs. [1-9], we find the only solution is V1 = V2 = V3.

It is therefore **not possible** to select nonzero values for the voltage sources and meet the specifications.

36. Define a clockwise mesh current i_y in the mesh containing the 10 A source. Then, define clockwise mesh currents i_1, i_2 and i_x, respectively, in the remaining meshes, starting on the left, and proceeding towards the right.

By inspection, $i_y = 10$ A [1]

Then,

$-3 + (8 + 4)i_1 - 4i_2 = 0$ [2]

$-4i_1 + (4 + 12 + 8)i_2 - 8i_3 = 0$ [3]

$-8i_2 + (8 + 20 + 5)i_x - 20i_x = 0$ [4]

Solving, $\boxed{i_x = 6.639 \text{ A}}$

37. Define CW mesh currents i_1, i_2 and i_3 such that $i_3 - i_2 = i$.
Our mesh equations then are:

$-2 + 8i_1 - 4i_2 - i_3 = 0$ [1]
$0 = 5i_2 - 4i_1$ [2]
$0 = 5i_3 - i_1$ [3]

Solving, i_1 = 434.8 mA, i_2 = 347.8 mA, and i_3 = 86.96 mA

Then, $i = i_3 - i_2 = \boxed{-260.8 \text{ mA}}$

38. In the lefthand mesh, we define a clockwise mesh current and name it i_2.
Then, our mesh equations may be written as:

$4 - 2i_1 + (3 + 4)i_2 - 3i_1 = 0$ [1]
$-3i_2 + (3 + 5)i_1 + 1 = 0$ [2]

(note that since the dependent source is controlled by one of our mesh currents/variables/unknowns, these two equations suffice.)

Solving, $i_2 = -902.4$ mA so $P_{4\Omega} = (i_2)^2(4) = \boxed{3.257 \text{ W}}$

39. Define clockwise mesh currents i_1, i_2, i_3 and i_4. By inspection $i_1 = 4$ A and $i_4 = 1$ A.
(a) Define v_x across the dependent source with the bottom node as the reference node. Then,

$3i_2 - 2(4) + v_x = 0$ [1]
$-v_x + 7i_3 - 2 = 0$ [2]

We note that $i_3 - i_2 = 5i_x$, where $i_x = i_4 - i_3$.
Thus, $-i_2 + 6i_3 = 5$ [3]

We first add Eqs. [1] and [2], so that our set of equations becomes:

$3i_2 + 7i_3 = 10$ [1']
$-i_2 + 6i_3 = 5$ [3]

Solving, $i_2 = 1$ A and $i_3 = 1$ A. Thus, $P_{1\Omega} = (1)(i_2)^2 = \boxed{1 \text{ W}}$

(b) Using nodal analysis, we define V_1 at the top of the 4 A source, V_2 at the top of the dependent source, and V_3 at the top of the 1 A source. The bottom node is our reference node.

Then,

$$4 = \frac{V_1}{2} + \frac{V_1 - V_2}{1}$$
$$5i_x = \frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{5}$$
$$-1 = \frac{V_3}{2} + \frac{V_3 - V_2}{5}$$

and

$i_x = -V_2/2$

Solving, $V_1 = 6$ V and $V_2 = 5$ V

Hence, $P_{1\Omega} = (V_1 - V_2)^2/1 = \boxed{1 \text{ W}}$

40. Define a clockwise mesh current i_1 for the mesh with the 2 V source; a clockwise mesh current i_2 for the mesh with the 5 V source, and clockwise mesh current i_3 for the remaining mesh.

Then, we may write

$-2 + (2 + 9 + 3)i_1 + 1 = 0$

which can be solved for $\boxed{i_1 = 71.43 \text{ mA}}$

By inspection, $i_3 = -0.5v_x = -0.5(9i_1) = \boxed{321.4 \text{ mA}}$

For the remaining mesh, $-1 + 10i_2 - 10i_3 - 5 = 0$ or $\boxed{i_2 = 921.4 \text{ mA}}$

41. We define four clockwise mesh currents. In the top left mesh, define i_1. In the top right mesh, define i_2. In the bottom left mesh, define i_3 (note that $i_3 = i_x$). In the last mesh, define i_4. Then, our mesh equations are:

$14i_1 - 7i_2 + 9 = 0$ [1]
$5i_3 - i_4 - 9 = 0$ [2]
$11i_2 + 0.2i_x = 0$ [3]
$5i_4 - 4i_2 - i_3 = 0.1v_a$ [4]

where $v_a = -7i_1$.

Solving, $i_1 = -660.0$ mA, $i_2 = -34.34$ mA, $i_3 = 1.889$ A and $i_4 = 442.6$ mA.

$\boxed{\text{Hence, } i_x = i_3 = 1.889 \text{ A and } v_a = 462.0 \text{ mV}}$

42. Our best approach here is to define a supermesh with meshes 1 and 3. Then,

$$-1 + 7i_1 - 7i_2 + 3i_3 - 3i_2 + 2i_3 = 0 \qquad [1]$$
$$-7i_1 + (7 + 1 + 3)i_2 - 3i_3 = 0 \qquad [2]$$
$$i_3 - i_1 = 2 \qquad [3]$$

Solving, $\boxed{i_1 = -1.219 \text{ A},\ i_2 = -562.5 \text{ mA and } i_3 = 781.3 \text{ mA}}$

43. In the one remaining mesh, we define a clockwise mesh current i_2. Then, a supermesh from meshes 1 and 3 may be formed to simplify our analysis. Hence,

$-3 + 10i_1 + (i_2 - i_3) + 4i_3 + 17i_3 = 0$ [1]
$-10i_1 + 16i_2 - i_3 = 0$ [2]
$-i_1 + i_3 = 5$ [3]

Solving, $i_1 = -3.269$ A, $i_2 = -1.935$ A and $\boxed{i_3 = 1.731 \text{ A}}$

Hence, $P_{1\Omega} = (i_2 - i_3)^2(1) = \boxed{13.44 \text{ W}}$

44. Define (left to right) three clockwise mesh currents i_2, i_3 and i_4. Then, we may create a supermesh from meshes 2 and 3. By inspection, $i_4 = 3$ A.

Our mesh/supermesh equations are:

$7 + 5i_2 - 5i_1 + 11i_3 - 11i_1 + (1)i_3 - (1)i_4 + 5i_3 = 0$ [1]
$-5i_2 + (3 + 5 + 10 + 11)i_1 - 11i_3 = 0$ [2]
$i_3 - i_2 = 9$ [3]

Solving, $\boxed{i_1 = -874.3 \text{ mA}}$
$i_2 = -7.772$ A
$i_3 = 1.228$ A
$i_4 = 3$ A

Thus, the 1 Ω resistor dissipates $P_{1\Omega} = (1)(i_4 - i_3)^2 = \boxed{3.141 \text{ W}}$

45. Our three equations are

$7 + 2200i_3 + 3500i_2 + 3100(i_2 – 2) = 0$
$–i_1 + i_2 = 1$
$i_1 – i_3 = 3$

Solving, $\boxed{i_1 = 705.3 \text{ mA}, i_2 = 1.705 \text{ A and } i_3 = –2.295 \text{ A}}$

46. By inspection, the unlabelled mesh must have a clockwise mesh current equal to 3 A. Define a supermesh comprised of the remaining 3 meshes. Then,

$-3 + 3i_2 - 3(3) + 5i_1 - 5(3) + 8 - 2 + 2i_3 = 0$ [1]

We also write $i2 - i_1 = 1$ [2]

and $i_1 - i_3 = -2$ [3]

Solving, $\boxed{i_1 = 1.4 \text{ A, } i_2 = 2.4 \text{ A and } i_3 = 3.4 \text{ A}}$

47. By inspection, $\boxed{i_1 = 5 \text{ A.}}$

$i_3 – i_1 = v_x/3$ but $v_x = 13i_3$. Hence, $\boxed{i_3 = –1.5 \text{ A.}}$

In the remaining mesh,

$–13i_1 + 36i_2 – 11i_3 = 0$ so $\boxed{i_2 = 1.347 \text{ A.}}$

48. Define clockwise mesh current i_2 in the top mesh and a clockwise mesh current i_3 in the bottom mesh. Next, create a supermesh from meshes 2 and 3. Our mesh/supermesh equations are:

$-1 + (4 + 3 + 1)i_1 - 3i_2 - (1)i_3 = 0$ [1]
$(1)i_3 - (1)i_1 + 3i_2 - 3i_1 - 8 + 2i_3 = 0$ [2]
and
$i_2 - i_3 = 5i_1$ [3]

(Since the dependent source is controlled by a mesh current, there is no need for additional equations.)

Solving, $i_1 = 19$ A and hence $P_{supplied} = (1)i_1 = \boxed{19 \text{ W}}$

49. Define clockwise mesh currents i_1, i_2 and i_3 so that $i_2 - i_3 = 1.8v_3$.

We form a supermesh from meshes 2 and 3 since they share a (dependent) current source.

Our supermesh/mesh equations are then

$-3 + 7i_1 - 4i_2 - 2i_3 = 0$ [1]
$-5 + i_3 + 2(i_3 - i_1) + 4(i_2 - i_1) = 0$ [2]
Also,
$i_2 - i_3 = 1.8v3$ where $v_3 = i_3(1) = i_3$. Hence, $i_2 - i_3 = 1.8i_3$ [3]

Solving Eqs. [1-3], $\boxed{i_1 = -2.138 \text{ A}, i_2 = -1.543 \text{ A and } i_3 = 551.2 \text{ mA}}$

50. With the top node naturally associated with a clockwise mesh current i_a, we name (left to right) the remaining mesh currents (all defined flowing clockwise) as i_1, i_2 and i_3, respectively.

We create a supermesh from meshes ‘a’ and ‘2’, noting that $i_3 = 6$ A by inspection.

Then,

$-4 + 3i_1 - 2i_a = 0$ [1]

$2i_a + 3i_a - 3i_1 + 10i_a + 4i_a - 4(6) + 5i_2 - 5(6) = 0$ [2]

Also, $i_2 - i_a = 5$ [3]

Solving, $i_a = 1.5$ A. Thus, $P_{10\Omega} = 10(i_a)^2 = \boxed{22.5 \text{ W}}$

51. (a) 4;

(b) Technically 5, but 1 mesh current is available "by inspection" so really just 4. We also note that a supermesh is indicated so the actual number of "mesh" equations is only 3.

(c) With nodal analysis we obtain i_5 by Ohm's law and 4 simultaneous equations. With mesh/supermesh, we solve 4 simultaneous equations and perform a subtraction. The difference here is not significant.

In the case of $v_{7\Omega}$, we could define the common node to the 3 A source and 7 W resistor as the refernce and obtain the answer with no further arithmetic steps. Still, we are faced with 4 simultaneous equations with nodal analysis so mesh analysis is still preferable in this case.

52. (a) Without employing the supernode technique, 4 nodal equations would be required. With supernode, only 3 nodal equations are needed plus a simple KVL equation. (Then simple division is necessary to obtain i_5).

(b) Although there are 5 meshes, one mesh current is available by inspection, so really only 4 mesh equations are required.

(c) The supernode technique is preferable here regardless; it requires fewer simultaneous equations.

53. (a) Nodal analysis requires 2 nodal equations and 2 simple subtractions
Mesh analysis requires 2 mesh equations and 2 simple multiplications

Neglecting the issues associated with fractions and grouping terms, neither appears to have a distinct advantage.

(b) Nodal analysis: we would form a supernode so 2 nodal equations plus one KVL equation. v_1 is available by inspection, v_2 obtained by subtraction.

Mesh analysis: 1 mesh equation, v_1 available by Ohm's law, v_2 by multiplication.

Mesh analysis has a slight edge here as no simultaneous equations required.

54. (a) Mesh analysis: Define two clockwise mesh currents i_1 and i_2 in the left and right meshes, respectively. A supermesh exists here.

Then, $2i_1 + 22 + 9i_2 = 0$ [1]
and $-i_1 + i_2 = 11$ [2]

Solving, $i_1 = 11$ A and $i_2 = 0$. Hence, $\boxed{v_x = 0.}$

(b) Nodal analysis: Define the top left node as v_1, the top right node as v_x.

We form a supernode from nodes 1 and x. Then,

$11 = v_1/2 + v_x/9$ [1]
and $v_1 - v_x = 22$ [2]

Solving, $v_1 = 22$ V and $\boxed{v_x = 0}$

(c) In terms of simultaneous equations, there is no real difference between the two approaches. Mesh analysis did require multiplication (Ohm's law) so
nodal analysis had a very slight edge here.

55. (a) Nodal analysis: 1 supernode equation, 1 simple KVL equation. v_1 is a nodal voltage so no further arithmetic required.

(b) Mesh analysis: 4 mesh equations but two mesh currents available "by inspection" so only two mesh equations actually required. Then, invoking Ohm's law is required to obtain v_1.

(c) Nodal analysis is the winner, but it has only a slight advantage (no final arithmetic step). The choice of reference node will not change this.

56. (a) Using nodal analysis, we have 4 nodal voltages to obtain although one is available by inspection. Thus, $\boxed{\text{3 simultaneous equations}}$ are required to obtain v_1, from which we may calculate $P_{40\Omega}$.

(b) Employing mesh analysis, the existence of four meshes implies the need for 4 simultaneous equations. However, 2 mesh currents are available by inspection, hence only 2 simultaneous equations are needed. Since the dependent source relies on v_1, simple subtraction yields this voltage ($0.1\ v_1 = 6 - 4 = 2$ A).

Thus, $\boxed{v_1 \text{ can be obtained with NO simultaneous equations.}}$

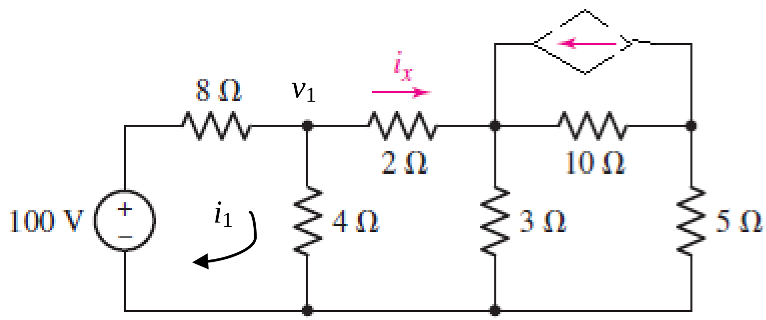
$\boxed{\text{Mesh wins.}}$

57. (a) Nodal analysis: 2 nodal equations, plus 1 equation for each dependent source that is not controlled by a nodal voltage = 2 + 3 = 5 equations.

(b) Mesh analysis: 3 mesh equations, 1 KCL equation, 1 equation for each dependent source not controlled by a mesh current = 3 + 1 + 2 = 6 equations.

58. <Design> One possible solution:

Replace the independent current source of Fig. 4.28 with a dependent current source.



(a) Make the controlling quantity $8v_1$, i.e. depends on a nodal voltage.

(b) Make the controlling quantity $8i_1$, i.e. depends on a mesh current.

59. Referring to the circuit of Fig. 4.34, our two nodal equations are

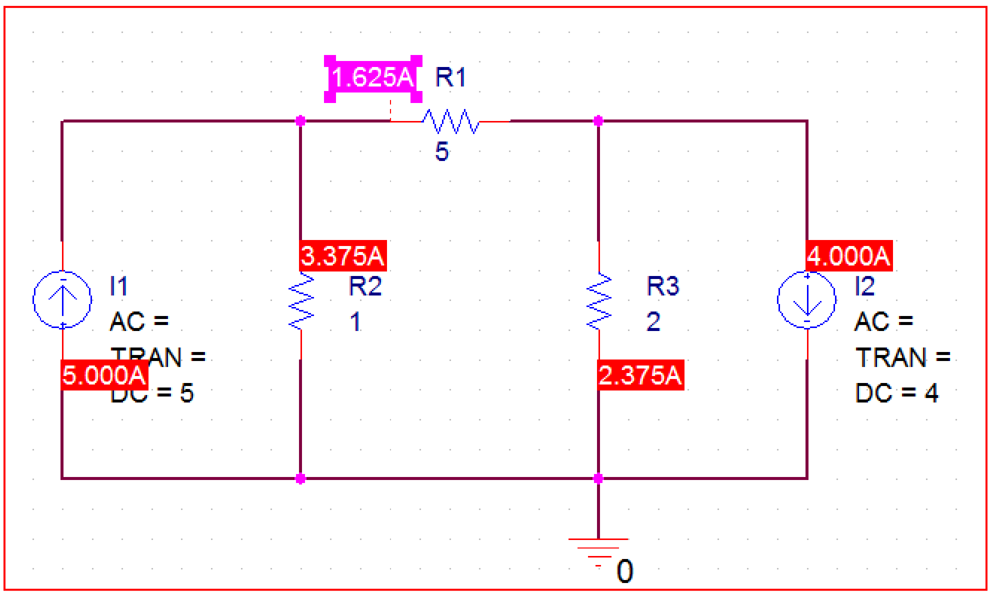
$$5 = \frac{v_1}{1} + \frac{v_1 - v_2}{5} \qquad [1]$$

$$-4 = \frac{v_2}{2} + \frac{v_2 - v_1}{5} \qquad [2]$$

Solving, v_1 = 3.375 V and v_2 = -4.75 V.
Hence,

$$i = (v_1 - v_2)/5 = \boxed{1.625\text{ A}}$$

In PSpice,



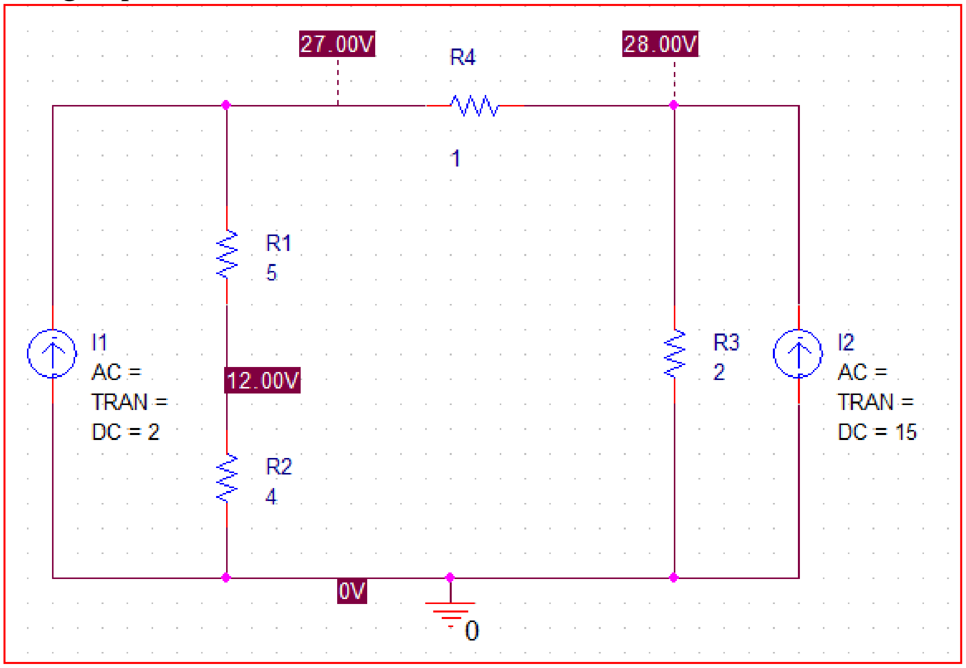
60. Referring to Fig. 4.36, our two nodal equations are:

$$2 = \frac{v_1}{9} + \frac{v_1 - v_2}{1} \Rightarrow 10v_1 - 9v_2 = 18$$

$$15 = \frac{v_2}{2} + \frac{v_2 - v_1}{1} \Rightarrow -2v_1 + 3v_2 = 30$$

Solving, v_1 = 27 V and v_2 = 28 V. Thus, $\boxed{v_1 - v_2 = -1 \text{ V}}$

Using PSpice,



61. Referring to Fig. 4.39, our nodal equations are then

$$8+4=\frac{v_1-v_3}{3}+\frac{v_1-v_2}{3} \qquad [1]$$

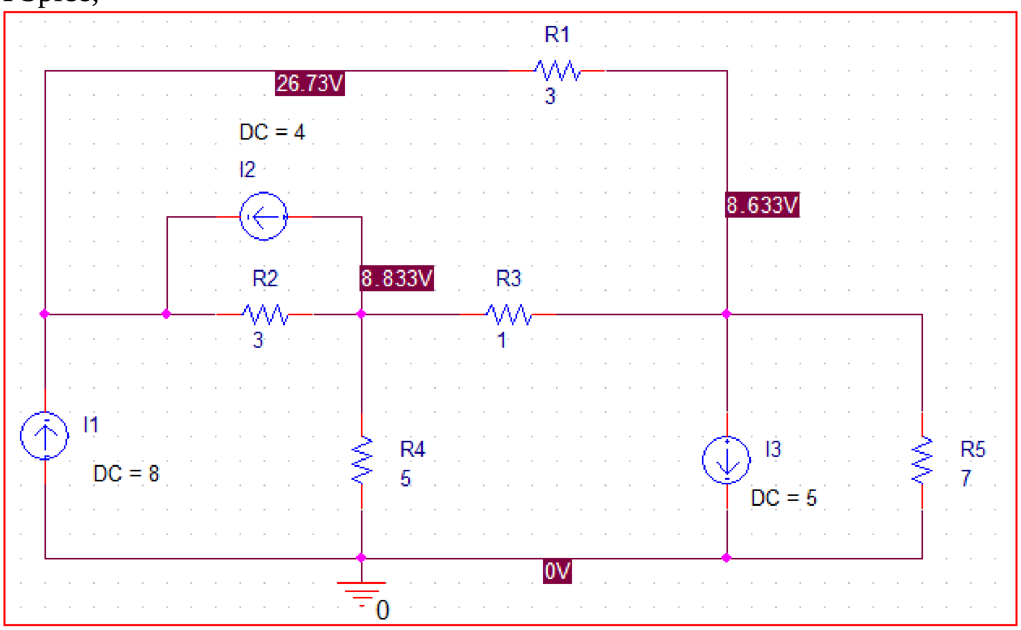
$$-4=\frac{v_2}{5}+\frac{v_2-v_1}{3}+\frac{v_2-v_3}{1} \qquad [2]$$

$$-5=\frac{v_3-v_1}{3}+\frac{v_3-v_2}{1}+\frac{v_3}{7} \qquad [3]$$

Solving, v_1 = 26.73 V, v_2 = 8.833 V, v_3 = 8.633 V

$v_{5\Omega} = v_2 = \boxed{8.833 \text{ V}}$

Within PSpice,



62. Referring to Fig. 4.41, first, we note that it is possible to separate this circuit into two parts, connected by a single wire (hence, the two sections cannot affect one another). For the left-hand section, our nodal equations are:

$$2 = \frac{v_1}{2} + \frac{v_1 - v_3}{6} \qquad [1]$$

$$-2 = \frac{v_2}{5} + \frac{v_2 - v_3}{2} + \frac{v_2 - v_4}{10} \qquad [2]$$

$$1 = \frac{v_3 - v_1}{6} + \frac{v_3 - v_2}{2} + \frac{v_3 - v_4}{5} \qquad [3]$$

$$0 = \frac{v_4 - v_2}{10} + \frac{v_4 - v_3}{5} + \frac{v_4}{5} \qquad [4]$$

Solving,

v_1 = 3.078 V, v_2 = –2.349 V, v_3 = 0.3109 V, v_4 = -0.3454 V

For the right-hand section, our nodal equations are:

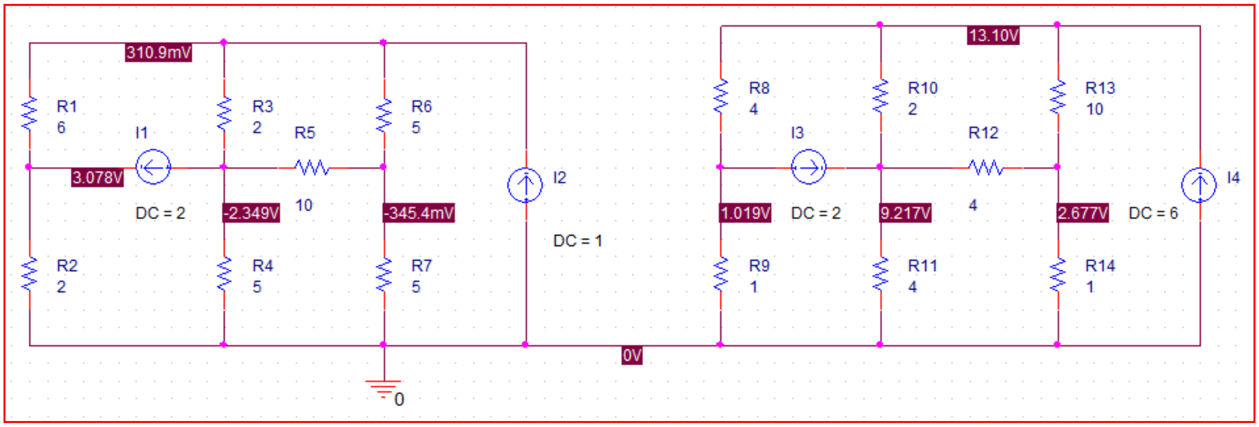
$$-2 = \frac{v_5}{1} + \frac{v_5 - v_7}{4} \qquad [1]$$

$$2 = \frac{v_6}{4} + \frac{v_6 - v_8}{4} + \frac{v_6 - v_7}{2} \qquad [2]$$

$$6 = \frac{v_7 - v_5}{4} + \frac{v_7 - v_6}{2} + \frac{v_7 - v_8}{10} \qquad [3]$$

$$0 = \frac{v_8}{1} + \frac{v_8 - v_6}{4} + \frac{v_8 - v_7}{10} \qquad [4]$$

Solving, v_5 = 1.019 V, v_6 = 9.217 V, v_7 = 13.10 V, v_8 = 2.677 V



63. Referring to Fig. 4.43, select the bottom node as the reference node. The top node is designated as v_1, and the center node at the top of the dependent source is designated as v_2. Our nodal equations are:

$$1 = \frac{v_1 - v_2}{5} + \frac{v_1}{2} \qquad [1]$$

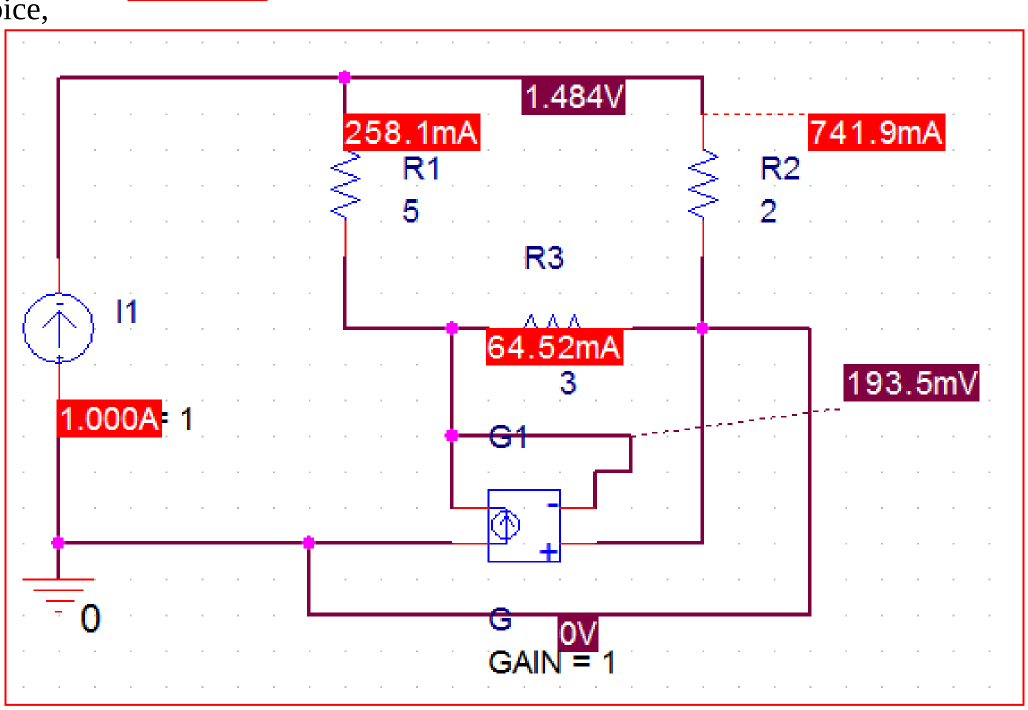
$$v_x = \frac{v_2}{3} + \frac{v_2 - v_1}{5} \qquad [2]$$

We have two equations in three unknowns, due to the presence of the dependent source. However, $v_x = -v_2$, which can be substituted into Eq. [2]. Solving,

$v_1 = 1.484$ V and $v_2 = 0.1936$ V

Thus, $i_1 = v_1/2 = \boxed{742 \text{ mA}}$

By PSpice,



64. (a) PSpice output file (edited slightly for clarity)

```
.OP
    V1 1 0 DC 40
    R1 1 2 11
    R2 0 2 10
    R3 2 3 4
    R4 2 4 3
    R5 0 3 5
    R6 3 4 6
    R7 3 9 2
    R8 4 9 8
    R9 0 9 9
    R10 0 4 7

NODE  VOLTAGE   NODE  VOLTAGE   NODE  VOLTAGE   NODE  VOLTAGE

(  1)  40.0000 (  2)  8.3441 (  3)  4.3670 (  4)  5.1966

(  9)  3.8487
```

(b) Hand calculations: nodal analysis is the only option.

$$0 = \frac{v_2 - 40}{11} + \frac{v_2}{10} + \frac{v_2 - v_3}{4} + \frac{v_2 - v_4}{3}$$

$$0 = \frac{v_3 - v_2}{4} + \frac{v_3}{5} + \frac{v_3 - v_4}{6} + \frac{v_3 - v_9}{2}$$

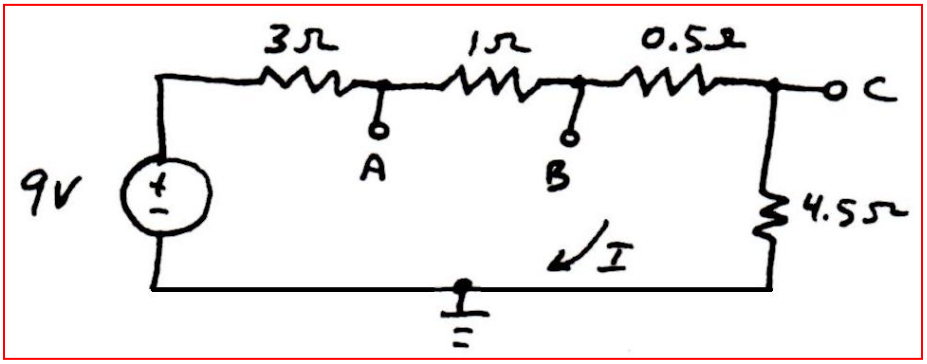
$$0 = \frac{v_4 - v_3}{6} + \frac{v_4}{7} + \frac{v_4 - v_9}{8} + \frac{v_4 - v_2}{3}$$

$$0 = \frac{v_9 - v_4}{8} + \frac{v_9 - v_3}{2} + \frac{v_9}{9}$$

Solving,

v_1 = 40 V (by inspection); v_2 = 8.344 V; v_3 = 4.367 V; v_4 = 5.200 V; $\boxed{v_9 = 3.849\text{ V}}$

65. <Design> (a) One possible solution:

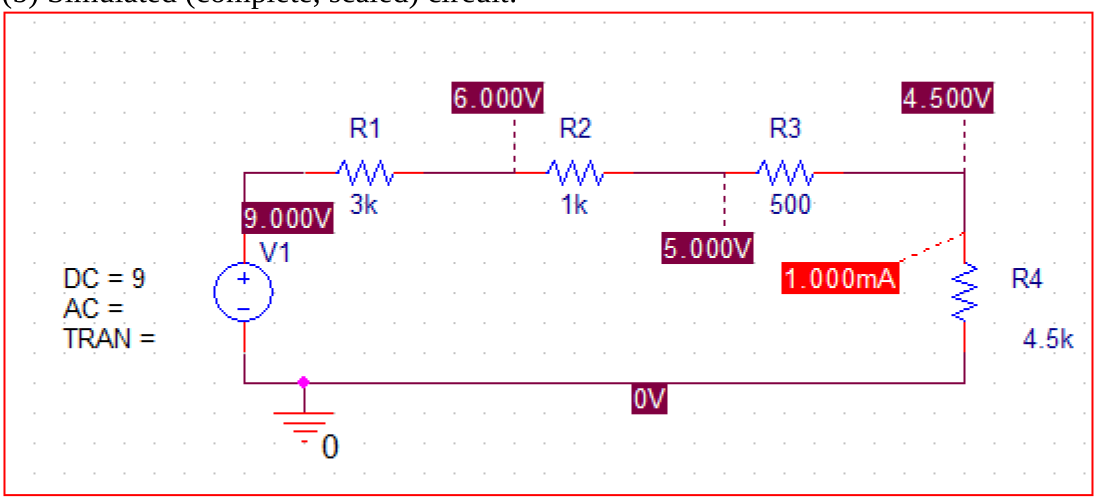


This circuit provides 1.5 V across terminals A and C;
4.5 V across terminal C and the reference terminal;
5 V across terminal B and the reference terminal.

Although this leads to I = 1 A, which is greater than 1 mA, it is almost 1000 times the required value. Hence, scale the resistors by 1000 to construct a circuit from 3 kΩ, 1 kΩ, 500 Ω, and 4.5 kΩ. Then I = 1 mA.

Using standard 5% tolerance resistors, 3 kΩ = 3 kΩ; 500 Ω = 1 kΩ || 1 kΩ;
1 kΩ = 1 kΩ; 4.5 kΩ = 2 kΩ + 2 kΩ.

(b) Simulated (complete, scaled) circuit:



66. (a) $\boxed{\text{The bulbs must be connected in parallel, or they would } \textit{all} \text{ be unlit.}}$

(b) Parallel-connected means each bulb runs on 12 V dc. A power rating of 10 mW then indicates each bulb has resistance $(12)^2/(10\times10^{-3})$ = 14.4 kΩ

Given the high resistance of each bulb, the resistance of the wire connecting them is negligible.

SPICE OUTPUT: (edited slightly for clarity)

```
.OP
V1 1 0 DC 12
R1 1 0 327.3
****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
******************************************************************************
NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
(    1)   12.0000
    VOLTAGE SOURCE CURRENTS
  NAME        CURRENT
   V1        -3.666E-02

  TOTAL POWER DISSIPATION   4.40E-01  WATTS
  JOB CONCLUDED
```

(c) The equivalent resistance of 44 parallel-connected lights is

$$R_{eq} = \frac{1}{\left[(44)\left(\frac{1}{14400}\right)\right]} = 327.3\ \Omega$$

. This would draw $(12)^2/327.3$ = $\boxed{440\text{ mW.}}$

(A more straightforward, but less interesting, route would be to multiply the per-bulb power consumption by the total number of bulbs).

67. <Design> One possible solution:
We select nodal analysis, with the bottom node as the reference terminal. We then assign nodal voltages v_1, v_2, v_3 and v_4 respectively to the top nodes, beginning at the left and proceeding to the right.

We need $v_2 - v_3 = (2)(0.2) = 400$ mV
Arbitrarily select $v_2 = 1.4$ V, $v_3 = 1$ V.

So, element B must be a 1.4 V voltage source, and element C must be a 1 V voltage source. Choose A = 1 A current source, D = 1 A current source; F = 500 mV voltage source, E = 500 mV voltage source.

68. (a) If the voltage source lies between any node and the reference node, that nodal voltage is readily apparent simply by inspection.

(b) If the current source lies on the periphery of a mesh, i.e. is not shared by two meshes, then that mesh current is readily apparent simply by inspection.

(c) Nodal analysis is based upon conservation of charge.

(d) Mesh analysis is based upon conservation of energy.

69. (a) Although mesh analysis yields i_2 directly, it requires three mesh equations to be solved. Therefore, nodal analysis has a slight edge here since the supernode technique can be invoked.

We choose the node at the "+" terminal of the 30 V source as our reference. We assign nodal voltage v_A to the top of the 80 V source, and v_C to the "-" terminal of that source. v_B, the nodal voltage at the remaining node (the "-" terminal of the 30 V source), is seen by inspection to be -30 V (v_B = -30 [1]). Our nodal equations are then

$$0 = \frac{v_A - v_B}{10} + \frac{v_C}{30} + \frac{v_C}{40} \quad [2]$$

and $v_A - v_C = 80$ [3]

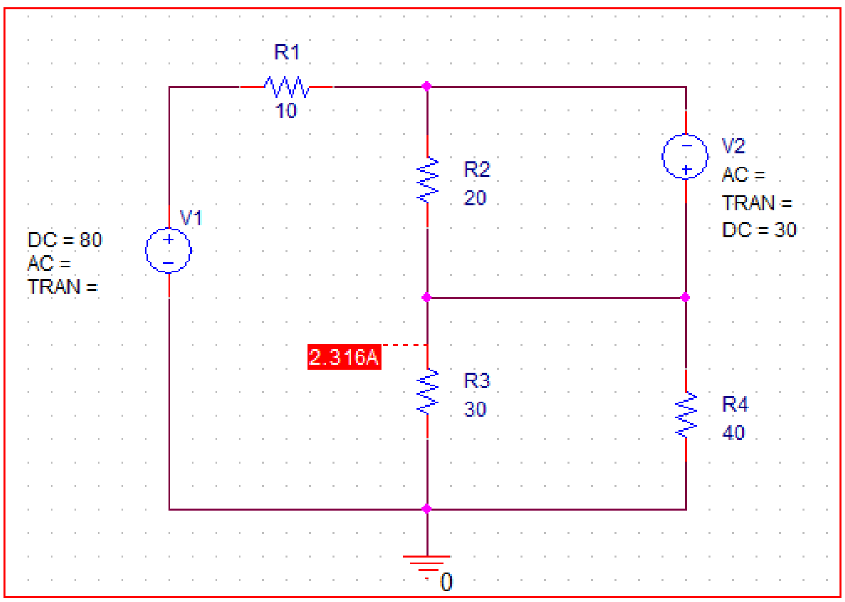
Solving,

$v_A = 10.53$ V, $v_B = -30$ V, and $v_C = -69.47$ V.

Hence

$$i_2 = -v_C/30 = \boxed{2.316\text{ A}}$$

(b)



70. (a) Let's try both and see.

Nodal analysis:

Assign v_1 to the top node of the diamond; the bottom node is our reference node. Then v_2 is assigned to the lefthand node of the diamond; v_3 is assigned to the remaining node. We can form a supernode from nodes 1,2, and 3, so

$$0 = \frac{v_1 - 80}{10} + \frac{v_2}{30} + \frac{v_3}{40} \quad [1] \quad \text{and}$$

$$v_3 - v_1 = 30 \quad [2] \text{ and}$$

$$v_2 - v_3 = 2.5 \quad [3]$$

Solving, $\boxed{v_3 = 68.95\text{ V}}$

Mesh analysis:

$$-80 + (10 + 20 + 30)i_1 - 20i_2 - 30i_3 = 0 \quad [4]$$

$$20i_2 - 20i_1 - 30 - 2.5 = 0 \quad [5]$$

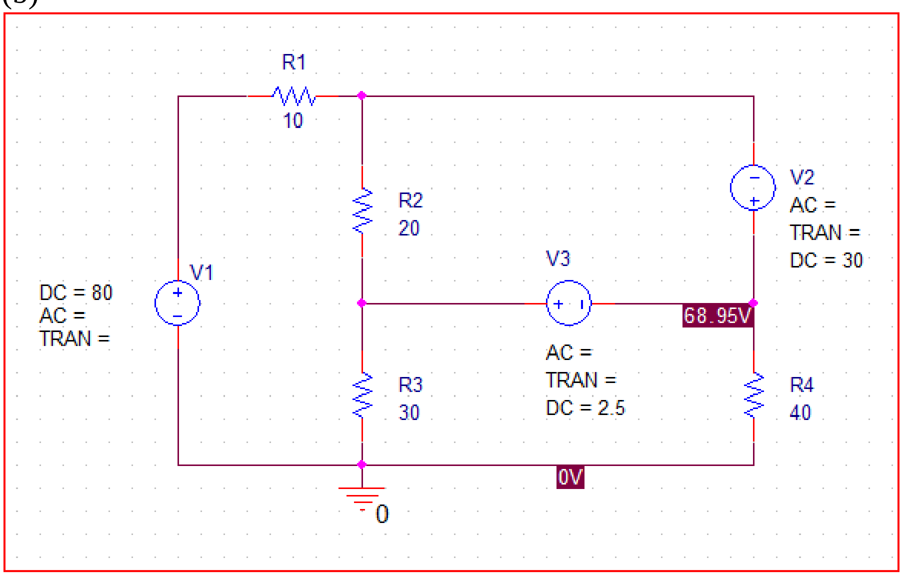
$$(30 + 40)i_3 - 30i_1 + 2.5 = 0 \quad [6]$$

Solving, $i_3 = 1.724$ A

Hence, $v_3 = 40i_3 = 68.95$ V

There does not appear to be a clear winner here. Both methods required writing and solving three simultaneous equations, practically speaking. Mesh analysis then required multiplication to obtain the voltage desired, but is that really hard enough to give nodal analysis the win? Let's call it a draw.

(b)



(c) In this case, perhaps mesh analysis will look slightly more attractive, but via nodal analysis Ohm's law yields i_2 easily enough.

Perhaps all this change does it make it more of a draw.

1. (a) $f_{linear} = \boxed{1 + x}$

(b)

x	f_{linear}	e^x	%error
10^{-6}	1.000001	1.000001000	-
10^{-4}	1.001	1.00100005	-
10^{-2}	1.01	1.010050167	0.005%
0.1	1.1	1.105170918	0.5%
1	2	2.718281828	26%

(c) Somewhat subjectively, we note that the relative error is less than 0.5% for $x < 0.1$ so use this as our estimate of what constitutes "reasonable."

2. $y(t) = 4\sin 2t \approx 4(2t) = 8t$

(a) $\textit{Define \%error} \triangleq 100 \times \left| \dfrac{8t - 4\sin 2t}{4\sin 2t} \right|$

t	$8t$	$4\sin 2t$	%error
10^{-6}	8×10^{-6}	8.000×10^{-6}	0% (to 4 digits)
10^{-4}	8×10^{-4}	8.000×10^{-4}	0% (to 4 digits)
10^{-2}	8×10^{-2}	0.07999	0.01%
10^{-1}	8×10^{-1}	0.7947	0.7%
1.0	8.0	3.637	55%

(b) This linear approximation holds well (< 1% relative error) even up to t = 0.1. Above that value and the errors are appreciable.

3. $i_8\big|_{6\text{ A only}} = 6\frac{3}{3+8} = \boxed{\frac{18}{11}\text{ A}}$. $i_8\big|_{2\text{ V only}} = 2\frac{8}{3+8} = \boxed{\frac{16}{11}\text{ A}}$

4. (a) We replace the voltage source with a short circuit and designate the downward current through the 4 Ω resistor as i'.

Then, i' = (10)(9)/(13) = 6.923 A

Next, we replace the current source in the original circuit with an open circuit and designate the downward current through the 4 Ω resistor as i".
Then, i" = 1/13 = 0.07692 A

Adding, i = i' + i" = $\boxed{7.000\ \text{A}}$

(b) The 1 V source contributes (100)(0.07692)/7.000 = $\boxed{\text{1.1\% of the total current.}}$

(c) $I_x(9)/13 = 0.07692$. Thus, $\boxed{I_x = 111.1\ \text{mA}}$

5. (a) Replacing the 5 A source with an open circuit, $\left.i_x\right|_{3\text{ A only}} = -3\dfrac{14}{14+10} = \boxed{-1.75\text{ A}}.$

Replacing the 3 A source with an open circuit, $\left.i_x\right|_{5\text{ A only}} = -5\dfrac{5}{19} = \boxed{-1.316\text{ A}}.$

(b) -I(5/19) = -1.75. Thus, $\boxed{\text{I = 6.65 A.}}$

6. (a) Open circuiting the 4 A source leaves 5 + 5 + 2 = 12 Ω in parallel with the 1 Ω resistor. Thus, $v_1\big|_{7\text{ A}} = (7)(1\|12) = (7)(0.9231) = \boxed{6.462\text{ V}}$

Open circuiting the 7 A source leaves 1 + 5 = 6 Ω in parallel with 5 + 2 = 7 Ω. Assisted by current division,

$$v_1\big|_{4\text{ A}} = (1)\left[-4\frac{7}{7+7}\right] = \boxed{-2.154\text{ V}}$$

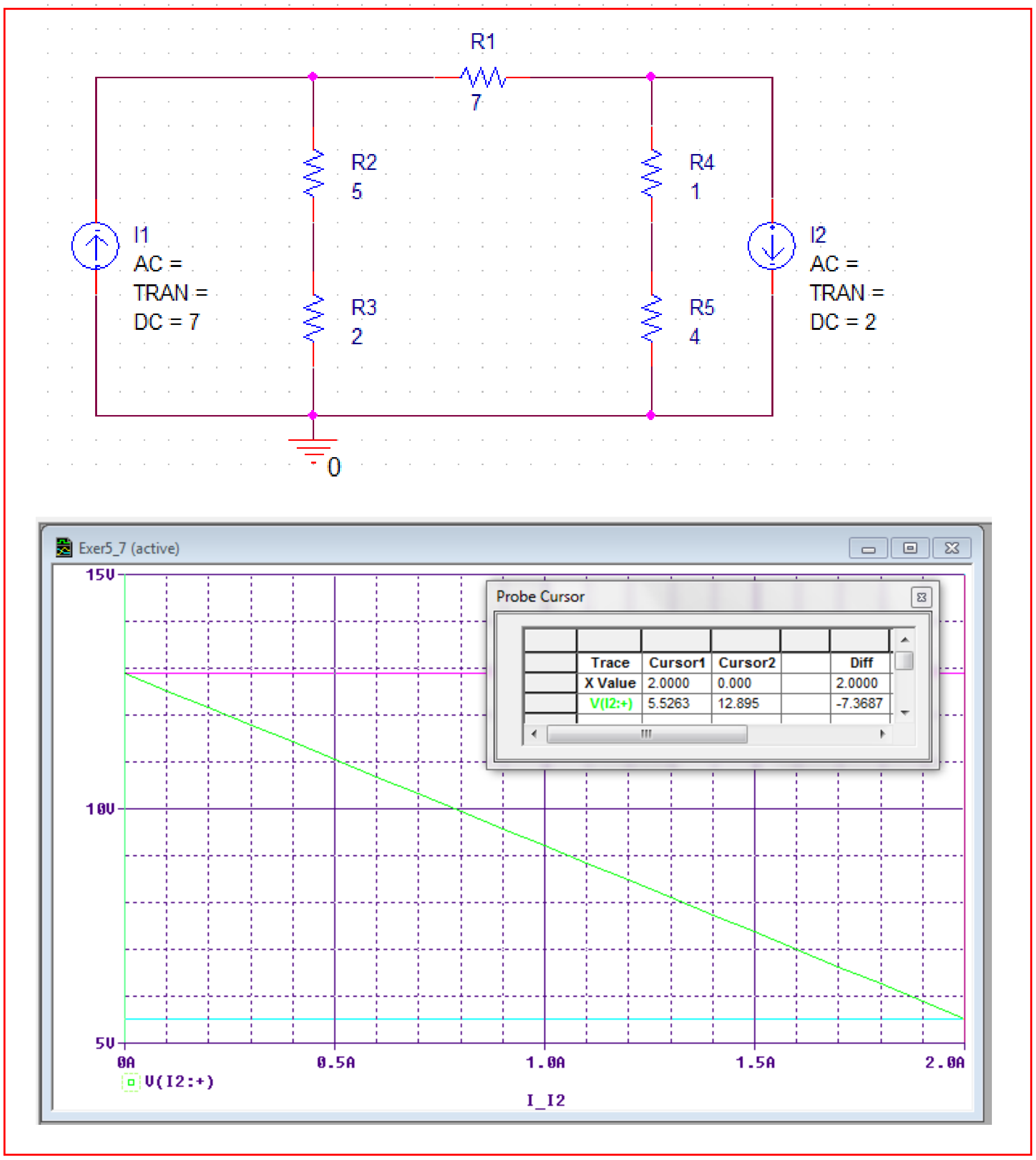
Thus, $v_1 = 6.462 - 2.154 = 4.308$ V

(b) $\boxed{\text{Superposition does not apply to power – that's a nonlinear quantity.}}$

7. (a) $$v_2\big|_{7\text{ A}} = (5)\left[7\frac{7}{19}\right] = \boxed{12.89\text{ V}}$$

$$v_2\big|_{2\text{ A}} = (5)\left[-2\frac{14}{19}\right] = \boxed{-7.368\text{ V}}$$

(b) We see from the simulation output that the 7 A source alone contributes 12.89 V. The output with both sources on is 5.526 V, which agrees within rounding error to our hand calculations (5.522 V).



8. (a) $\boxed{4\text{ V} \rightarrow 8\text{ V};\ 10\text{ V} \rightarrow 20\text{ V}}$

(b) $\boxed{4\text{ V} \rightarrow -4\text{ V};\ 10\text{ V} \rightarrow -10\text{ V}}$

(c) not possible; superposition does not apply to power.

9. $$v_x = 12\frac{3\|2}{(3\|2)+1} + (-15)\frac{1\|3}{(1\|3)+2} = \boxed{2.454\ \text{V}}$$

$$v_x' = 6\frac{3\|2}{(3\|2)+1} + (-10)\frac{1\|3}{(1\|3)+2} = \boxed{0.5454\ \text{V}}$$

$$v_x'' = 6\frac{3\|2}{(3\|2)+1} + (-5)\frac{1\|3}{(1\|3)+2} = \boxed{1.909\ \text{V}}$$

$$v_x' + v_x'' = 2.455\ \text{V} \quad \text{(agrees within rounding error)}$$

10. (a) With the right-hand voltage source short-circuited and the current source open-circuited, we have 2 || 5 = 10/7 Ω

By voltage division, $v_x\big|_{\text{lefthand 4 V}} = (4)\dfrac{1}{3+1+10/7} = 0.7368\text{ V}$

With the other voltage source short-circuited and the current source open-circuited, we have (3 + 1) ||5 = 2.222 Ω.

$v_{5\Omega} = 4\dfrac{2.222}{2.222+2} = 2.105\text{ V}$. Then, $v_x\big|_{\text{righthand 4 V}} = -2.105\dfrac{1}{4} = -0.5263\text{ V}$

Finally, with both voltage sources short-circuited, we find that

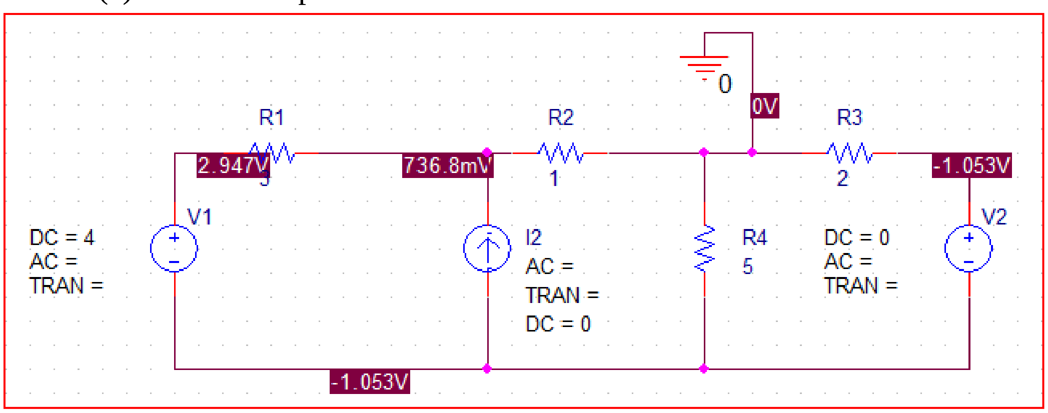
$$v_x\big|_{2\text{ A}} = (1)\left[2\frac{3}{3+1+10/7}\right] = 1.105\text{ V}$$

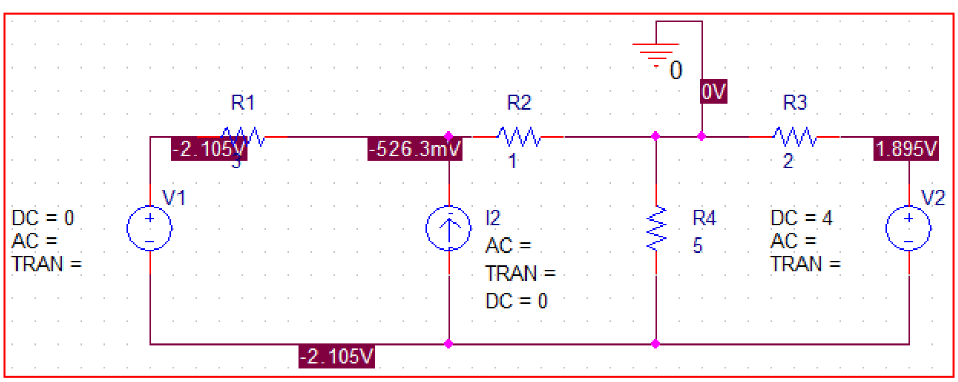
Adding these three terms together, $\boxed{v_x = 1.316\text{ V}}$

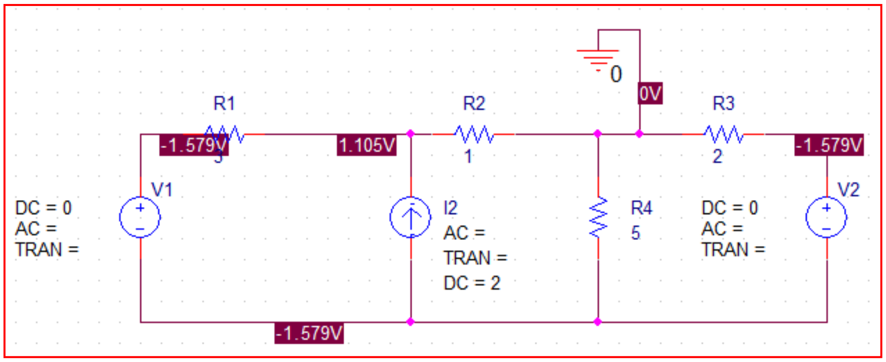
(b) (0.9)(1.316) = 0.7368 + 1.105*k* – 0.5263

Solving, *k* = 0.8814. Hence, we should reduce the 2 A source to 2*k* = $\boxed{1.763\text{ A}}$

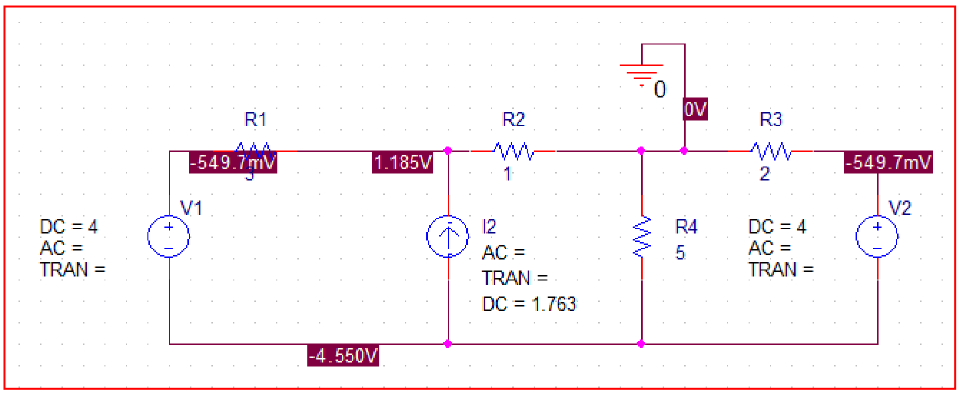
(c) Our three separate simulations:







Our reduced voltage alternative:



11. We select the bottom node as the reference, then identify v_1 with the lefthand terminal of the dependent source and v_2 with the righthand terminal.

Via superposition, we first consider the contribution of the 1 V source:

$$\frac{v_1' - 1}{5000} + \frac{v_1'}{7000} + \frac{v_2'}{2000} = 0 \text{ and}$$

$$\left(1 + \frac{0.2}{7000}\right) v_1' - v_2' = 0$$

Solving, $v_1' = 0.237$ V

Next, we consider the contribution of the 2 A source:

$$\frac{v_1''}{5000} + \frac{v_1''}{7000} + \frac{v_2''}{2000} = -2 \text{ and}$$

$$\left(1 + \frac{0.2}{7000}\right) v_1'' - v_2'' = 0$$

Solving, $v_1'' = -2373$ V. Adding our two components, $v_1 = -2373$ V.

Thus, $i_x = v_1/7000 = \boxed{339 \text{ mA}}$

12. We note that the proper label for the voltage source is 4 V.

With the current source open-circuited, we name the top of the 2 Ω resistor v_1, identify the current through it as i_1', and the voltage across the 3 Ω resistor as v'.Applying nodal analysis yields

$$0 = \frac{v_1 - 4}{7} + \frac{v_1}{2} + \frac{v_1 - v'}{1} \qquad \text{[1] and}$$

$$0.4 i_1' = \frac{v'}{3} + \frac{v' - v_1}{1} \qquad \text{[2] where } i_1' = \frac{v_1}{2}.$$

Solving, $\boxed{v' = 692.3 \text{ mV}}$, v_1 = 769.2 mV so i_1' = 384.6 mA

We next short circuit the voltage source, name the node at the top of the 2 Ω resistor v_2, the current through it i_1", and the voltage across the 3 Ω resistor v". Then,

$$-6 = \frac{v_2}{7} + \frac{v_2}{2} + \frac{v_2 - v''}{1} \qquad \text{[1] and}$$

$$6 + 0.4 i_1'' = \frac{v''}{3} + \frac{v'' - v_2}{1} \qquad \text{[2] where } i_1'' = \frac{v_2}{2}$$

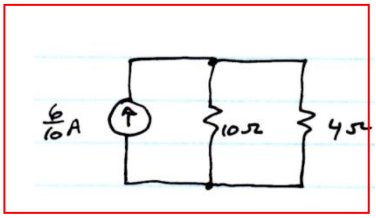
Solving, $\boxed{v'' = 2.783 \text{ V}}$, v_2 = –2.019V so i_2" = –1.001 A.

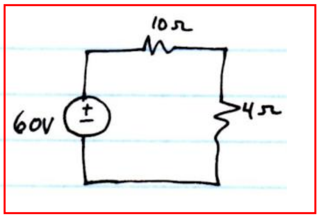
Thus, $v = v' + v''$ = 3.375 V

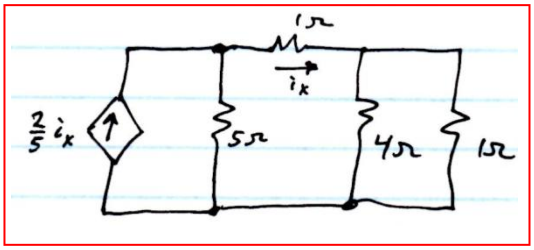
(b) $i_1 = i_1' + i_1''$ = –625 mA

and $P_{2\Omega} = 2(i_1)^2 = \boxed{781.3 \text{ mW}}$

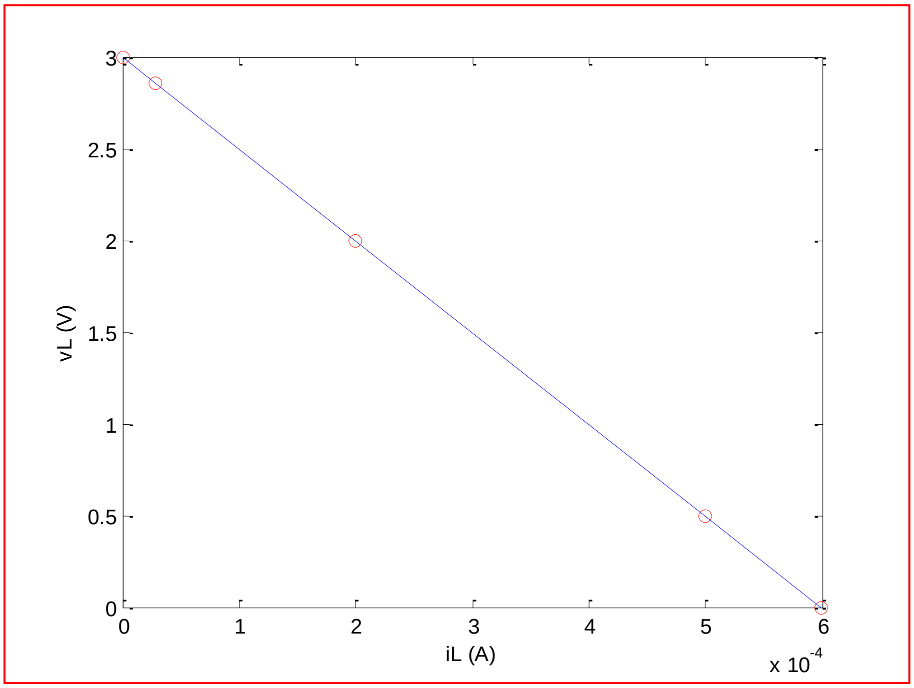
13.







14. $v_L = 3\frac{R}{R+5000}; \; i_L = \frac{3}{5000+R}$



15. We cannot involve the 5 Ω resistor in any transforms as we are interested in its current.

Hence, combine the 7 and 4 Ω to obtain 11 Ω; Transform to 9/11 A curent source in parallel with 11 Ω.

3 + 9/11 = 42/11 in parallel with 1 Ω.

No further simplification is advisible although 5 || 11 = 3.44 Ω. Hence,

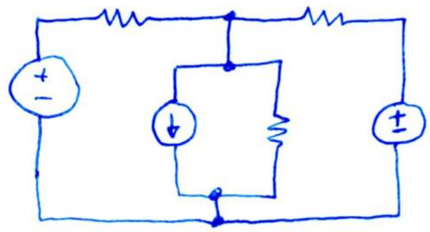
$V_{5\Omega}$ = (42/11)(3.44) = 13.13 V so I = 13.13/5 = $\boxed{2.63\text{ A}}$

16. For the circuit depicted in Fig. 5.22a, $i_{7\Omega} = (5 - 3)/7 = \boxed{285.7 \text{ mA.}}$

For the circuit depicted in Fig. 5.22c, $i_{7\Omega} = (5 - 3)/7 = \boxed{285.7 \text{ mA}}$

Thus, the power dissipated by this resistor is unchanged since it is proportional to $(i_{7\Omega})^2$.

17. (a) The transform available to us is clearer if we first redraw the circuit:



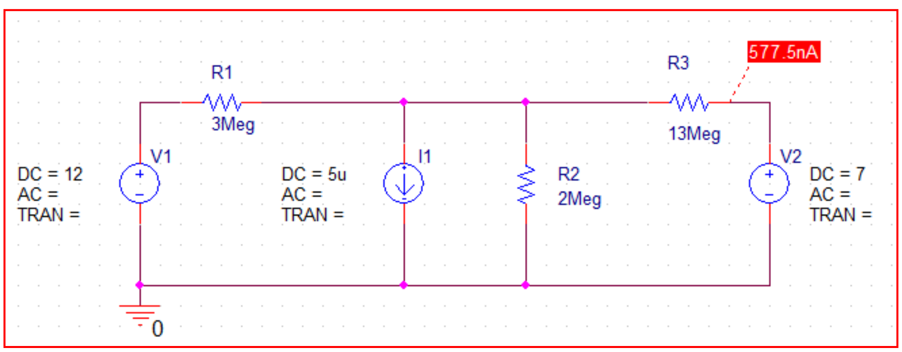
We can replace the current source / resistor parallel combination with a 10 V voltage source ("-" terminal at the top node) in series with a 2 M□ resistor. The circuit is easily analyzed with mesh analysis:

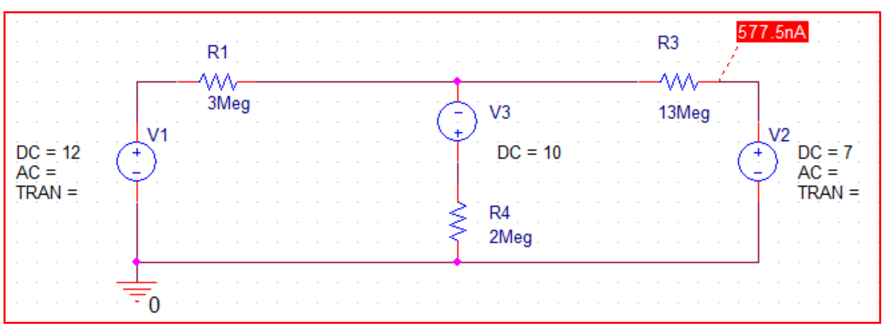
$$-22+5\times10^6 i_1 - 2\times10^6 i = 0 \quad [1]$$

$$17+15\times10^6 i - 2\times10^6 i_1 = 0 \quad [2]$$

Solving, $\boxed{i = -577.5 \text{ nA}}$

(b)





18. Perform the following steps in order:

Combine the 27 μA and 750 kΩ to obtain 20.25 V in series with 750 kΩ in series with 3.5 MΩ.

Convert this series combination to a 4.25 MΩ resistor in parallel with a 4.765 μA source, arrow up.

Convert the 15 V/ 1.2 MΩ series combination into a 12.5 μA source (arrow down) in parallel with 1.2 MΩ. This appears in parallel with the current source from above as well as the 7 MΩ and 6 MΩ.

Combine: 4.25 MΩ || 1.2 MΩ || 7 MΩ = 0.8254 MΩ. This, along with the -12.5 μA + 4.765 μA yield a -7.735 μA source (arrow up) in parallel with 825.4 kΩ in parallel with 6 MΩ.

Convert the current source and 825.4 kΩ resistor into a -6.284 V source in series with 825.4 kΩ and 6 MΩ.

Then, $P_{6M\Omega} = \left[\dfrac{-6.384}{6\times 10^6 + 825.4\times 10_3} \right] \left(6\times 10^6\right) = \boxed{5.249\ \mu\text{W}}$

19. (a) We combine the 1 Ω and 3 Ω resistors to obtain 0.75 Ω. The 2 A and 5 A current sources can be combined to yield a 3 A source.

These two elements can be source-transformed to a (9/4) V voltage source ("+" sign up) in series with a 0.75 Ω resistor in series with the 7 V source and the far-left 3 Ω resistor.

(b) In the original circuit, we define the top node of the current sources as v_1 and the bottom node is our reference node.

Then nodal analysis yields $(v_1 + 7)/3 + v_1/1 + v_1/3 = 5 – 2$

Solving, $v_1 = 2/5$ V and so the clockwise current flowing through the 7 V source is

$i = (-7 – v_1)/3 = -37/15$. Hence, $\boxed{P_{7V} = 17.27\text{ W}}$

Analyzing our transformed circuit, the clockwise current flowing through the 7 V source is (-7 – 9/4)/3.75 = -37/15 A.

Again, $\boxed{P_{7V} = 17.27\text{ W}}$.

20. (a) We start at the left, switching between voltage and current sources as we progressively combine resistors.

12/47 = 0.2553 A in parallel with 47 Ω and 22 Ω

477 || 22 = 14.99 Ω

Back to voltage source: (0.2553)(14.99) = 3.797 V in series with 14.99 Ω.

Combine with 10 Ω to obtain 24.99 Ω. Back to current source: 3.797/24.99 = 0.1519 A in parallel with 24.99 Ω and 7 Ω. 24.99 || 7 = 5.468 Ω

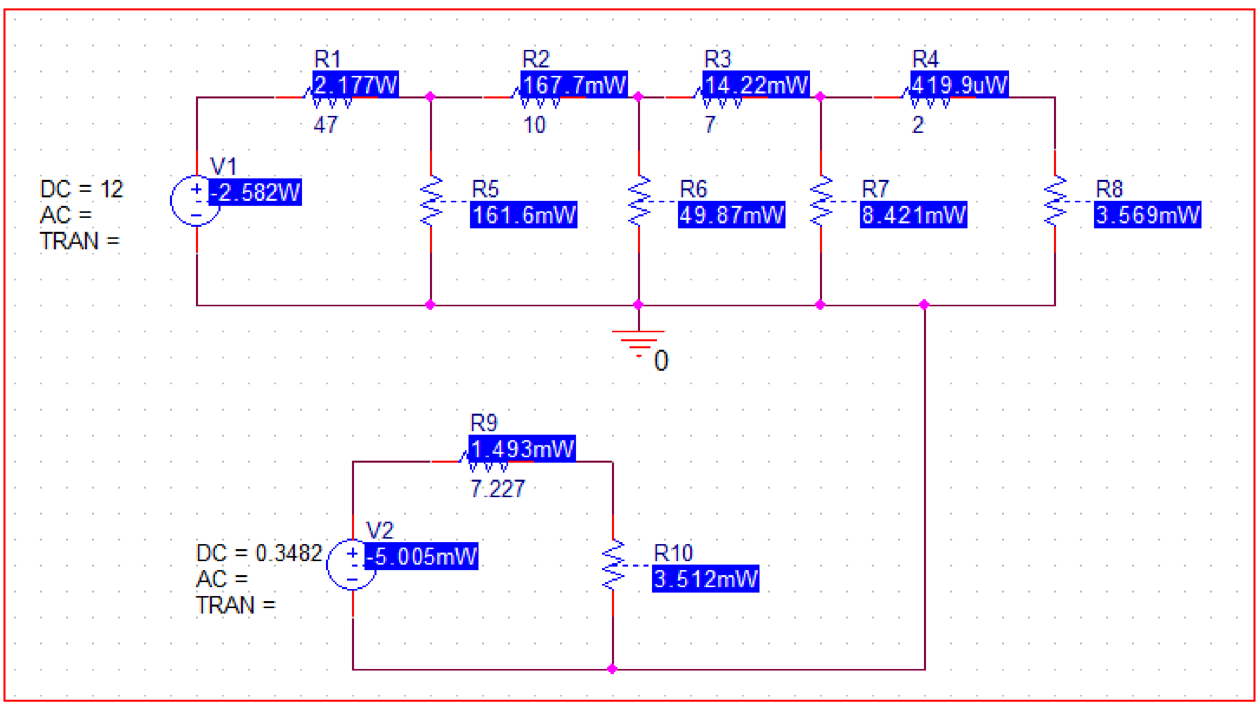
Back to voltage source: (0.1519)(5.468) = 0.8306 V in series with 5.468 Ω. Combine with next 7 Ω to obtain 12.468 Ω. Back to current source: 0.8306/12.468 = 0.06662 A in parallel with 12.468 Ω and 9 Ω.

12.468 ||9 = 5.227 Ω. Back to voltage source: (0.06662)(5.22) = 0.3482 V in series with 5.227 Ω. Combine with 2 Ω to yield 7.227 Ω.

We are left with a 0.3482 V source in series with 7.227 Ω and 17 Ω.

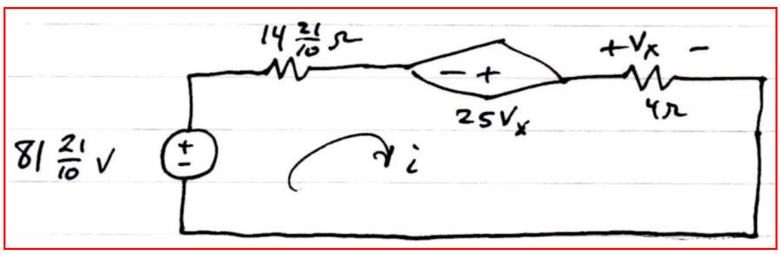
(b) Thus, I_x = 0.3482/(17 + 7.227) = 14.37 mA and $P_{17\Omega} = 17(I_x)^2 =$ $\boxed{3.510\text{ W}}$

(c) We note good agreement with our answer in part (b) but some rounding errors creep in due to the multiple source transformations.



21. We combine the 3 V source and 3 Ω resistor to obtain 1 A in parallel with 3 Ω in parallel with 7 Ω. Note that 3 || 7 = 21/10 Ω. We transform the 9 A current source into an 81 V source, "+" reference on the bottom, in series with 9 Ω.

We combine the two 10 Ω resistors in parallel (5 Ω) and the dependent current source to obtain a dependent voltage source, "+" reference on the right, controlled by 25 V_x. This source is in series with 14 Ω. Transforming the current source back to a voltage source allows us to combine the 81 V source with (1)(21/10) V to obtain $81\frac{21}{10}$ V. We are left with an independent voltage source $81\frac{21}{10}$ V in series with $14\frac{21}{10}$ Ω in series with the dependent voltage source, in series with the 4 Ω resistor:



Then defining a clockwise current i,

$$-81\frac{21}{10}+14\frac{21}{10}i-25V_x+4i=0$$

where

$$V_x=4i$$

Solving, i = –1.04 A so $\boxed{V_x = -4.160\text{ V}}$

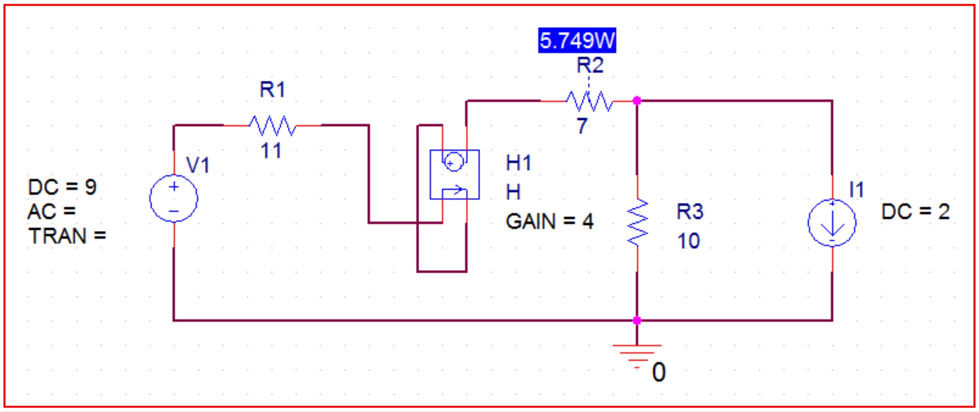
22. (a) Because the controlling current flows through the dependent source as well as the 7 Ω, we cannot transform the dependent voltage source into a dependent current source; doing so technically loses I_1.

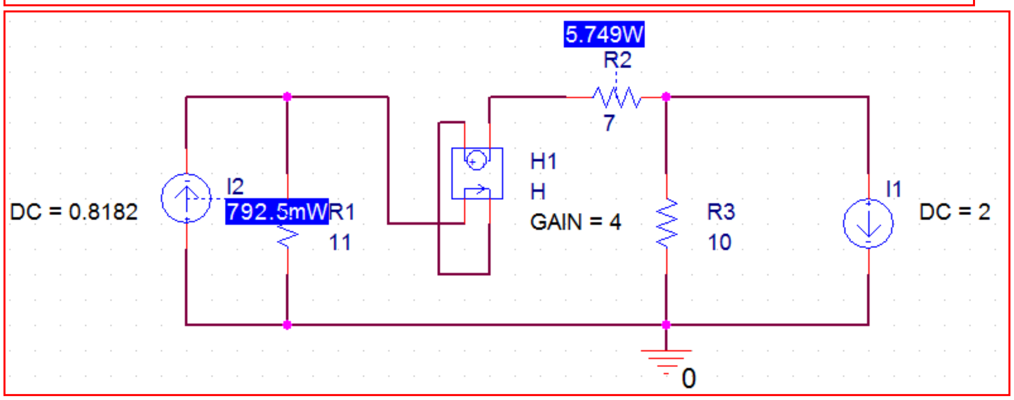
Thus, the only simplification is to replace the voltage source and 11 Ω resistor with a (9/11) A current source (arrow up) in parallel with an 11 Ω resistor.

(b) $28I_1 - (9/11)(11) - 10(2) + 4I_1 = 0$

Solving, $I_1 = 29/32$ A. Hence, $P_{7\Omega} = 7(I_1)^2 = \boxed{5.75 \text{ W}}$

(c)





23. The 2 Ω resistor and the bottom 6 Ω resistor can be neglected as no current flows through either. Hence, $V_1 = V_0$.

We transform the dependent current source into a dependent voltage source ("+" reference at the bottom) in series with a 6 Ω resistor.

Then, defining a clockwise mesh current i,

$+72V_1 – 0.7 + 13\,i = 0$

Also, $V_1 = 7i$ so i = 1.354 mA and $V_0 = V_1 = \boxed{9.478 \text{ mV}}$

24. The independent source may be replaced by a (2/6) A current source, arrow pointing up, in parallel with 6 Ω. The dependent voltage source may be replaced by a dependent current source (arrow pointing up) controlled by v_3. This is in turn in parallel with 2 Ω.

No simplification or reduction of components is really possible here.

Choose the bottom node as the reference noded. Name the top left node v_a and the top right node v_b. Then,

$$\frac{2}{6} - 4v_3 = \frac{v_a}{6} + \frac{v_a - v_b}{3}$$

and

$$v_3 = \frac{v_b - v_a}{3} + \frac{v_b}{2}$$

Since $v_3 = v_a - v_b$, we can solve to obtain $\boxed{v_3 = 67.42 \text{ mV}}$

25. (a) $V_{th} = 9(3/5) = 27/5 = \boxed{5.4\text{ V}}$

$R_{th} = 1 + 2||3 = \boxed{2.2\ \Omega}$

(b) By voltage division, $V_L = V_{th}\,(R_L/R_L + R_{th})$

So:

R_L	V_L
1 Ω	1.688 V
3.5 Ω	3.313 V
6.257 Ω	3.995 V
9.8 Ω	4.410 V

26. (a) Remove R_L; Short the 9 V source.

R_{th} seen looking into the terminals = 1 + 3||2 = $\boxed{2.2\ \Omega}$

V_{oc} = 9(3)(3 + 2) = $\boxed{5.4\text{ V} = V_{th}}$

(b) We have a voltage divider circuit so

$V_L = V_{th}(R_L)/(R_{th} + R_L)$

R_L (Ω)	V_L (V)
1	1.688
3.5	3.316
6.257	4.000
9.8	4.410

27. (a) We remove R_L and replace it with a short. The downward current through the short is then

$i_{sc} = 0.8/(2.5 + 0.8) = \boxed{242.4\text{ mA}}$

Returning to the original network, open circuit the current source, remove R_L. Looking into the open terminals we find $R_N = 2 \,\|\, (2.5 + 0.8) = \boxed{1.245\ \Omega}$

(b) $R_{TH} = R_N = \boxed{1.245\ \Omega}$

$V_{th} = (i_{sc})(R_N) = \boxed{301.8\text{ mV}}$

(c)

RL (Ω)	i_L(mA)
0	242.4
1	134.4
4.923	48.93
8.107	32.27

28. (a) 1.1 k || 2.3 k = 744 Ω; 2.5 k || 744 = 573.4 Ω

$$0 = \frac{v_1 - 4.2}{1800} + \frac{v_1}{2500} + \frac{v_1 - v_{oc}}{744} \quad [1]$$

$$0 = \frac{v_{oc}}{2500} + \frac{v_{oc} - v_1}{744} \quad [2]$$

Solving, $\boxed{v_{oc} = 1.423 \text{ V}} = v_{th}$

$$i_{sc} = \left(\frac{4.2}{1800}\right)\frac{744^{-1}}{1800^{-1} + 2500^{-1} + 744^{-1}} = \boxed{1.364 \text{ mA}}$$

Thus, $R_{th} = v_{oc}/i_{sc} = 1.04$ kΩ

(b) $P_{4.7k} = (4700)[v_{oc}/(R_{th} + 4700)]^2 = \boxed{289\ \mu\text{W}}$

29. (a) 1.1 k || 2.3 k = 744 Ω; 2.5 k || 744 = 573.4 Ω

$$0 = \frac{v_1 - 4.2}{1800} + \frac{v_1}{2500} + \frac{v_1 - v_{oc}}{744} \quad [1]$$

$$0 = \frac{v_{oc}}{2500} + \frac{v_{oc} - v_1}{744} \quad [2]$$

Solving, $\boxed{v_{oc} = 1.423\text{ V}}$ = v_{th}

$$i_{sc} = \left(\frac{4.2}{1800}\right)\frac{744^{-1}}{1800^{-1} + 2500^{-1} + 744^{-1}} = \boxed{1.364\text{ mA}}$$

Thus, $R_{th} = v_{oc}/i_{sc}$ = 1.04 kΩ

(b) $P_{1.7k}$ = $(1700)[v_{oc}/(R_{th} + 1700)]^2$ = $\boxed{459\ \mu\text{W}}$

30. (a) Define three clockwise mesh currents i_1, i_2 and i_3, respectively in the three meshes, beginning on the left. Short the opn terminals together. Then, create a supermesh:

$$-0.7 + 45i_1 + 122i_1 - 122i_2 = 0 \quad [1]$$

$$-122i_1 + (122 + 75)i_2 + 220i_3 = 0 \qquad [2]$$

$$i_3 - i_2 = 0.3 \qquad [3]$$

Solving, $i_{sc} = i_3 = 100.3$ mA

Short the voltage source, open circuit the current source, and look into the open terminals:

$R_{th} = 220 + 75 + 45||122 = \boxed{328\ \Omega}$

Thus, $V_{th} = R_{th}(i_{sc}) = \boxed{32.8\text{ V}}$

(b) $P_{100\Omega} = (100)[V_{th}/(100 + R_{th})]^2 = \boxed{587.3\text{ mW}}$

(c) Only the second mesh equations needs to be modified:

$$-122i_1 + (122 + 75)i_2 + 220i_3 + 100i_3 = 0 \qquad [2']$$

Solving, $i_3 = 76.83$ mA and so $P_{100\Omega} = (100)(i_3)^2 = \boxed{590\text{ mW}}$

31. Select the top of the R_4 resistor as the reference node. v_1 is at the top of R_5, v_2 is at the "+" of v_{oc} and v_3 is at the "-" of v_{oc}. The bottom node is the negative reference of v_{oc}.

Then $i_1 = \frac{v_1}{R_2} + \frac{v_1 - v_3}{R_5} + \frac{v_1 - v_2}{R_3}$ [1]

$0 = \frac{v_2 - v_1}{R_3} + \frac{v_2}{R_1}$ [2]

$0 = \frac{v_3 - v_1}{R_5} + \frac{v_3}{R_4}$ [3] Solving,

$$v_{th} = v_{oc} = v_2 - v_3 = \boxed{\frac{R_2(R_1R_5 - R_3R_4)i_1}{R_1R_2 + R_1R_4 + R_2R_3 + R_1R_5 + R_2R_4 + R_2R_5 + R_3R_4 + R_3R_5}}$$

Next, short the open terminals and define four clockwise mesh currents i_1, i_2, i_3, and i_4. i_1 is the top mesh, i_3 is the bottom left mesh, i_4 is the bottom right mesh, and i_2 is the remaining mesh. Then

$-R_2i_2 + (R_2 + R_4 + R_5)i_3 - R_5i_4 = 0$ [1]

$-R_3i_1 - R_5i_3 + (R_3 + R_5)i_4 = 0$ [2]

$-R_2i_3 + R_2i_2 + R_1i_1 + R_3i_1 - R_3i_4 = 0$ [3] and $i_2 - i_1 = i_x$ [4]

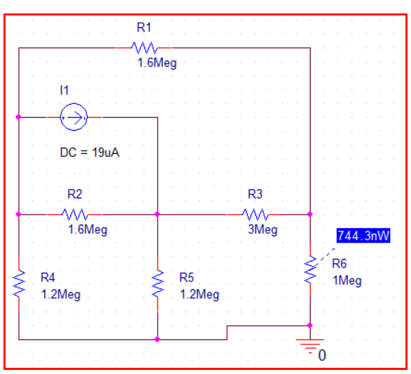
Solving, $i_{sc} = i_4 =$

$$\frac{R_2\left(R_1R_5 - R_3R_4\right)}{R_1R_2R_3 + R_1R_2R_5 + R_1R_3R_4 + R_1R_3R_5 + R_2R_3R_4 + R_1R_4R_5 + R_2R_4R_5 + R_3R_4R_5}i_1$$

Then, the ratio of v_{th} and i_{sc} yields R_{th}:

$$\boxed{\frac{R_1R_2R_3 + R_1R_2R_5 + R_1R_3R_4 + R_1R_3R_5 + R_2R_3R_4 + R_1R_4R_5 + R_2R_4R_5 + R_3R_4R_5}{R_1R_2 + R_1R_4 + R_2R_3 + R_1R_5 + R_2R_4 + R_2R_5 + R_3R_4 + R_3R_5}}$$

(b) $V_{oc} =$ –2.2964 V; R_{th} = 1.66 MΩ. Hence $P_{1M\Omega} = \left[\frac{-2.296}{1.66\times10^6 + 10^6}\right]^2 \left(10^6\right) \boxed{= 745 \text{ nW}}$

(c) 

32. Define three clockwise mesh currents i_1, i_2, i_3 starting on the left.
Then

$2 + 2i_1 + 6i_2 - i_3 = 0$ [1]

$-i_2 + 4i_3 + 4 = 0$ [2]

$i_2 - i_1 = 2$ [3]

Solving,
i_2 = 129 mA. Hence, $v_{oc} = v_x = 5i_2 = \boxed{645.2 \text{ mV}}$

Next, short the voltage sources and open circuit the current source. Then,
$R_{th} = 5 \parallel (2 + 3) = \boxed{2.5\ \Omega}$

33. (a) Employ nodal analysis:

$$2 = \frac{v_1 + 2}{2} + \frac{v_1 - v_2}{5} \qquad [1]$$

$$0 = \frac{v_2 - v_1}{5} + v_2 + \frac{v_2 - 4}{3} \qquad [2]$$

Solving, v_1 = 54/31 V and v_2 = 34/31 V. Thus, $V_{TH} = v_X = v_1 - v_2$ = 645.2 mV

By inspection, R_{TH} = 5 || [2 + 1||3] = 1.774 Ω

Thus, $i_N = v_{TH}/R_{TH}$ = $\boxed{363.7 \text{ mA}}$ and $R_N = R_{TH}$ = $\boxed{1.774\ \Omega}$

(b) i_{load} = (0.3637)(1.774)/(5 + 1.74) = 95.25 mA

$P_{load} = 5(i_{load})^2$ = $\boxed{45.36 \text{ mW}}$

(c) $\boxed{363.7 \text{ mA}}$

34. (a) Define three clockwise mesh currents i_1, i_2, i_3, respectively, starting on the left, in addition to i_{sc} which flows through the shorted leads once R_L is removed.

By inspection, i_1 = 0.3 A, and $i_{sc} = i_3$ since the 6 kΩ resistor is shorted here.

Then, $-7000(0.3) + (12\times10^3)i_2 = 0$ [1]

$-2.5 + 1000i_3 - 1000i_2 = 0$ [2]

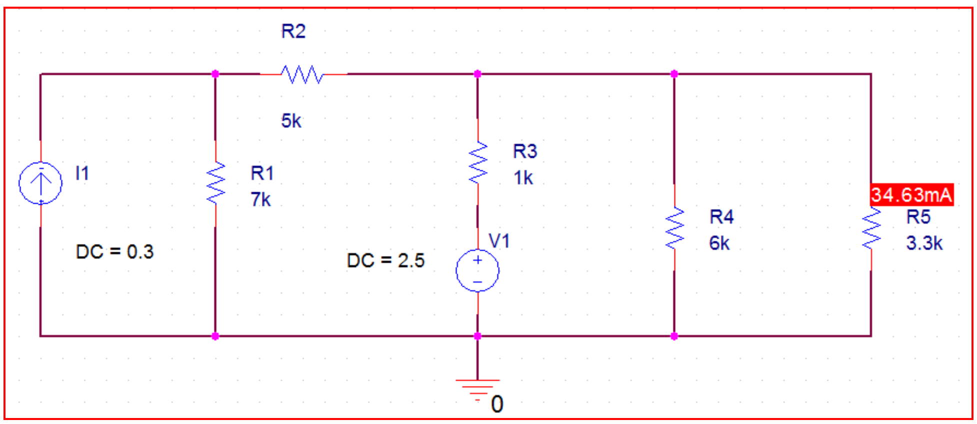
Solving, $i_{sc} = i_3 = \boxed{177.5 \text{ mA}}$

Looking into the open terminals with the sources zeroed,

R_{th} = 6000 || 10000 || 120000 = $\boxed{800\ \Omega}$

(b) $i_{R_L} = i_{sc}\dfrac{R_{TH}}{R_L + R_{TH}} = \boxed{34.63 \text{ mA}}$

(c)



35. (a) We select the bottom node as the reference node. The top left node is then –2 V by inspection; the next node is named v_1, the next v_2, and the far right node is v_{oc}.

$$0 = \frac{v_1 + 2}{10} + \frac{v_1}{7} + \frac{v_1 - v_2}{20} \qquad [1]$$

$$0 = \frac{v_2 - v_1}{20} + \frac{v_2}{7} \qquad [2]$$

Solving,

$$v_2 = v_{oc} = -185.3 \text{ mV}$$

Next, we short the output terminals and compute the short circuit current. Naming the three clockwise mesh currents i_1, i_2 and i_{sc}, respectively, beginning at the left,

$$2 + 17i_1 - 7i_2 = 0 \qquad [1]$$

$$-7i_1 + 34i_2 - 7i_{sc} = 0 \qquad [2]$$

$$-7i_2 + 37i_{sc} = 0 \qquad [3]$$

Solving, isc = -5.2295 mA.

Hence

$$R_{TH} = \frac{v_{oc}}{i_{sc}} = \boxed{35.43\ \Omega}$$

(b) Connecting a 1 A source to the dead network, we can simplify by inspection, but performing nodal analysis anyway:

$$0 = \frac{v_1}{10} + \frac{v_1}{7} + \frac{v_1 - v_2}{20} \qquad [1]$$

$$0 = \frac{v_2 - v_1}{20} + \frac{v_2}{7} + \frac{v_2 - v_{test}}{30} \qquad [2]$$

$$1 = \frac{v_{test} - v_2}{30} \qquad [3]$$

Solving, v_{test} = 35.43 V hence R_{TH} = 35.43/1 = $\boxed{35.43\ \Omega}$

(c) Connecting a 1 A source, we can write three mesh equations after defining clockwise mesh currents:

$$0 = 17i_1 - 7i_2 \qquad [1]$$

$$0 = -7i_1 + 34i_2 - 7i_3 \qquad [2]$$

$$-1 = -7i_2 + 37i_3 \qquad [3]$$

Solving,

i_3 = –28.23 mA. Thus, $R_{TH} = 1/(-i_3)$ = $\boxed{35.42\ \Omega}$

36. (a) We can ignore the 3 Ω resistor to determine v_{oc}. Then, $i_{4\Omega} = (1)(2)/(2 + 5) = 2/7$ A. Hence, $v_{oc} = 4i_{4\Omega} = 1.143$ V

i_{sc}: A source transformation is helpful here, yielding 2 V in series with 2 Ω. Then noting that $3 \| 4 = 1.714\ \Omega$, $V_{3\Omega} = 2(1.714)/(3 + 1.714) = 0.7272$ V
Hence,
$i_{sc} = v_{3\Omega}/3 = 242.4$ mA

Consequently, $R_{TH} = v_{oc}/i_{sc} = \boxed{4.715\ \Omega}$

(b) Connect the 1 A source as instructed to the dead network, and define v_x across the source. Then $v_x = (1)(3 + 4 \|3) = 4.714$ V.
Hence,

$$R_{TH} = \boxed{4.714\ \Omega}$$

(c) Connect the 1 V source to the dead network as instructed, and define i_x flowing out of the source. Then, $i_x = [3 + 3 \|4]^{-1} = 1/4.714$ A.

Consequently, $R_{TH} = 1/i_x = \boxed{4.714\ \Omega}$

37. (a) With the terminals open-circuited, we select the bottom node as our reference and assign nodal voltages v_1, v_2, and v_3 to the top nodes, respectively beginning at the left. Then,

$$222 = \frac{v_1}{6} + \frac{v_1 - v_2}{17} \qquad [1]$$

By inspection, $v_2 = 20$ V [2]

$$-33 = \frac{v_3 - v_2}{9} + \frac{v_3}{4} + \frac{v_3}{2} \qquad [3]$$

Solving,

$v_{oc} = v_3 = -35.74$ V

Next, we short the output terminals and compute isc, the downward flowing current: KCL requires that $i_{sc} = 20/9 - 33 = -30.78$ A

Hence, $R_{TH} = v_{oc}/i_{sc} = \boxed{1.161\ \Omega}$

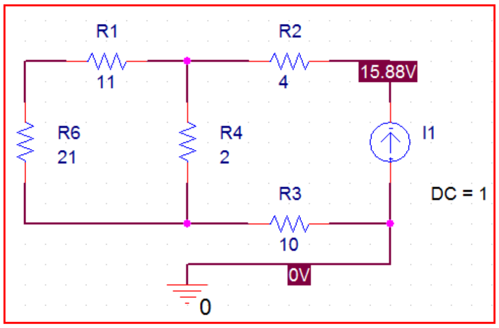
(b) By inspection, $R_{TH} = 2 \| 4 \| 9 = \boxed{1.161\ \Omega}$

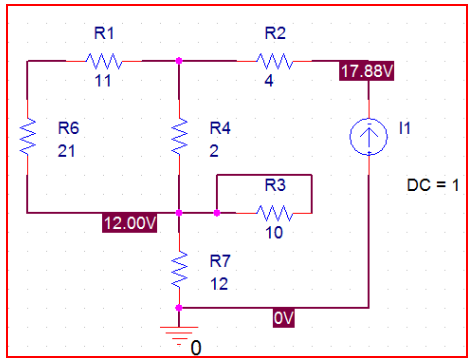
38. (a) Between terminals *a* and *b*, $R_{TH} = 4 + (11 + 21) \parallel 2 + 10 = \boxed{15.88\ \Omega}$

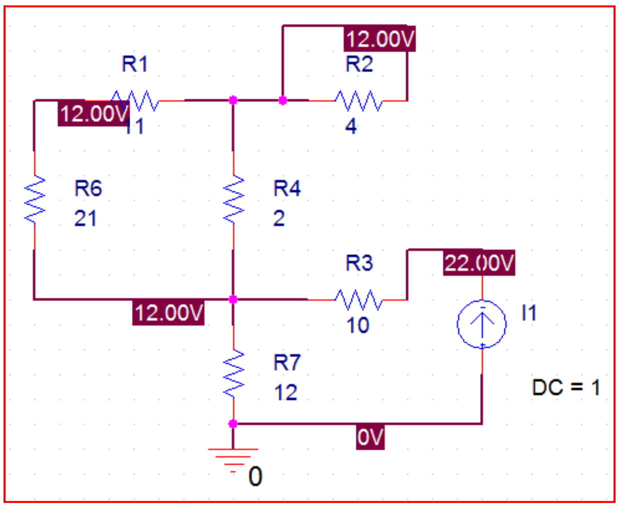
(b) Between terminals *a* and *c*, $R_{TH} = 4 + (11 + 21) \parallel 2 + 12 = \boxed{17.88\ \Omega}$

(c) Between terminals *b* and *c*, $R_{TH} = 10 + 12 = \boxed{22\ \Omega}$

(d) Note that the magnitude of R_{TH} is the same as that of the voltage across the 1 A source.







39. We connect a 1 A source across the open terminals of the dead network, and compute the voltage v_x which develops across the source.

By nodal analysis, $1 + 10v_x = v_x/21$. Solving, $v_x = 0.1005$ V.
Hence,
$R_{TH} = v_x/1 = \boxed{-100.5 \text{ m}\Omega}$

(the dependent source helps us achieve what appears to be a negative resistance!)

40. We short the terminals of the network and compute the short circuit current. To do this, define two clockwise mesh currents (the 1500 Ω resistor is shorted out).

$-500i_x + 2i_x + 2500i_2 = 0$ [1]

$-i_2 - i_x = 0.7$ [2]

Solving, $i_2 = i_{sc} = i_N = \boxed{-116.3 \text{ mA}}$

Next, we zero out the independent source and connect a 1 A test source across the terminals *a* and *b*. Define v_x across the current source with the "+" reference at the arrow head of the current source. Then,

$1 = \frac{v_x}{1500} + \frac{v_1}{3000}$ [1]

$v_1 - v_x = 2i_x$ [2]

$\frac{v_1}{3000} = i_x$ [3]

Solving, $v_x = 998.8$ V, so $R_{TH} = v_x/1 = \boxed{999.8\ \Omega}$

41. We define nodal voltage v_1 at the top left node, and nodal voltage v_2 at the top right node. The bottom node is our reference node. By nodal analysis,

$$-0.02v_1 = \frac{v_1}{10\times10^3} + \frac{v_2}{20\times10^3} \quad [1]$$

and $v_2 - v_1 = 1$ [2]

Solving,

$v_1 = -2.481$ mV $= v_{oc} = v_{TH}$

Next, short the 1 V independent source and connect a 1 A source across the open terminals. Define v_{test} across the source with the "+" reference at the arrow head of the source.

Then

$$1 - 0.02v_1 = \frac{v_1}{10\times10^3} + \frac{v_1}{20\times10^3} \quad [1]$$

$$v_1 = v_{test} = 49.63 \text{ V}$$

Hence,

$R_{TH} = v_{test}/1 = 49.63\ \Omega$

$P_{R_L} = \left(\dfrac{-2.481\times10^{-3}}{R_L + 49.63}\right)^2 R_L$. Plugging in resistor values,

(a) $\boxed{5.587 \text{ nW}}$

(b) $\boxed{1.282 \text{ nW}}$

(c) $\boxed{578.5 \text{ pW}}$

42. Connect a 1 A source across the open terminals with the arrow pointing into terminal *a*. Next define vab with the "+" reference at terminal *a* and the "-" reference at terminal *b*. Define three clockwise mesh currents i_1, i_2, and i_3, respectively, beginning with the leftmost mesh.

By inspection, $i_3 = -1$ A [1]

Also by inspection, $i_1 = 0.11 v_{ab}$ [2]

Then,

$-11i_1 + 32i_2 + 0.5v_{ab} + 15i_2 - 15i_3 = 0$ [3]

and

$v_{ab} = 15(i_2 - i_3)$ [4]

Solving, i_2 = -0.1197 A

Hence, v_{ab} = 13.20 V and $R_{TH} = v_{ab}/1 = \boxed{13.20\ \Omega}$

Since there is no independent source in the network, this represents both the Thévenin and Norton equivalent.

43. Connect a 1 A source to the open terminals, and select the bottom node as the reference terminal. Define v_1 at the top of the 1 A source. Then

$1 = v_1/10^6$ so $v_1 = 10^6$ V and $R_{TH} = v_1/1 = \boxed{1\ \text{M}\Omega}$

44. Disconnect the two elements left of the dashed line. Then apply a 1 A test source to the open terminals and define v_x across the 1 A source such that the "+" reference is at the arrow head of the source. By nodal analysis,

$$1 = \frac{v_x}{2\times 10^6} + \frac{v_x - v_2}{r_\pi} \qquad [1]$$

$$0.02 v_\pi = \frac{v_2 - v_x}{r_\pi} + \frac{v_2}{1000} + \frac{v_2}{2000} \qquad [2] \text{ and}$$

$$v_\pi = v_x - v_2 \qquad [3]$$

Solving,

$$v_x = \frac{2\times 10^6 (43 r_\pi + 2000)}{43 r_\pi + 6.002\times 10^6}. \text{ Hence, } R_{TH} = \frac{v_x}{1} = \boxed{\frac{2\times 10^6 (43 r_\pi + 2000)}{43 r_\pi + 6.002\times 10^6}}$$

45. First, we determine v_{TH} by employing nodal analysis:

$$0 = \frac{-v_d - v_{in}}{R_1} + \frac{-v_d}{R_i} + \frac{-v_d - v_{out}}{R_f} \qquad [1]$$

$$0 = \frac{v_{out} - Av_d}{R_o} + \frac{v_{out} + v_d}{R_f} \qquad [2]$$

Solving,

$$\boxed{v_{out} = \frac{\left(R_o - AR_f\right)R_i}{R_1R_f + R_1R_i + R_1R_o + R_fR_i + R_iR_o + AR_1R_i}v_{in}}$$

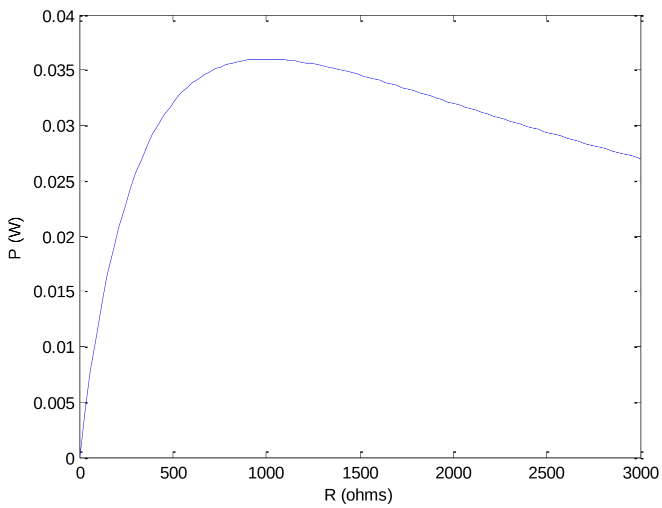
We now find R_{TH} by injecting 1 A of current into the dead network and determining the voltage which develops:

$$0 = \frac{-v_d}{R_1} + \frac{-v_d - v_{out}}{R_f} - \frac{v_d}{R_i} \qquad [1] \text{ and}$$

$$1 = \frac{v_{out} + v_d}{R_f} + \frac{v_{out} - Av_d}{R_o} \qquad [2]$$

Solving, $R_{TH} = \frac{v_{out}}{1} = v_{out} = \boxed{\frac{R_o(R_iR_f + R_1R_f + R_1R_i)}{R_iR_o + R_1R_o + R_iR_f + R_1R_f + R_1R_i + AR_1R_i}}$

46. (a) $P_R = \left[\dfrac{12}{R_s + R}\right]^2 R = \dfrac{144R}{(R_s+R)^2} = \left(\dfrac{144}{1000}\right)\left(\dfrac{R}{1000}\right)\left(\dfrac{1}{1+\dfrac{R}{1000}}\right)^2$



(b) $\dfrac{dP_R}{d\left(\dfrac{R}{R_s}\right)} = \dfrac{144}{R_s}\left[\dfrac{\left(1+\dfrac{R}{R_s}\right)^2\left(2\dfrac{R}{R_s}\right) - 2\left(1+\dfrac{R}{R_s}\right)\left(\dfrac{R}{R_s}\right)^2}{\left(1+\dfrac{R}{R_s}\right)^4}\right]$

We see from the graph that maximum power is transferred when R = 1000 Ω = R_s.

47. (a) Define a clockwise mesh current i. Then $5i + 4 + 2 = 0$ and $i = –6/5$ A

$v_{out} = 2 + 2i = 2 – 12/5 = \boxed{–400 \text{ mV}}$

By inspection of the dead network, $R_{TH} = 2 \parallel 3 = \boxed{1.2\ \Omega}$

(b) Choose $R_{out} = R_{TH} = \boxed{1.2\ \Omega}$

48. (a) A quick source transformation and we have all voltage sources. Then, remove R_{out} and short the open terminals. Mesh analysis yields

$$-4 + 1000i_1 + 2000i_1 + 2 - 2000i_2 = 0 \quad [1]$$
$$-2000i_1 + 2000i_2 - 2 + 3 = 0 \quad [2]$$

Solving,

$$i_N = i_2 = \boxed{500\ \mu\text{A}}$$

Next, zero out all sources, remove R_{out}, and look into the open terminals.

R_{TH} = 1000 || 2000 = $\boxed{667\ \Omega}$

(b) Maximum power is obtained for $R_{out} = R_{TH} = \boxed{667\ \Omega}$

49. Yes, it would theoretically result in maximum power transfer. Since we're charged for the energy we use (power multiplied by time), this would cost the consumer a fortune. In reality, we don't want all the power the utility can provide – only the amount we need!

50. We need only R_{TH}. Setting all sources to zero, removing R_L, and looking into the terminals,

$$R_{TH} = 5 \parallel 2 \parallel 3 = 967.7 \text{ m}\Omega$$

$\boxed{\text{Setting } R_L = R_{TH} = 967.7 \text{ m}\Omega \text{ achieves maximum power delivery.}}$

51. We first perform two source transformations such that a voltage source with value $9R_s$ appears in series with a 6 V source, R_s and a 3 Ω resistor.

(a) $P_{9\Omega} = 9\left(\dfrac{9R_s + 6}{3 + R_s + 9}\right)^2 = \boxed{81\text{ W}}$

(b) If $3 + R_s = 9\ \Omega$, maximum power is delivered.
Hence,
$\boxed{R_s = 6\ \Omega \text{ and } P_{9\Omega} = 361\text{ W}}$

52. (a) Define clockwise mesh current i. Then

$-0.1v_2 + 2i - 5 + 7i + 3.3i = 0$ where $v_2 = 3.3i$

Hence,

$$-0.1(3.3i) + 12.3i = 5$$

Solving,

i = 417.7 mA and so v_2 = 1.378 V = v_{TH}

Consequently, P = $(v_2)^2/3.3$ = $\boxed{575.4 \text{ mW}}$

(b) Find R_{TH} by connecting a 1 A source across the open terminals.

$1 = \dfrac{v_2 - 0.1v_2}{9}$. Solving, v_2 = 10 V.

Thus, $R_{TH} = v_2/1 = 10\ \Omega$

$\boxed{\text{Hence, replace the 3.3 } \Omega \text{ resistor in the original circuit with 10 } \Omega.}$

53. We connect a 1 A source across the open terminals and define v_{test} across the source such that its "+" reference corresponds to the head of the current source arrow. Then, after defining nodal voltages v_1 and v_2 at the top left and top right nodes, respectively,

$1 = \frac{v_1}{5} + \frac{v_1}{8}$ and so v_1 = 3.077 V

$-1 + 0.2v_1 = \frac{v_2}{10}$ so v_2 = -3.846 V

By KVL, $v_{test} = v_1 - v_2$ = 6.923 V so $\boxed{R_{TH} = 6.923\ \Omega.}$ We select this value for R_L to ensure maximum power transfer.

54. We zero out the current source and connect 1 A to terminals *a* and *b*. Define clockwise mesh currents i_1, i_2, i_3 and i_4 left to right, respectively.

By inspection $i_4 = -1$ A

Then

$100i_1 + 50i_2 - 50i_3 = 0$ [1]

$-50i_2 + 80i_3 - 10(-1) = 0$ [2]

and $i_2 - i_1 = 0.1v_{ab}$ [3]

and $v_{ab} = 10(i_3 + 1)$ [4]

Solving, v_{ab} = 17.78 V. Hence, $R_{TH} = v_{ab}/1$ = 17.78 Ω.

Select 17.78 Ω then to obtain maximum power transfer.

55. We note that the equations which describe the two equivalent circuits are already developed and provided as Eqs. 23-24 and Eqs. 25-26, respectively. Equating terms most directly results in the equations for R_1, R_2 and R_3.

The next step is to divide those equations to find the following ratios:

$$\frac{R_1}{R_2} = \frac{R_A}{R_C}; \ \frac{R_1}{R_3} = \frac{R_B}{R_C} \text{ and } \frac{R_2}{R_3} = \frac{R_B}{R_A}.$$

These three equations yield two equations for RA, two for RB and two for RC, which may be equated (respectively) to obtain:

$$\frac{R_3}{R_2} R_B - \frac{R_1}{R_2} R_C = 0$$

$$\frac{R_2}{R_3} R_A - \frac{R_1}{R_3} R_C = 0$$

$$\frac{R_2}{R_1} R_A - \frac{R_3}{R_1} R_B = 0$$

Solving, we find that

$$\boxed{R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}, \ R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}, \ R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}}$$

56. For the first circuit, we compute $\sum = 33 + 17 + 22 = 71\ \Omega$.

Then,

$R_1 = (33)(17)/\Sigma = 7.901\ \Omega$
$R_2 = (17)(21)/\Sigma = 5.028\ \Omega$
$R_3 = (21)(33)/\Sigma = 9.761\ \Omega$

For the second circuit, $\sum = 1.1 + 4.7 + 2.1 = 7.9\ \text{k}\Omega$

Then,

$R_1 = (1.1)(4.7)/7.9 = 654.4\ \Omega$
$R_2 = (4.7)(2.1)/7.9 = 1.249\ \text{k}\Omega$
$R_3 = (21.)(1.1)/7.9 = 292.4\ \Omega$

57. Left: $\boxed{R_A = 76.71\ \Omega;\ R_B = 94.76\ \Omega;\ R_C = 48.82\ \Omega}$

Right: $\boxed{R_A = 8.586\ \text{k}\Omega;\ R_B = 3.836\ \text{k}\Omega;\ R_C = 15.03\ \text{k}\Omega}$

58. We begin by converting the bottom section to a T network (R_1, R_2, R_3)

$\Sigma = 2 + 3 + R = 5 + R$
$R_1 = 2R/\Sigma = 2R/(5 + R)$
$R_2 = 3R/\Sigma = 3R/(5 + R)$
$R_3 = (3)(2)/\Sigma = 6/(5 + R)$

We now have 30 Ω in series with R_1, 10 Ω in series with R_2. Those branches are in parallel. The total is in series with R_3.
The new network then is equivalent to

$$(30 + R_1) \parallel (10 + R_2) + R_3 = \left(30+\frac{2R}{5+R}\right)\|\left(10+\frac{3R}{5+R}\right)+\frac{6}{5+R}$$

$$= \frac{300(5+R)+110R+\dfrac{6R^2}{5+R}}{40(5+R)+5R}+\frac{6}{5+R}=9$$

Solving, $\boxed{R = 5.5\ \Omega \text{ (rounded)}}$

59. $R_A = 42\ \Omega$; $R_B = 200\ \Omega$; $R_C = 68\ \Omega$.
Then $R_1 = 27.10\ \Omega$, $R_2 = 43.87\ \Omega$, and $R_3 = 9.213\ \Omega$

The new network is then $(100 + 27.1) \parallel (R + 43.87) + 9.213 = 70.6\ \Omega$

Solving, $\boxed{R = 74.86\ \Omega}$

60. Define $R_x = R \parallel R_B$

$R_A = R_B = R_C = 3R^2/R = 3R$

Thus, $R_x = (3R)(R)/(3R + R) = (3/4)R$

Define $R_{11} = (3R)(0.75R)/(3R + 0.75R + 3R) = R/3$

$R_{22} = (0.75R)(3R)/(3R + 0.75R + 3R) = R/3$

$R_{33} = (3R)(3R)/(3R + 0.75R + 3R) = 4R/3$

Combine series resistances then define

$R_{AA} = (R^2 + 4R^23 + 4R^2/3) = 11R/3$

$R_{BB} = (R^2 + 4R^2/3 + 4R^2/3) = 11R/4$

$R_{CC} = (R^2 + 4R^2/3 + 4R^2/3)/R = 11R/3$

The equivalent resistance is then $2[(4R/3) \parallel (11R/3)] \parallel (11R/4) = \boxed{2514\ \Omega}$

61. We can neglect the 110 Ω resistor (no current flow).

Identify the 11, 23 and 31 Ω as R1, R2, R3 and convert to a Δ network with
RA1 = 56.83 Ω
RB1 = 42.16 Ω
RC1 = 118.82 Ω

Identify the 55, 46, and 61 Ω as R1, R2, R3 and convert to a Δ network with
RA2 = 188.93 Ω
RB2 = 142.48 Ω
RC2 = 158.02 Ω

Then RC1 || 31 = 24.59 Ω
RB1 || RB2 = 32.53 Ω
25 || RA2 = 22.08 Ω

We are now left with a network identical to that in FIg. 5.46, in parallel with the 63 Ω resistor. Converting the upper network to a Y network and simplifying, we obtain $\boxed{R_{TH} = 25.68\ \Omega.}$

62. We begin by noting that (6 + 12) || 20 = 9.474 Ω.

Define Σ = R1R2 + R2R3 + R3R1 = (9)(5) + (5)(6) + (6)(0) = 129

Then RA = Σ/R2 = 25.8 Ω
RB = Σ/R3 = 21.5 Ω
RC = Σ/R1 = 14.3 Ω

After this conversion, we have RC || 4 = 3.126 Ω, RA || 3 = 2.688 Ω.

Now define Σ = RAA + RBB + RCC = 2.688 + 21.5 + 2.126 = 27.31 Ω.
Then R11 = 2.116, R22 = 2.461 and R33 = 0.0377. This last resistance appears in series with 10 Ω.

Performing one last conversion,

Define Σ = (2.116)((2.461) + (2.461)(10.31) + (10.31)(2.116) = 52.40
Ra = Σ/2.461 = 21.29 Ω
Rb = Σ/10.31 = 5.082 Ω
Rc = Σ/2.116 = 24.76 Ω

By inspection,

R_{Th} = Rb || [(Rc||7) + (Ra||9.474)] = $\boxed{3.57\ \Omega}$

63. (a) $\boxed{V_{oc} = -606.7 \text{ mV} \quad R_{Th} = 6.098 \text{ W}}$

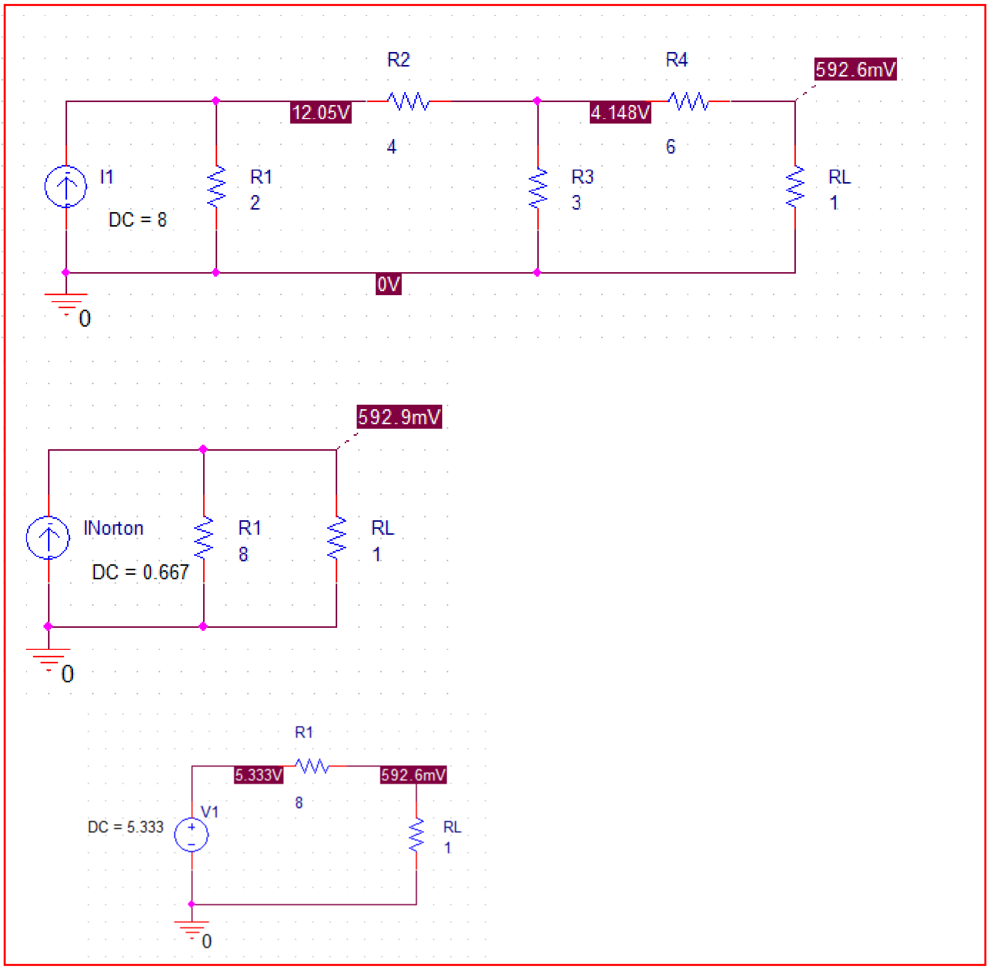
(b) $P_{1\Omega} = [0.6067/7.098]^2 (1) = \boxed{7.306 \text{ mW}}$

64. (a) By inspection, $R_{Th} = 6 + 6\|3 = 8\ \Omega$

Then, $V_{oc} = 16(3)/(6 + 3) = 5.333$ V
$I_N = V_{oc}/R_{Th} = 5.333/8 = 666.7$ mA

Thus, the Thévenin equivalent is 5.333 V in series with 8 Ω and the Norton equivalent is 666.7 mA in parallel with 8 Ω

(b) The three simulations agree within acceptable rounding error.



65. (a) Although this network may be simplified, it is not possible to replace it with a three-resistor equivalent.

(b) See (a).

66. The wording points to the need for a Thévenin (Norton) equivalent. Simplifying using Δ-Π conversion, note 1 k || 7 k = 875 Ω; 10 k || 2.2 k = 1.803 kΩ

R1 = (10)(4)/19 = 2.105 kΩ
R2 = (4)(5)/19 = 1.053 kΩ
R3 = (5)(10)/19 = 2.632 kΩ

By inspection, R_{Th} = R3 + (R1 + 875) || (R2 + 1803) = 4090 Ω

V_{Th} = (3.5)(R2 + 1803)/ (R1 + 875 + R2 + 1803) = 1.713 V

Then, $P_{abs} = \left(\frac{V_{Th}}{R_{Th}+R}\right)^2 \frac{1}{R}$

(a) $\boxed{\text{175 nW}}$

(b) $\boxed{\text{1.67 nW}}$

(c) $\boxed{\text{24.38 pW}}$

(d) $\boxed{\text{2.02 aW}}$

67. (a) $\boxed{\text{Change the 25 V source to 10 V, then the two legs are identical.}}$

(b) By KVL, $-10 + 15i + 15i + 10 = 0$

Solving, $i = 0$ Therefore $V_{th} = 10$ V.

By inspection, $\boxed{R_{TH} = 7.5\ \Omega}$

68. As constructed, we may find the power delivered to the 2.57 Ω resistor by first employing mesh analysis. Define clockwise mesh currents i1 on the left and i2 on the right, respectively.

$-20 + 10 + 30i1 - 15i2 = 0$
$-10 + 13.57i2 - 15i1 = 0$

Solving, i2 = 2.883 A. Thus, $P_{2.57\Omega} = (i_2)^2(2.57) = 21.36$ W

We need twice this, or 42.72 W so i2 must be 4.077 A

Superposition may not be applied to power in the general case, but we can argue that if each source provides the same current to the load, each contributes equally to the power delivered to the load. Noting that 15 || 2.57 = 2.194 Ω,

$$I_{"25V"} = \frac{\left[V\left(\frac{2.194}{15+2.194}\right)\right]}{2.57} = \frac{4.077}{2}$$

Which requires the 25 V source to be replaced with a 41.06 V source.

Similarly,

$$I_{"10V"} = \frac{\left[V\left(\frac{2.194}{15+2.194}\right)\right]}{2.57} = \frac{4.077}{2}$$

which requires the 10 V source to be replaced with a 41.06 V source.

69. (a) $R_{TH} = 5 \parallel [1.8 + 5.4 + 3] = \boxed{3.355\ \Omega}$

(c) Retain $R_{TH} = 3.355\ \Omega$

Power to load is three times too large, so the voltage is $\sqrt{3}$ times too large, so reduce all sources by $\dfrac{1}{\sqrt{3}}$:

1.2 A becomes 692.8 mA
0.8 A becomes 461.9 mA
0.1 A becomes 57.74 mA

70. We first simplify the circuit and obtain its Thévenin equivalent.

Choose the bottom node as the reference. Designate the top left nodal voltage V_1 and the top right nodal voltage V_2. Then

$$-0.4 + \frac{V_1}{8.4} + \frac{V_1 - V_2}{1.8} = 0$$

$$0.1 + \frac{V_2}{5} + \frac{V_2 - V_1}{1.8} = 0$$

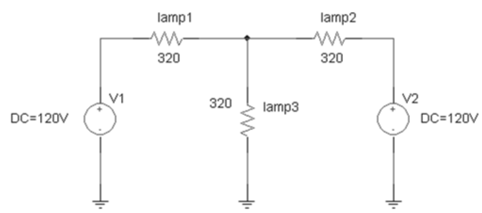
Solving, $V_2 = V_{Th} = 769.7$ mV

By inspection $R_{Th} = 5 \parallel (8.4 + 1.8) = 3.355\ \Omega$

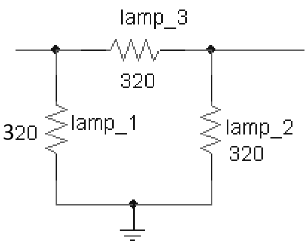
To precisely mimic the behavior of the circuit at the open terminals, the battery should have an open circuit voltage of 769.7 mV, and an intenral series reistance of 3.355 Ω.

There is no way to specify tolerance without knowing the details of the actual load.

71. To solve this problem, we need to assume that "45 W" is a designation that applies when 120 Vac is applied directly to a particular lamp. This corresponds to a current draw of 375 mA, or a light bulb resistance of 120/ 0.375 = 320 Ω.



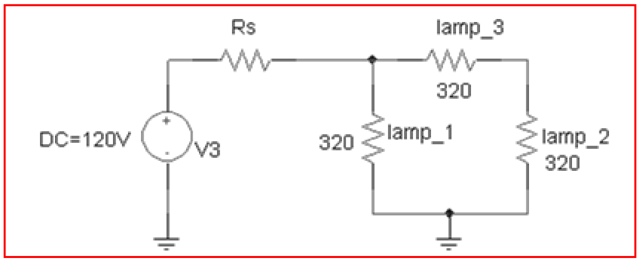
Original wiring scheme



New wiring scheme

In the original wiring scheme, Lamps 1 & 2 draw (40)2 / 320 = 5 W of power each, and Lamp 3 draws (80)2 / 320 = 20 W of power. Therefore, none of the lamps is running at its maximum rating of 45 W. We require a circuit which will deliver the same intensity after the lamps are reconnected in a Δ configuration. Thus, we need a total of 30 W from the new network of lamps.

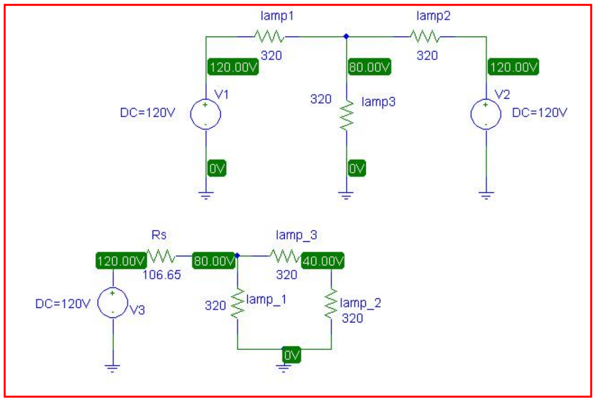
There are several ways to accomplish this, but the simplest may be to just use one 120-Vac source connected to the left port in series with a resistor whose value is chosen to obtain 30 W delivered to the three lamps.



In other words,

$$\frac{\left[120\,\dfrac{213.3}{Rs+213.3}\right]^2}{320}+2\,\frac{\left[60\,\dfrac{213.3}{Rs+213.3}\right]^2}{320}\;=\;30$$

Solving, we find that we require $\boxed{\text{Rs} = 106.65\ \Omega}$, as confirmed by the PSpice simulation below, which shows that both wiring configurations lead to one lamp with 80-V across it, and two lamps with 40 V across each.



72. (a) Source transformation can be used to simplify either nodal or mesh analysis by having all sources of one type. Otherwise, repeated source transformations can in many instances be used to reduce the total number of components, provided none of the elements involved are of interest.

(b) If the transformations involve an element whose voltage or current is of interest, since that information will be lost.

(c) We do, indirectly, as their controling variables will be scaled.

(d) This is the same as replacing the source with a short circuit, so theoretically any current value is possible.

(e) This is the same as replacing the sources with an open circuit, so theoretically any voltage is possible.

73. (a) Define a nodal voltage V_1 at the top of the current source I_S, and a nodal voltage V_2 at the top of the load resistor R_L. Since the load resistor can safely dissipate 1 W, and we know that

$$P_{R_L} = \frac{V_2^2}{1000}$$

then $V_2\big|_{max} = 31.62\text{ V}$. This corresponds to a load resistor (and hence lamp) current of 32.62 mA, so we may treat the lamp as a 10.6-Ω resistor.

Proceeding with nodal analysis, we may write:

$$I_S = V_1/200 + (V_1 - 5V_x)/200 \qquad [1]$$

$$0 = V_2/1000 + (V_2 - 5V_x)/10.6 \qquad [2]$$

$$V_x = V_1 - 5V_x \quad \text{or} \quad V_x = V_1/6 \quad [3]$$

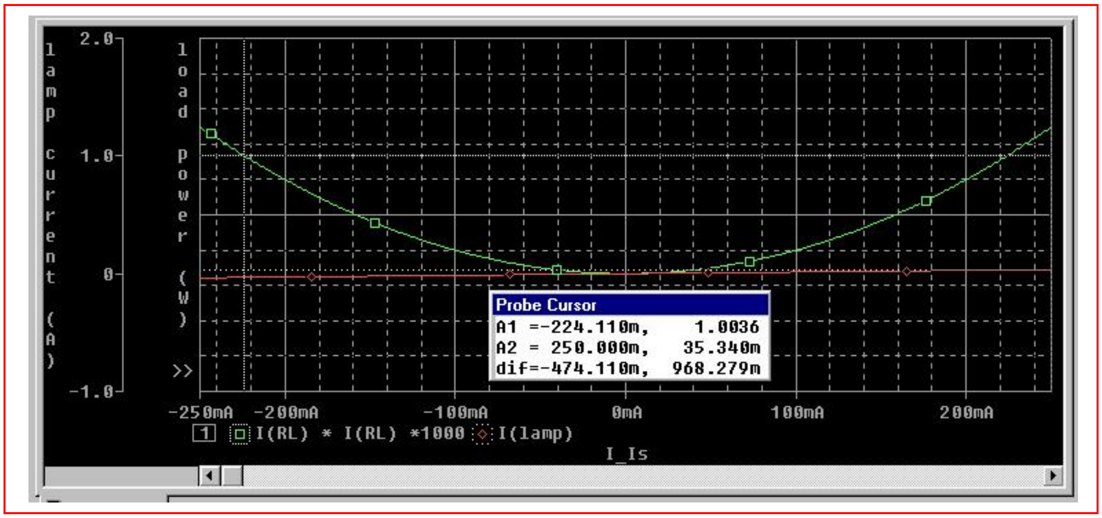
Substituting Eq. [3] into Eqs. [1] and [2], we find that

$$7V_1 = 1200 I_S \qquad [1]$$
$$-5000V_1 + 6063.6V_2 = 0 \qquad [2]$$

Substituting $V_2\big|_{max} = 31.62\text{ V}$ into Eq. [2] then yields $V_1 = 38.35$ V, so that

$I_S|_{max} = (7)(38.35)/1200 = \boxed{223.7\text{ mA.}}$

(b) PSpice verification.



The lamp current does not exceed 36 mA in the range of operation allowed (*i.e.* a load power of < 1 W.) The simulation result shows that the load will dissipate slightly more than 1 W for a source current magnitude of 224 mA, as predicted by hand analysis.

74. <Design> One possible solution:

Imax = 35 mA
Rmin = 47 Ω
Rmax = 117 Ω

With only 9 V batteries and standard resistance values available, we begin by neglecting the series resistance of the battery.

We choose a single 9 V battery in series with a resistor R and the LED.

Then, $I = 9/(R + R_{LED}) < 35$ mA

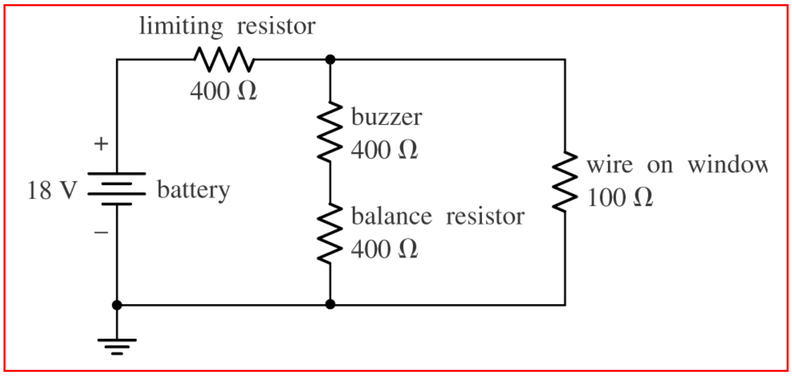
Or $R + R_{LED} > 9/35\times10^{-3}$

For safety, we design assuming the minimum LED resistance and so must selct

$R > 9/35\times10^{-3} – 47$ or $R > 210\ \Omega$

The closest standard resistance value is 220 Ω.

75. We note that the buzzer draws 15 mA at 6 V, so that it may be modeled as a 400-Ω resistor. One possible solution of many, then, is:



Note: construct the 18-V source from 12 1.5-V batteries in series, and the two 400-Ω resistors can be fabricated by soldering 400 1-Ω resistors in series, although there's probably a much better alternative…

1. Inverting amplifier so $v_{out}/v_{in} = –R_2/R_1$.

(a) $v_{out} = –100(5)/100 = \boxed{–5\ V}$

(b) $v_{out} = –200R_1/R_1 = \boxed{–200\ V}$

(c) $v_{out} = –47(20\sin 5t)/ 4.7 = \boxed{–200 \sin 5t\ V}$

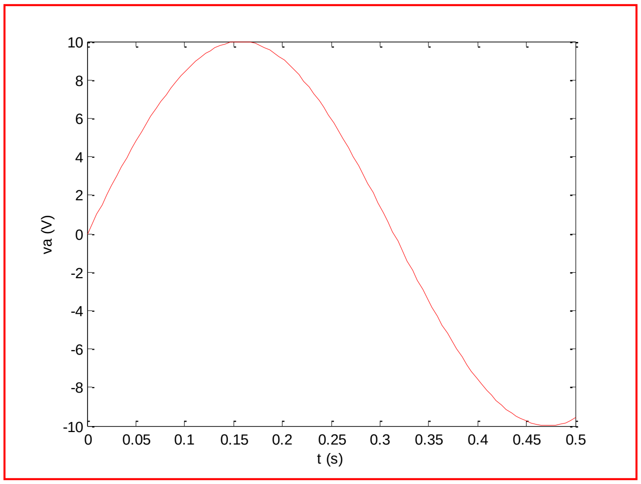
2. Inverting amplifier so vout/vin = –R2/R1.

$P_{100} = (\text{vout})^2/100$

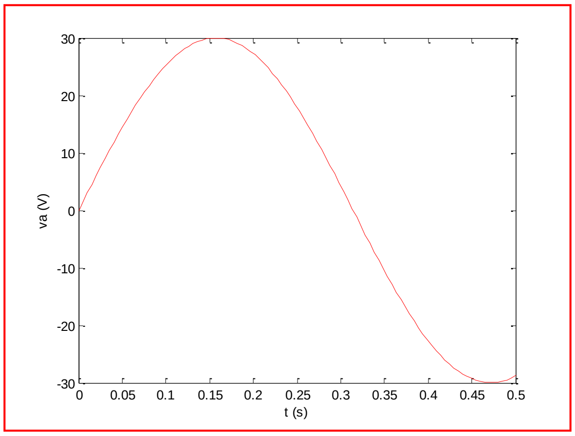
	R2/R1	vout (V)	P_{100} (W)
(a)	0.5	–2	$\boxed{0.04}$
(b)	22	–88	$\boxed{77.44}$
(c)	101/100	–404/100	$\boxed{0.163}$

3. Inverting op amp so vout = (1 + R2/R1)vin.

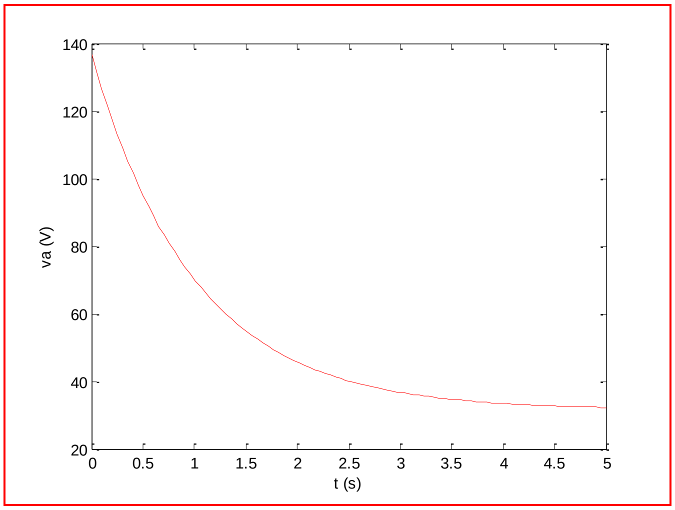
(a) vout = 10sin 10 t V



(b) vout = 30sin 10t V



(c) vout = $21(1.5 + 5e^{-t})$ V



4. Non-inverting op amp so $v_{out}/v_{in} = 1 + R_1/R_2$

(a) $v_{out} = (1 + 1)(5) = \boxed{10 \text{ V}}$

(b) $v_{out} = (1 + 0.1)(2) = \boxed{2.20 \text{ V}}$

(c) $R_1 = 1000\ \Omega$ is okay, but $R_2 = 0$ leads to shorting of voltage source. Thus, $\boxed{v_{out} = 0.}$

5. <Design> We note a typographical error in the first printing; the desired output is actually 4cos 5t V.

One possible solution:

(a) Need “gain” of 4/9. This is not possible in a noninverting configuration so we choose an inverting amplifier with $R_f/R_1 = 4/9$. Selecting $R_f = 4\ k\Omega$ yields $R_1 = 9\ k\Omega$.

(b) We have a source 9 cos 5t V with its positive terminal grounded and its negative terminal connected to R1. Then $v_{out} = (-4/9)(-9 \cos 5t) = 4 \cos 5t$ V.

6. <Design> One possible solution:

Since attenuation is required, only an inverting amplifier is appropriate. Thus, we need Rf/R1 = 5/9. For standard 10% resistor values, selecting Rf = 10 Ω leads to R1 = 18 Ω.

Next, connect the negative terminal of a 9 V source to R1, and ground the positive terminal of the source.

7. The feedback resistor is R1, so vout = $(1 + R1/R2)v_{in}$

We want $(1 + 50/R2)^2 v_{in}^2 = 250$

(a) $\boxed{R2 = 23.12\ \Omega}$

(b) $\boxed{R2 = 5.241\ \Omega}$

Now we need $(1 + 50/R2)^2 v_{in}^2 = 110$

(c) $\boxed{45.55\ \Omega;\ 8.344\ \Omega}$

8. $v_{in} = R_p i_{in}$, inverting amplifier

$v_{out} = -(R_3/R_p)v_{in} = -R_3 i_{in}$

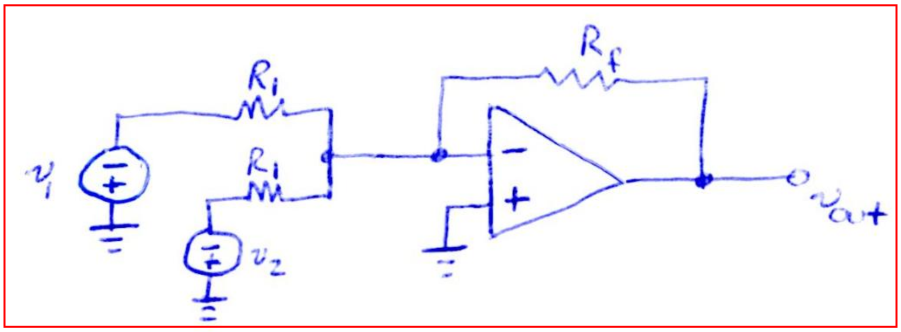
(a) $\boxed{-1\text{ V}}$

(b) $\boxed{-17.0\text{ V}}$

(c) Regardless of component values chosen above, the circuit is electrically equivalent to the inverting amplifer circuit depicted in Fig. 6.3.

9. <Design> One possible solution:

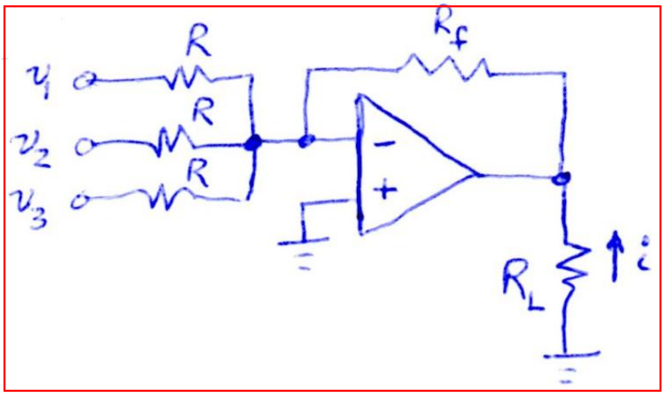
(a) Implement the circuit shown below, with $\boxed{R_f = 2\ \text{k}\Omega \text{ and } R_1 = 1\ \text{k}\Omega.}$



(b) vout = – (Rf/R1)(–v1 – v2) = $\boxed{2(v1 + v2)}$.

10. <Design> One posible solution: Define v1, v2, v3 as being with respect to ground (i.e. think of them as nodal voltages, with ground as the reference).

(a) Implement the following, with all resistors as 1 Ω. Define i as upwards through R_L.

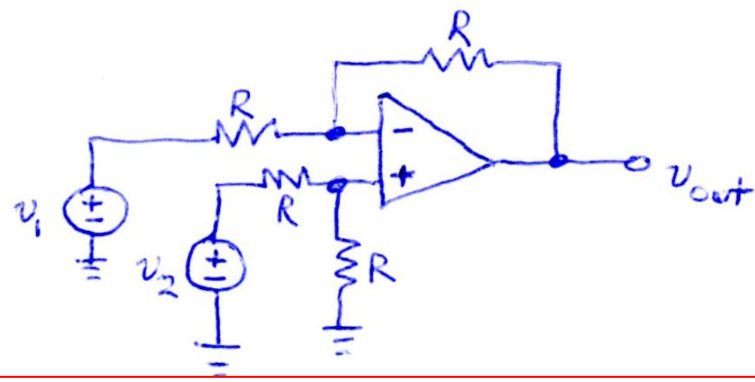


(b) vout = -(Rf/R)(v1 + v2 + v3) = -(v1 + v2 + v3)

i = -vout/RL = v1 + v2 + v3

11. <Design> One possible solution:

(a) Implement the circuit below, with all resistors equal to 1.5 kΩ.
That's done by using two of the 1.5 kΩ resistors, three 50 Ω resistors in series, and 4 6 kΩ resistors in parallel.



(b) This is a classic difference amplifier with $v_{out} = v_2 - v_1$, since all resistor values are equal.

12. The two 850 Ω resistors may be combined to 1700 Ω. Peform a source transformation on the current source so that a 10 V source is in series with 10 kΩ, connected to the non-inverting input.
No current flows through the 1 MΩ so it is neglected.

By KCL, $0 = (10 – 9)/100 + (10 – V1)/1700$

Solving, $\boxed{V1 = 27.0\text{ V}}$

13. Using nodal analysis,

$$0 = \frac{v_- - v_+}{R_1} + \frac{v_- - v_{out}}{R_f}$$

$$v_+ = \frac{R_3}{R_2 + R_3} v_2 \therefore \text{ since } v_+ = v_-$$

$$\frac{\left(\frac{R_3}{R_2 + R_3}\right) v_2 - v_1}{R_1} + \frac{\left(\frac{R_3}{R_2 + R_3}\right) v_2 - v_{out}}{R_f} = 0$$

Solving for v_{out} leads to:

$$\boxed{v_{out} = \left(\frac{R_f}{R_1} + 1\right)\left(\frac{R_3}{R_2 + R_3}\right) v_2 - \frac{R_f}{R_1} v_1}$$

14. (a) $\boxed{\text{No current can flow into either input pin of an ideal op amp.}}$

(b) $\boxed{\text{There can be no voltage difference between the input pins of an ideal op amp.}}$

15. We have a non-inverting amplifier so with the assistance of a source transformation,

$$v_{out} = \left(1+\frac{R_f}{Rx}\right)\left(-R_y I_s\right) = \left(1+\frac{500}{1000}\right)\left(4.7\times10^{3}\right)\left(2\times10^{-3}\right) = \boxed{-14\text{ V}}$$

16. We note a typographical error in the 1st printing: I_s should be –10 mA.

Then, with the assistance of a source transformation,

$$v_{out} = \left(1 + \frac{R_f}{Rx}\right)\left(-R_y I_s\right)$$

$$2 = \left(1 + \frac{R_f}{250}\right)(500)\left(10 \times 10^{-3}\right)$$

Solving,

$\boxed{R_f = 350\ \Omega}$

17. Noting that the output stage is an inverting amplifier,

$$v_{out} = -\left(\frac{3}{1}\right)\left(-10^{-3}v_\pi\right)(1000) = -3v_\pi$$

(a) $\boxed{-5.4545 \cos 100t \text{ mV}}$

(b) $\boxed{-5.4545 \sin (4t + 19^o) \text{ V}}$

18. The first stage is an invertign amplifer which puts (2)(-5/10) = -1 V across the 10 Ω resistor.

The second stage is also an inverting amplifer which multiples the voltage across the 10 Ω resistor by -2000/Rx.

Thus, vout = (-2000/Rx)(-1) = $\boxed{2\text{ V}}$

19. Left stage is an inverting amplifer with gain -5/10 hence (-5/10)(2) = -1 V appears across the 10 Ω resistor.

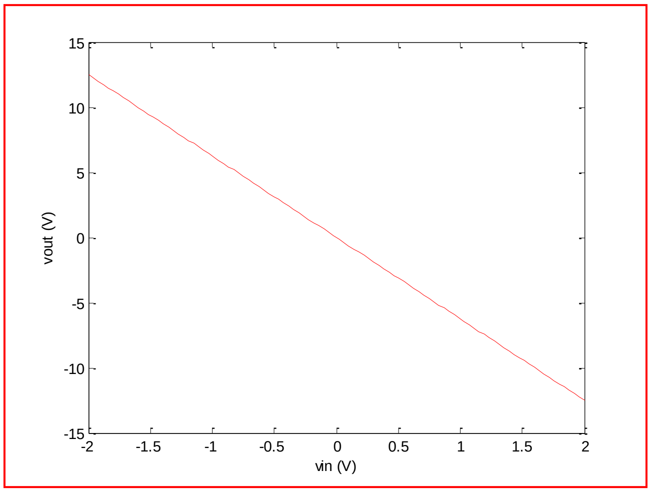
The right hand stage is also an inverting amplifer, now with gain -2000/Rx (Rx in ohms).

Thus, $v_{out} = (-1)(-2000/Rx) = 10$ so $\boxed{Rx = 200\ \Omega}$

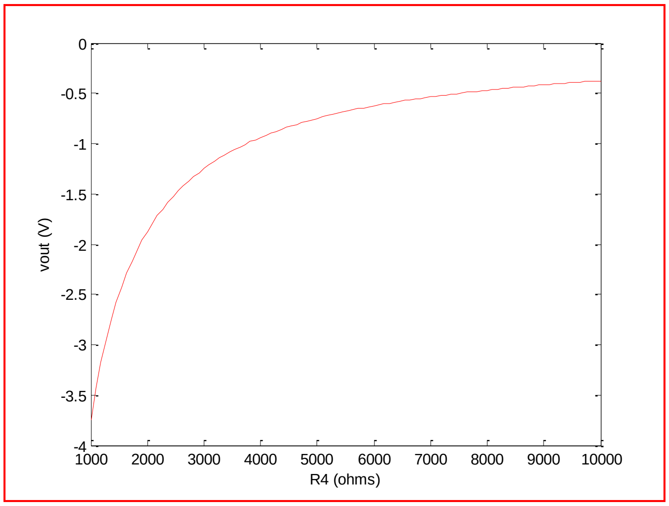
20. The left-hand stage provides (1 + 15/10)vin = 2.5vin to the second stage.

Hence, the second stage provides an output voltage (-5000/R4)(2.5vin)

(a)



(b)



21. On the right we have a difference amplifer where $v_{out} = v_1 - v_{left}$ (vleft = output of left-hand stage).

So, $v_{left} = (1.5)(-1500/500) = -4.5$ V

Hence, v_{out} =

(a) $0 - (-4.5) = \boxed{4.5 \text{ V}}$

(b) $1 - (-4.5) = \boxed{5.5 \text{ V}}$

(c) $-5 - (-4.5) = \boxed{-0.5 \text{ V}}$

(d) $\boxed{2\sin 100t + 4.5 \text{ V}}$

22. Stage 1 delivers vin(1 + 15/10) = 2.5vin to stage 2

Stage 2 delivers (-5000/2000)(2.5vin) = -6.25vin to stage 3

Stage 3 delivers (-1500/500)(-6.25vin) = 18.75vin to stage 4

Stage 4 delivers v1 – 18.75vin = vout

Thus,

(a) vout = 1 – 37.5 = $\boxed{-36.5\text{ V}}$

(b) vout = $\boxed{-18.75\text{ V}}$

(c) vout = -1 – 18.75 = $\boxed{-19.75\text{ V}}$

23. The last stage is merely a buffer and has no effect on the output.
The remainder is a summing amplifier with (defining R_f = 200 kΩ),

$$0 = \frac{0-v_1}{R_1} + \frac{0-v_2}{R_2} + \frac{0-v_3}{R_3} + \frac{0-v_{out}}{R_f}$$

Solving, $v_{out} = -R_f\left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}\right) = -200\times10^3\left(\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}\right)$

(a) v_{out} = $\boxed{-30.8\text{ V}}$

(b) v_{out} = $\boxed{0.8\text{ V}}$

24. <Design> One possible solution:

(a) Max sensor voltage = 5 V
Max summed voltage = 15 V
Adjust to obtain 2 V output when all three inputs are 3 V.

We take a summing amplifier with all resistor values set to 1 Ω. This output is fed into an inverting amplifer with R_1 = 15 Ω and feedback resistor R_f = 2 Ω

(b) The first stage provides a voltage equal to –(v1 + v2 + v3).

This is multipled by -2/15 by the second stage.

Check: When v1 = v2 = v3 = 0, vout = (-2/15)(0) = 0 (Ok)
When v1 = v2 = v3 = 5 V, vout = (-2/15)[-(5 + 5 + 5)] = 2 V (Ok)

25. <Design> One possible solution:

(a) We start with a difference amplifer as shown in Table 6.1 with all resistors set to 1 kΩ, but with v_2 designated as the input voltage to the inverting input and v_1 to the noninverting input. The output of this stage is taken to the input of an inverting amplifer with R1 = 1 kΩ and Rf = 10 kΩ

(b) $v_{out} = (-Rf/R1)(v_1 - v_2) = 10\,(v_2 - v_1)$.
For v1 = v2, vout = 0; for v1 – v2 = 1 the output is 10 V.

26. <Design> One possible solution:

(a) Maximum = 400,000 kg = 10 V
Sensor: 10 μV = 1 kg therefore 400,000 kg on one sensor yields 4 V

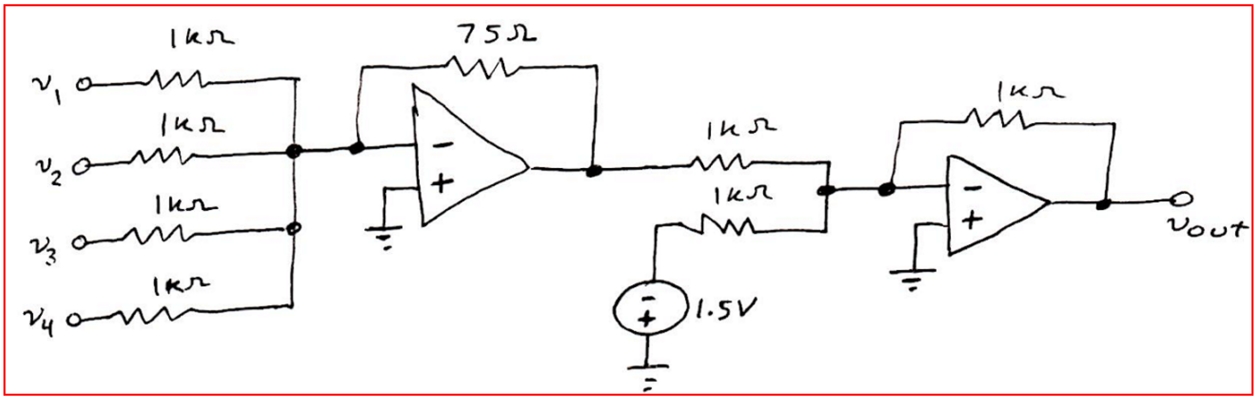
We take a general summing amplifier with all resistors set to 1 kΩ. The output of this stage is taken as the input voltage to an inverting amplifier with R_1 = 4 kΩ and R_f = 10 kΩ.

(b) The first stage sums the three sensor voltages v1, v2 and v3 to obtain –(v1 + v2 + v3) at the input to the second stage. The second stage multiplies this voltage by -2.5.
If v1 + v2 + v3 = 4 V (400,000 kg total), vout = -2.5(-4) = 10 V.

27. <Design> One possible solution:

(a) Designate the tank sensor output voltages as v1, v2, v3, v4. Each has a maximum value of 5 V. We desire 3 V when their sum is 20 V, and 1.5 V when their sum is zero.

Thus, we need a dc offset in addition to an adjusted sum of these voltages.



(b) The first stage sums the sensor voltages and attenuates the result such that -1.5 V is obtained at the stage output when each sensor voltage is 5 V. The second stage adds the necessary dc offset and inverts the sign of the output voltage. Thus, when all input voltages are equal to zero, vout = (-1)(0 – 1.5) = 1.5 V.
When all voltages are 5 V, vout = (-1)[-(75)(5 + 5 + 5 + 5)/1000 – 1.5) = 3 V

28. The left-hand stage is an inverting amplifer with output voltage vout = -(R2/R1)vin.

The middle stage is an inverting amplifer with output –(200/50)(-R2/R1)vin. R3 is irrelevant as long as it is greater than zero.

The last stage is a voltage follower and so does not affect the output.

Thus, 4 = -(200/50)(-R2/R1)(8)

$\boxed{\text{Arbitrarily set R1 = 8 k}\Omega\text{. Then R2 = 1 k}\Omega\text{. Arbitrarily set R3 = 1 k}\Omega\text{.}}$

29. The left-hand stage is a general difference amplifer. The right-hand stage is a simple inverting amplifer which multiplies the output of the differnce amplifer by (1 + R_6/R_4). Analyzing the left-hand stage then:

$$v_- = v_+ = v_{in}\left(\frac{R_3}{R_2 + R_3}\right)$$

So $0 = \dfrac{v_- - v_{out}\big|_{left-hand\ stage}}{R_4} + \dfrac{v_- - 1}{R_1}$

Substituting and solving for v_{out} yields

$$\boxed{v_{out} = (R_4 + R_6)\left[\left(\frac{1}{R_4} + \frac{1}{R_1}\right)v_{in}\left(\frac{R_3}{R_2 + R_3}\right) - \frac{1}{R_1}\right]}$$

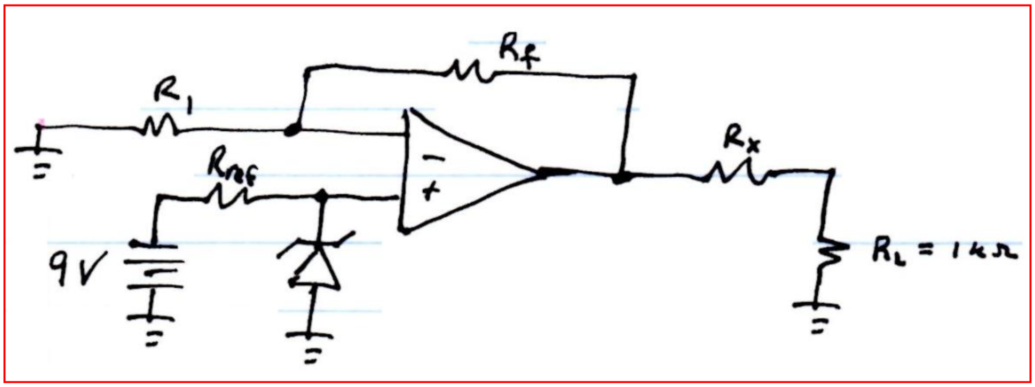
30. <Design> One possible solution:

We use a circuit such as the one in Fig. 6.19a, but with two 9 V batteries in series for a total of 18 V.

Then $R_{ref} = (18 - 10)/0.025 = 320\ \Omega$

31. <Design> One possible solution:

We can employ a voltage divider at the output as follows:



Then $R_{ref} = (9 - 5.1)/76\times10^{-3} = 51.3\ \Omega$

With 1 + Rf/R1 = 1, we set Rf= 0 and arbitrarily select R1 = 100 Ω

Finally,

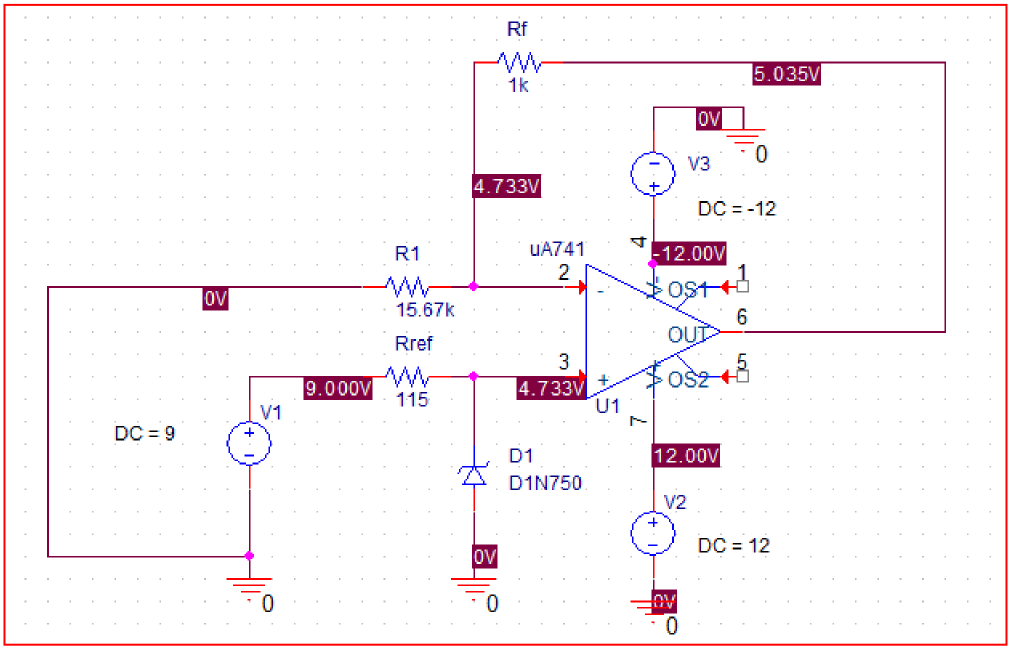
$$(5.1)\left(1+\frac{R_f}{R_1}\right)\left(\frac{R_L}{R_L+R_x}\right)=4$$

With R_L = 1 kΩ, we find Rx = 275 Ω

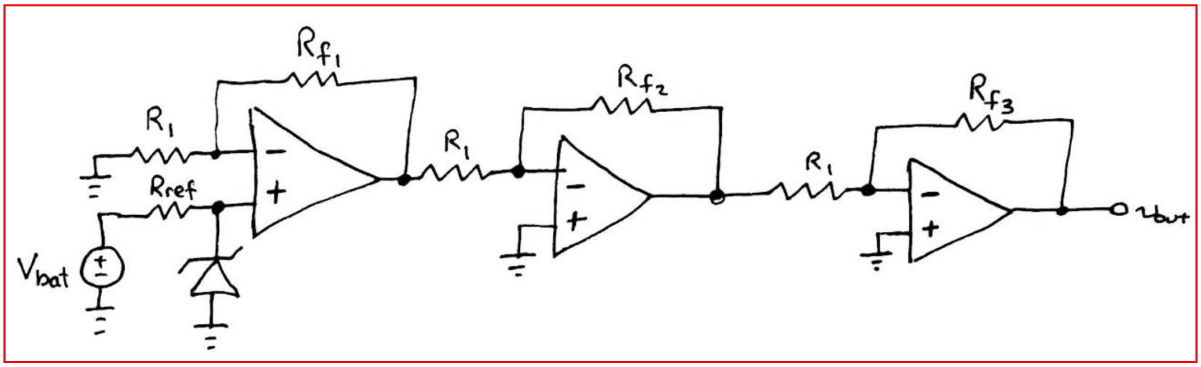
32. <Design> One possible solution:

(a) No diode is specified so we opt for the circuit of Fig. 6.20 and employ a 1N750, which has a Zener voltage of 4.7 V. We base our design on the simulation result of Fig. 6.19c, which shows a voltage of 4.733 V achieved at a current of 37.1 mA (R_{ref} = 115 Ω and 9 V source). Thus we need 1 + Rf/R1 = 5/4.8. For simplicity we select Rf = 1 kΩ so that R1 = 15.67 kΩ.

(b) With an infinite load we are accurate to better than 1% of the target value (5 V).

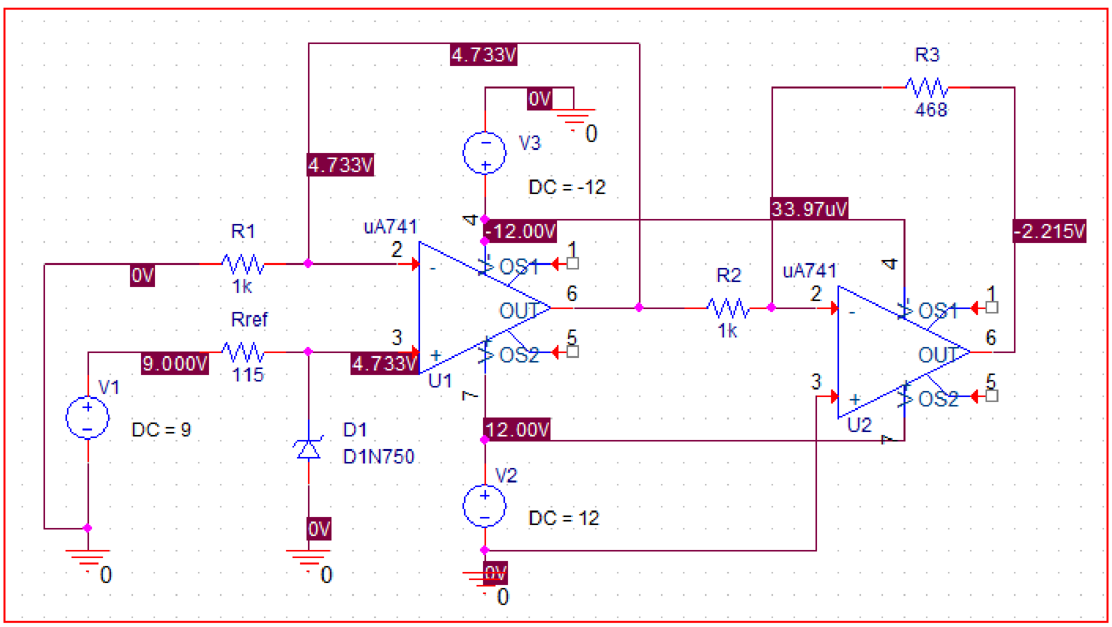


33. (a) A 1N750 Zener diode can be used to obtain a reference voltage of 4.7 V but the non-inverting circuit of Fig. 6.20 cannot achieve a gain of less than unity. So, instead, employ two cascaded inverting amplifiers, one to invert the sign of the voltage and one to reduce the voltage from 4.7 V to 2.2 V. The circuit then is



With V_{bat} = 9 V, R_{ref} = 115 Ω, R_1 = 1 kΩ, R_{f1} = 0, R_{f2} = 1 kΩ, and
$R_{f3} = (2.2/4.7)R_1 = 468\ \Omega$

(b) The evaluation version of PSpice will not allow the full circuit to be simulated, so we omit the middle stage (used only to invert the sign) and simulate the rest, confirming better than 1% accuracy with respect to our target value of 2.2 V.



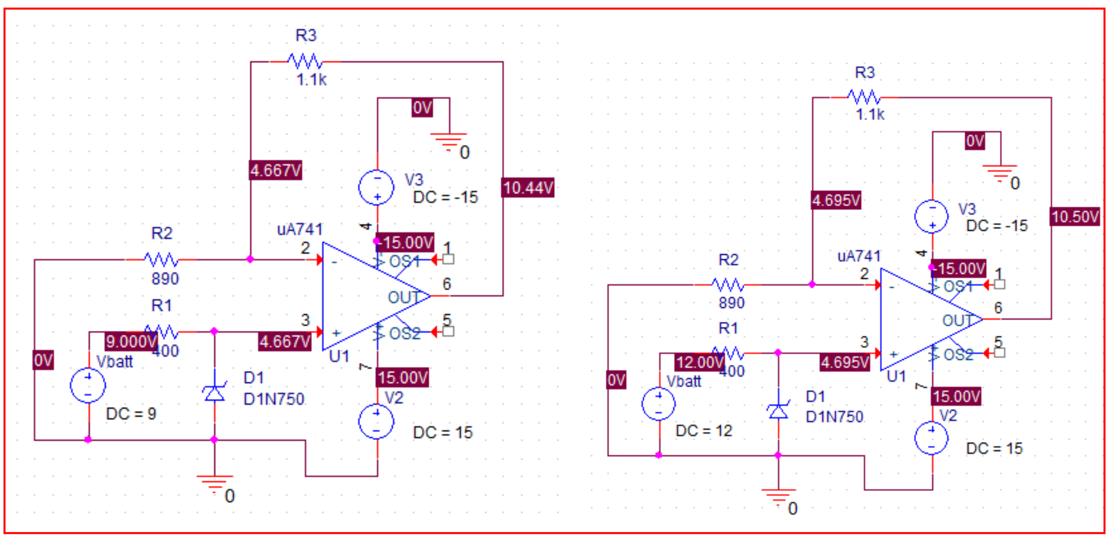
34. The output voltage of the circuit in Fig. 6.53 should be labelled V_1.

(a) Zener voltage is 4.7 V. For 9 V supply, the current through the 400 Ω resistor is $(9 – 4.7)/400 = 10.75$ mA

From Fig. 6.19b this current is reasonable for th diode to be in breakdwon. Since this is a non-inverting amplifer, $V_1 = (1100/890 + 1)(4.7) = \boxed{10.5 \text{ V}}$

(b) From Fig. 6.18b, 12 V results in breakdown but the current will be $(12 – 4.7)/400 = 18.3$ mA, which is still less than the maximum rated diode current; we expect $\boxed{V_1 = 10.5 \text{ V}}$ also.

(c) PSpice simulation: An output of 10.44 V is obtained for 9 V battery voltage; precisely 10.50 V was obtained for a 12 V battery voltage. In comparing the voltages at the input, we see the difference originates from a value closer to the design value (4.7 V) across the Zener diode.



35. <Design> One possible solution:

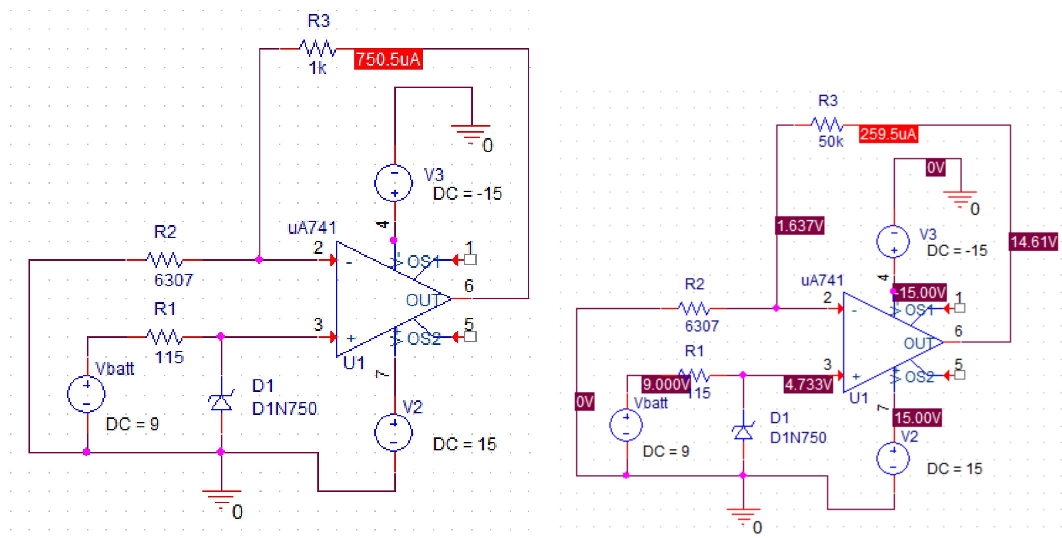
(a) We use a circuit such as that shown in Fig. 6.22, with a 9 V battery and 1N750 Zener diode. We increase the 100 Ω to 115 Ω. Name the 4.9 kΩ resistor R_1.

From Fig. 6.19c, we expect 4.73 V across the diode.

The voltage across R_L is then $(1 + R_L/R_1)(4.73) - 4.73 = 4.73(R_L/R_1)$
and the current through R_L is $(4.73R_L/R_1)/R_L = 4.73/R_1$

We want this to be 750 μA, requiring that $\boxed{R_1 = 6307\ \Omega}$

(b) Here we simulate the performance for R_L = 1 kΩ and R_L = 50 kΩ, and note a more elegant solution is possible using PARAM. For 1 kΩ, we find the circuit performs as desired, but at the other extreme (50 kΩ), the circuit fails as it enters saturation. The maximum resistance for ±18 V supplies, a typical upper limit for the μA741, is then 17.6 kΩ.



36. <Design> One possible solution:

(a) We use a circuit such as the one in Fig. 6.22. A 9 V battery is employed, but with a 1N4733 diode (5.1 V at 76 mA) is used. Rename the 4.9 k Ω resistor R1.

Instead of 100 Ω, we require (9 – 5.1)/0.076 = 51.3 Ω

The voltage across RL is (1 + RL/R1)(5.1) – 5.1 = 5.1RL/R1

Hence the current through RL is 5.1/R1 = 0.05 requiring R1 = 102 Ω

(b) This diode is not part of the standard library.

37. (a) We design a circuit based on that shown in Fig. 6.22 but using the specified diode instead. With a battery voltage of 24 V, assuming 20 V across the Zener diode and a diode current of 50% its maximum (12.5 mA), we compute

$R_{ref} = (24 - 20)/0.00625 = 640\ \Omega$

Then, since $I_s = V_{ref}/R_1$, we require $R_1 = 20/10\times10^{-3} = 2\ \text{k}\Omega$

(b) This diode is not part of the standard library.

38. Define nodeal voltages v_- and v_+ at the op amp input terminals. Then

$$0 = \frac{v_- - v_2}{R_1} + \frac{v_- - v_{out}}{R_2}$$

$$0 = \frac{v_+ - v_1}{R_3} + \frac{v_+}{R_L} + \frac{v_+ - v_{out}}{R_4}$$

With an ideal op amp, $v_- = v_+$.

Solving, $\boxed{v_{out} = \frac{\left(R_1R_4R_L + R_2R_4R_L\right)v_1 - (R_2R_3R_4 + R_2R_3R_L + R_2R_4R_L)v_2}{R_1R_3R_4 + R_1R_4R_L - R_2R_3R_L}}$

And $\boxed{I_L = \frac{v_+}{R_L} = \frac{R_1R_4v_1 - R_2R_3v_2}{R_1R_3R_4 + R_1R_4R_L - R_2R_3R_L}}$

39. Define two nodal voltages v_m and v_p at the inverting and non-inverting input terminals, respectively. For an ideal op amp, $v_m = v_p$. Then,

$$0 = \frac{v_p}{1000} + \frac{v_p - v_{out}}{500} \qquad [1]$$

$$0 = \frac{v_p - v_1}{1000} + \frac{v_p}{100} + \frac{v_p - v_{out}}{500} \qquad [2]$$

Solving, $v_p = v_1/10$ and $I_L = v_p/100 = \boxed{v_1/1000 \text{ (independent of } R_L)}$

40. (a) Employing nodal analysis,

$$0 = \frac{-v_d + 0.45}{250} + \frac{-v_d + 0.45 - v_{out}}{1400} - \frac{v_d}{2\times 10^6} \qquad [1]$$

$$0 = \frac{v_{out} + v_d - 0.45}{1400} + \frac{v_{out} + 2\times 10^5 v_d}{75} \qquad [2]$$

Solving, $\boxed{v_{out} = 2.970 \text{ V}}$

(b) The ideal model predicts $v_{out} = (0.45)(1 + 1400/250) = \boxed{2.970 \text{ V}}$
Thus, the ideal model is accurate to better than 0.1%.

41. (a) Employing nodal analysis and defining nodal voltages v_- and v_+ at the input,

$$0 = \frac{v_- - 2}{1500} + \frac{v_- - v_{out}}{1500} + \frac{v_- - v_+}{2\times10^6} \qquad [1]$$

$$0 = \frac{v_+ - 5}{1500} + \frac{v_+}{1500} + \frac{v_+ - v_-}{2\times10^6} \qquad [2]$$

$$0 = \frac{v_{out} - v_-}{1500} + \frac{v_{out} - 2\times10^5 v_d}{75} + \frac{v_{out}}{R_L} \qquad [3] \text{ and}$$

$$v_d = v_+ - v_- \qquad [4]$$

Solving,

$$\boxed{v_{out} = \frac{1.067\times10^4 R_L}{3.556\times10^3 R_L + 2.669}}$$

(b) The ideal model predicts $v_{out} = 5 - 2 = 3$ V

We note that for any appreciable value of R_L, the exact model reduces to

$(1.067\times10^4)/(3.56\times10^3)$ = 3.0006 V

so in this situation the ideal model is better than 0.1% accurate.

42. (a) *CMRR* is the ratio of differential mode gain to common mode gain. If the same signal is applied to both input terminals, ideally no output is generated. In reality, any common component of the input voltages will be amplified slightly.

(b) *Slew rate* is the rate at which the output voltage can respond to changes in the input voltage. This real limitation leads to distortion of the waveform above some frequency.

(c) *Saturation* refers to the inability of an op amp circuit to generate an ouptutp voltage larger than the supply voltage(s). If the input is too large, further increases do not lead to correspondingly larger output.

(d) *Feedback* refers to routing some portion of the output to the input. ***Negative*** fedback helps stabilize a circuit. ***Positive*** feedback can lead to oscillation.

43. (a) Using the ideal op amp model, we end up with an ideal voltage follower so the output is $\boxed{v_{out} = 2\text{ V.}}$

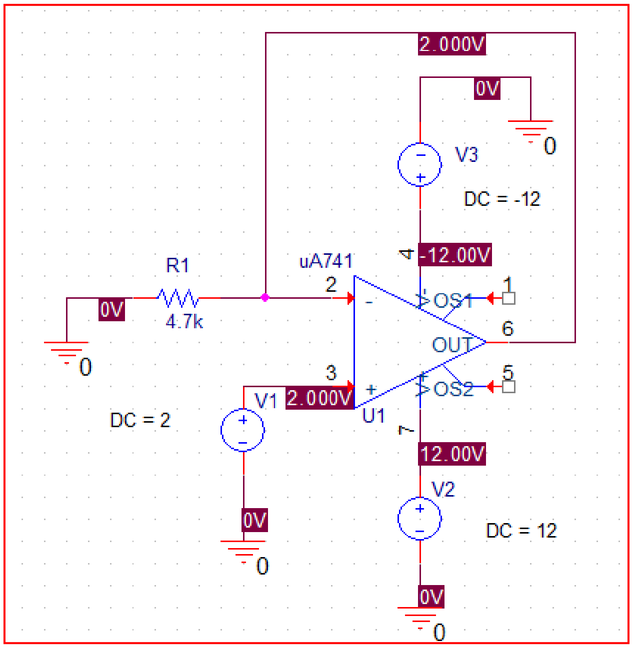
(b) Using parameters for a 741 op amp in conjunction with nodal analysis,

$$0 = \frac{v_{out}}{4700} + \frac{v_{out} - 2}{2\times10^6} + \frac{v_{out} - 2\times10^5 v_d}{75} \quad [1]$$

$$-2 + v_d + v_{out} = 0 \quad [2]$$

Solving, $\boxed{v_{out} = 1.99999\text{ V}}$

(c) PSpice simulation:



(d) $\boxed{\text{All three agree to at least four decimal places, so the ideal model is adequate in this instance.}}$

44. We identify the non-grounded side of the 250 Ω with the nodal voltage v_{m}., and assume zero output resistance and infinite input resistance.

For an ideal op amp, $\frac{v_{out}}{0.45} = 1 + \frac{1400}{250} = 6.60$

Applying nodal analysis to the detailed model,

$$0 = \frac{-v_d + 0.45}{250} + \frac{-v_d + 0.45 - Av_d}{1400}$$

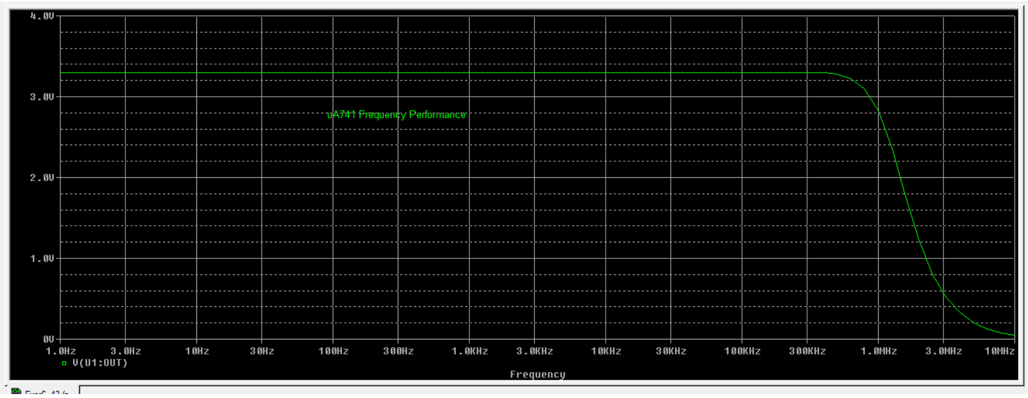
which can be solved to obtain $Av_d = \frac{14.85A}{5A+33}$.

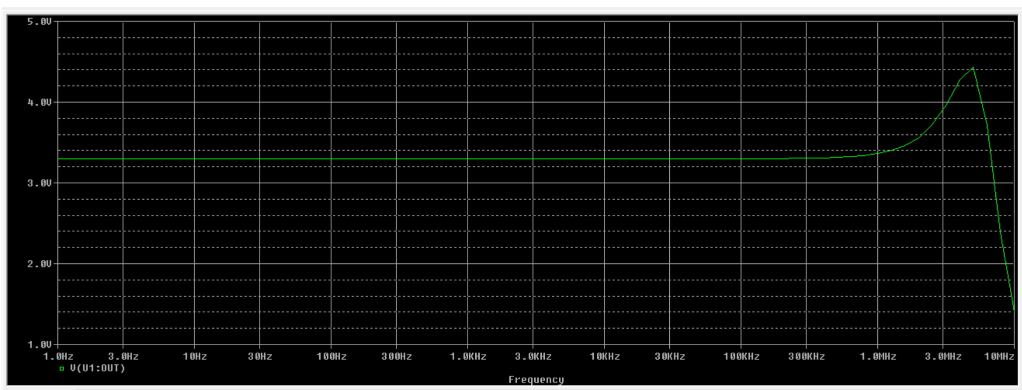
Thus, 2% of the ideal (closed-loop) value is

(0.98)(6.60) = Avd/0.45 and so $\boxed{A_{min} = 323}$

45. (a) The LF411 has a much higher slew rate (15 V/μs) than the μA741 (0.5 V/μs). Both are adequate at low frequencies but in the kHz range the LF411 will show better performance.

(b) Both op amps due well in this circuit up to 300 kHz, but the LF411 is performing well even at 1 MHz, although not perfectly at that frequency.





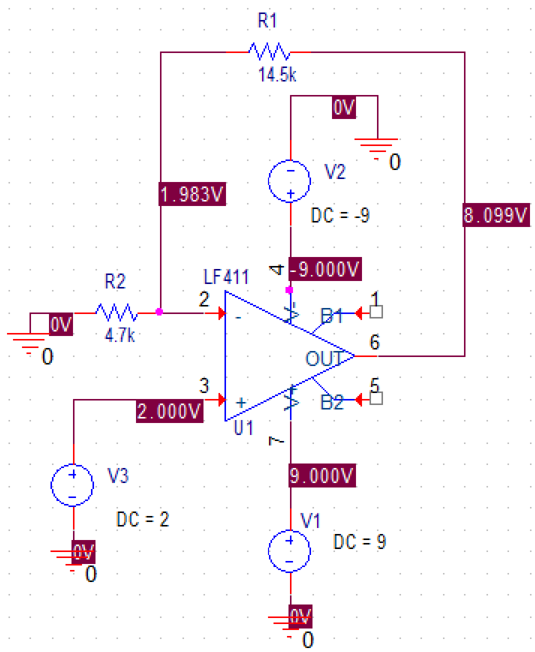
46. (a) This non-inverting amplifer has a gain of 1 + 470/4700 = 1.1
We therefore expect an output of (1.1)(2) = 2.2 V

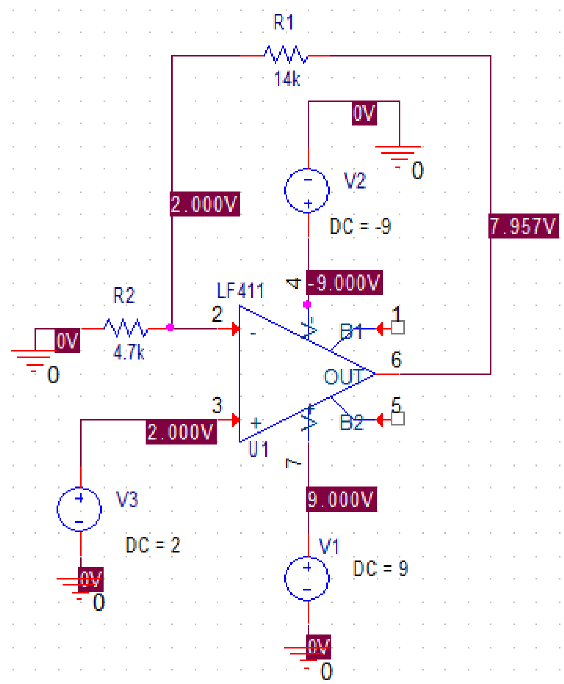
The output voltage cannot exceed the supply voltage, or 9 V in this case.
Thus, $(1 + R_f/4700)(2.2) = 9$

Solving, $\boxed{R_{f_{max}} = 14.5\text{ k}\Omega}$

(b) Summary, using LF411: For R_f = 14.5 kΩ, ideal model predicts 8.17 V, PSpice yields 8.099 V, so we're just barely in saturation with this value of resistance.

For R_f = 14 kΩ, v_{out} = 7.957 V as expected from the ideal model.





47. By nodal analysis,

$$0 = \frac{-v_d + 0.45}{250} + \frac{-v_d + 0.45 - v_{out}}{1400} + \frac{-v_d}{R_{in}} \qquad [1]$$

$$0 = \frac{v_{out} + v_d - 0.45}{1400} + \frac{v_{out} - Av_d}{R_o} \qquad [2]$$

Solving,

```
>> pretty(simplify(b.vout))

                        6300.0 Ro + 7425.0 A Rin
  -----------------------------------------------------------------
  16500.0 Rin + 14000.0 Ro + 56.0 Rin Ro + 2500.0 A Rin + 3500000.0
>> pretty(simplify(b.vd))

                          Rin (126.0 Ro + 37125.0)
  --------------------------------------------------------------------
  82500.0 Rin + 70000.0 Ro + 280.0 Rin Ro + 12500.0 A Rin + 17500000.0
```

Finally, we note the input bias current is simply v_d/R_{in}.

(a) μA741: $\boxed{v_d = 18.6\ \mu\text{V};\ \text{input bias current} = 9.31\ \text{pA}}$

(b) LF411: $\boxed{v_d = 2.98\ \mu\text{V};\ \text{input bias current} = 2.98\ \text{aA}}$

(c) AD549K: $\boxed{v_d = 31.2\ \mu\text{V};\ \text{input bias current} = 0.312\ \text{aA}}$

(d) OPA690 (note assuming $R_o = 0$): $\boxed{v_d = 707\ \mu\text{V};\ \text{input bias current} = 3.72\ \text{nA}}$

48. $\text{CMRR}_{\text{db}} = 20\log\left|\frac{A}{A_{CM}}\right|$ dB

where A = differential mode gain and A_{CM} = common mode gain.

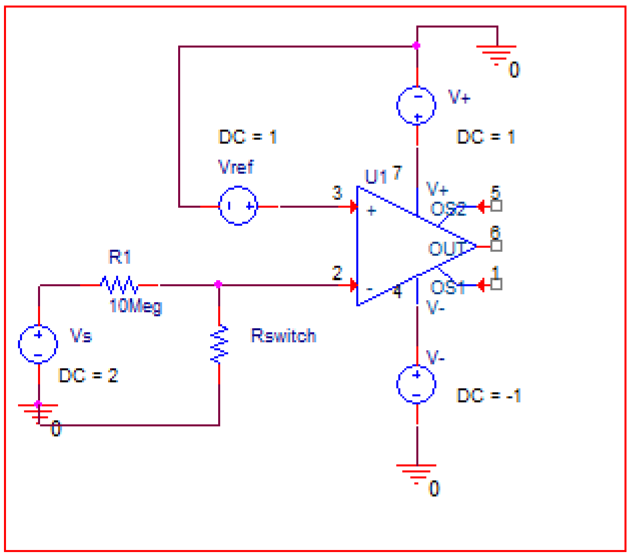
Solving, $A_{CM} = \dfrac{A}{10^{CMRR_{dB}/20}}$

Device	**A**	**CMRR_{db}**	**A_{CM} (V/V)**
µA741	2×10^5	90	6.3
LM324	105	85	5.6
LF411	2×10^5	100	2.0
AD549K	106	100	10.
OPA690	2800	65	1.6

49. <Design> One possible solution:

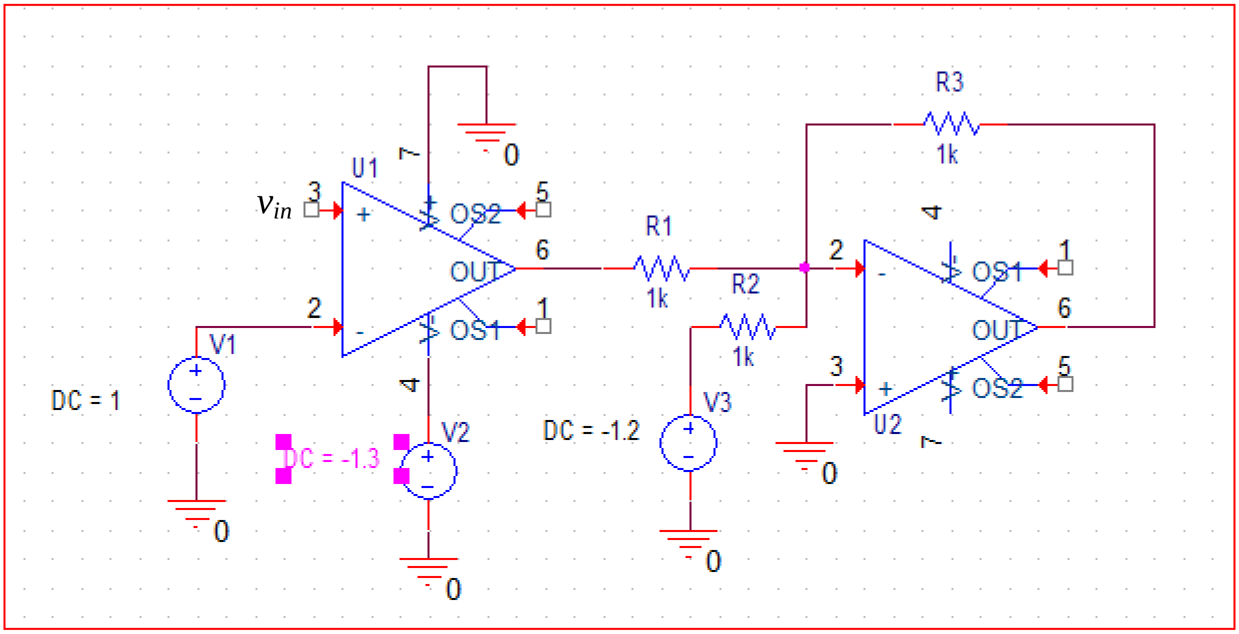
$$\text{Desired: } v_{out} = \begin{cases} -1 \text{ V, no finger } (R \geq 10 \text{ M}\Omega) \\ +1 \text{ V, finger present } (R < 10 \text{ M}\Omega) \end{cases}$$

We employ a voltage divider and a 1 V reference into a comparator circuit. With no finger present, a voltage greater than 1 V appears at the inverting input, so that the output voltage is -1 V. With the finger present, the voltage at the inverting input will drop below 50% of the driving voltage (2 V), so that +1 voltage will appear at the output.



50. <Design> One possible solution:

We build a two-stage circuit where a comparator has v_{in} applied to its non-inverting input, and a 1 V reference to the inverting input. The output of this stage is 0 V when v_{in} > 1 V and -1.3 V otherwise. This is summed with a -1.2 V reference source, the output of which is inverted and so is either +2.5 V for v_{in} > 1 V or +1.2 V otherwise.



51. From Eq. [23],

$$v_{out} = \frac{R_4}{R_3}\left(\frac{1+R_2/R_1}{1+R_4/R_3}\right)v_+ - \frac{R_2}{R_1}v_-$$

The differential input is $v_+ - v_-$ and hence the differential gain is

$$A_{dm} = \frac{v_{out}}{v_+ - v_-} = \frac{\frac{R_4}{R_3}\left(\frac{1+R_2/R_1}{1+R_4/R_3}\right)v_+ - \frac{R_2}{R_1}v_-}{v_+ - v_-} \qquad [1]$$

For common mode components we must average the inputs, and hence the common-mode gain is

$$A_{cm} = 2\frac{v_{out}}{v_+ + v_-} = (2)\frac{\frac{R_4}{R_3}\left(\frac{1+R_2/R_1}{1+R_4/R_3}\right)v_+ - \frac{R_2}{R_1}v_-}{v_+ + v_-} \qquad [2]$$

CMRR is defined as the absolute value of their ratio: CMRR = $\left|\frac{A_{dm}}{A_{cm}}\right|$.

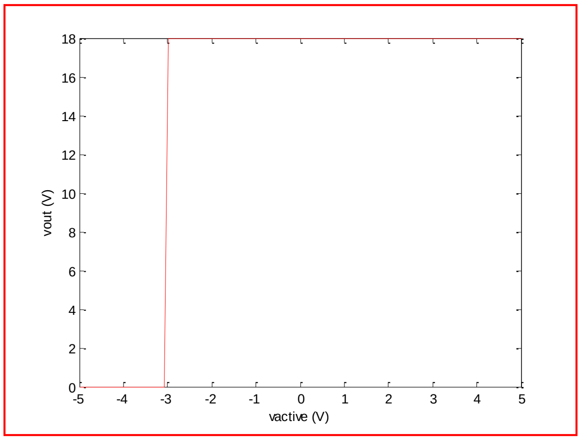
(a) $R_1 = R_3$ and $R_2 = R_4$. Eq. [1] above reduces to R_2/R_1.

Eq. [2] above reduces to $A_{cm} = 2\frac{R_2}{R_1}\left(\frac{v_+ - v_-}{v_+ + v_-}\right)$. Thus, CMRR = $\boxed{\frac{v_+ + v_-}{2(v_+ - v_-)}}$.

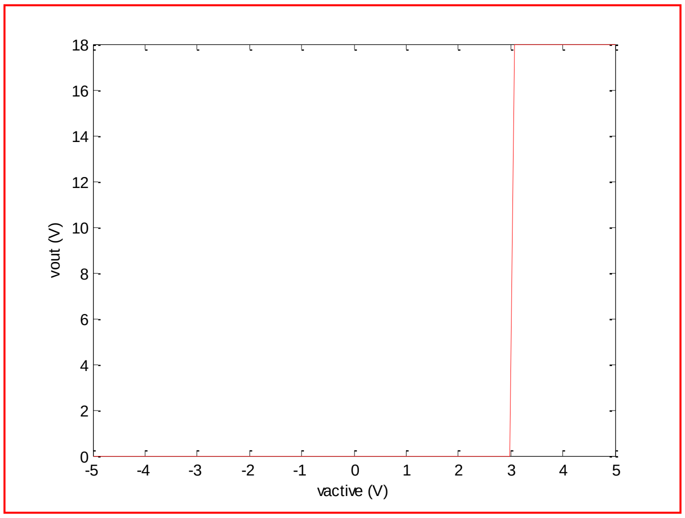
(b) All resistors are different. Then the above reduces to CMRR = $\boxed{\frac{v_+ + v_-}{2(v_+ - v_-)}}$

53. Since the reference voltage v_1 is applied to the inverting terminal, we expect the output to follow the negative supply voltage (0 V, here) until $v_{active} > v_{ref}$ at which point it follows the positive voltage supply.

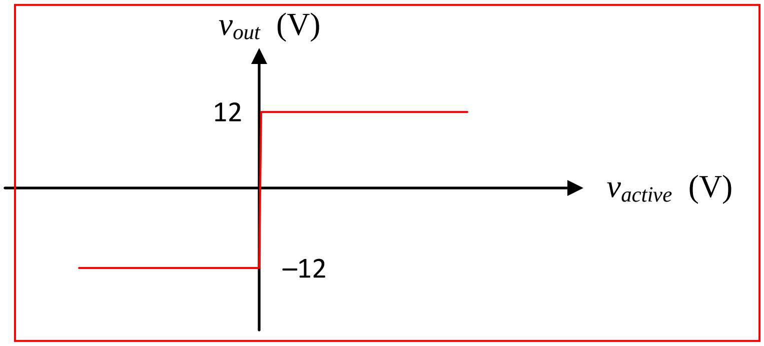
(a) Transition at v_{active} = -3 V



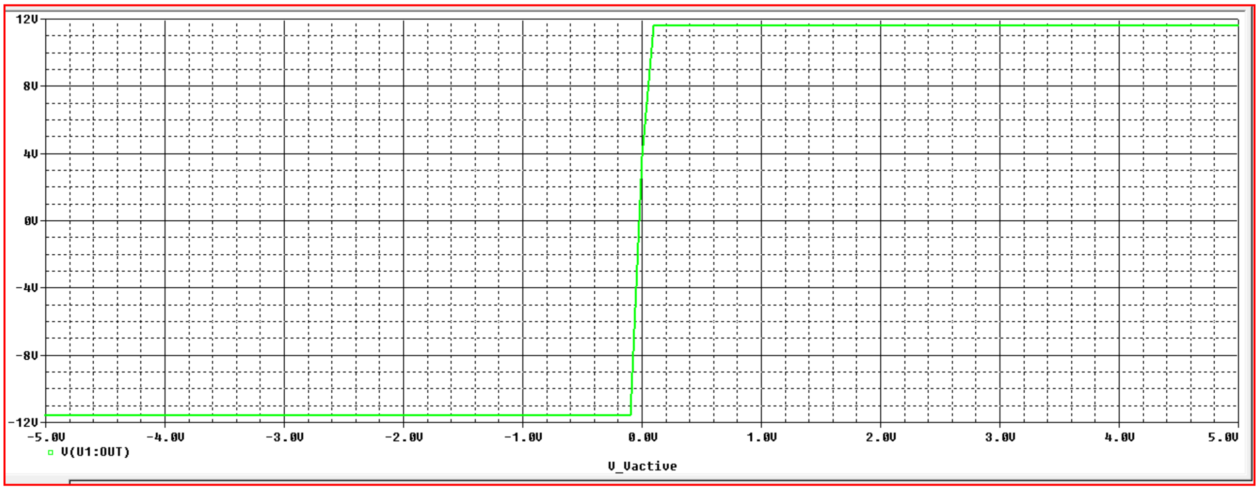
(b) Transition at v_{active} = +3 V



54. We have a comparator circuit with zero reference tied to the inverting input, and matched 12 V supplies. The expected (ideal) output is therefore:

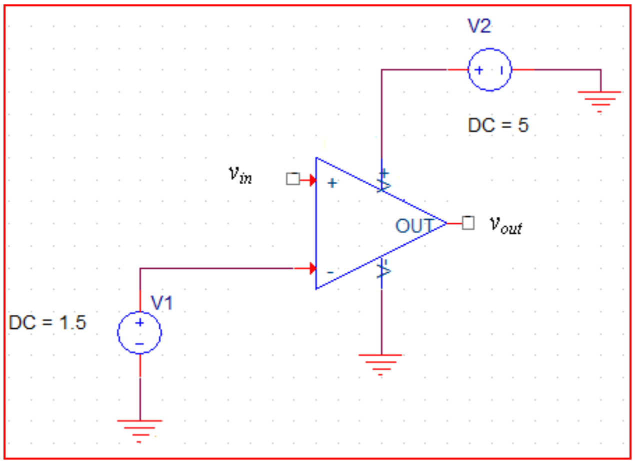


Simulating using a μA741 results in the following, which exhibits the expected shape with only a slight reduction in maximum and minimum voltages:



55. <Design> One possible solution:

This comparator circuit follows the negative supply voltage (0 V) until v_{in} exceeds 1.5 V, at which point 5 V appears at the output.



56. <Design>

We note that the short circuit should not appear across the + and – terminals of v_{out} as it appears in the first printing.

(a) We designate the bottom node as the reference node, then name the top node V_{in}, the node at the "+" terminal of V_{out} as V_A, and the remaining node V_B.

By voltage division, $V_A = V_{ref}\dfrac{R_2}{R_1+R_2}$ and $V_B = V_{ref}\dfrac{R_3}{R_3+R_{Gauge}}$. Thus,

$$V_{out} = V_A - V_B = V_{in}\left[\frac{R_2}{R_1+R_2} - \frac{R_3}{R_3+R_{Gauge}}\right].$$

(b) Set all four resistors equal; specify as R_G. Then,

$$V_{out} = V_{in}\left(\frac{R_G}{2R_G} - \frac{R_G}{R_G}\right) = 0$$

(c) One possible solution:

Connect -12 V to pin 4; connect +12 V to pin 7. Ground pin 2.
Employ a bridge circuit such as the one shown in Fig. 6.60(a) with the strain gauge in place of R_{Gauge} and all other resistors precisely 5000 Ω. Connect four 12 V supplies in series to botain V_{in} = 48 V. Then $V_{out} = V_{in}(2.5\times10^{-6}) = 1.2\times10^{-4}$ V. A 1 V signal requires a gain of $1/1.2\times10^{-4} = 8333$.

This value is in excess of our maximum gain of 1000 so connect a resistor having value $\dfrac{50.5\times10^3}{1000-1} = 50.55\ \Omega$ across pins 1 and 8. This provides a gain of 1000.

The voltage V_{out} is connected across pins 3 and 2 with the "+" reference at pin 3. We still require a gain of 8.333 so connect the pin 6 output to the non-inverting terminal of a non-inverting amplifer powered by ±12 V supplies. Then, set R_1 = 1 kΩ and R_f = 7.333 kΩ. This completes the design.

57. <Design> One possible solution:

(a) R_{TH} of the switch = $5/1\times10^{-3} = 5\ k\Omega$
We thus need a circuit with minimum gain $5/0.25 = 20$
Select a non-inverting amplifer circuit. Connect the microphone to the non-inverting input terminal, select $R_1 = 1\ k\Omega$ and so $R_f = 19\ k\Omega$.

(b) Although general speaking may not correspond to peak microphone voltage, we are not provided sufficient information to address this, and not that he gain may need adjusting in the final circuit.
Thus, we should connect the feedback resistor R_f in series with a variable 20 kΩ resistor, initially set to 0 Ω. This will allow us to vary the actual gain between $1 + R_f/R_1 = 1 + 19/1 = 20$ and $1 + 39/1 = 40$.

58. <Design> One possible solution:

We employ a summing amplifer such as that shown in Table 6.1, but with 5 inputs. We model each microphone as an ideal voltage source and connect a reistor R_1, R_2 etc with each.

We set $R_1 = R_2 = R_3 = R_4 = 1\ \text{k}\Omega$ and $R_f = 10\ \text{k}\Omega$

The lead singer's microphone is then connected in series with $R_5 = 10\ \text{k}\Omega$.

59. <Design> One possible solution:

R_f(dark) = 100 kΩ; R_f(light) = 10 kΩ, measured at 6 candela.
Deliver 1.5 V to R_L

Arbitrarily, select V_s = 1 V.
Assume the resistance scales linearly with light intensity, so

$$R_f\,(2\text{ candela}) = \left(\frac{100\times10^3 - 10\times10^3}{0-6}\right)(2) + 100\times10^3 = 70\text{ k}\Omega$$

With R_f = 70 kΩ and $v_{out} = (1 + R_f/R_1)V_s = 1.5$, $R_1 = 2R_f$ = 140 kΩ

Check:

0 candela leads to R_f = 100 kΩ, so v_{out} = (1 + 100/140)(1) = 1.7 V and it will *not* activate.

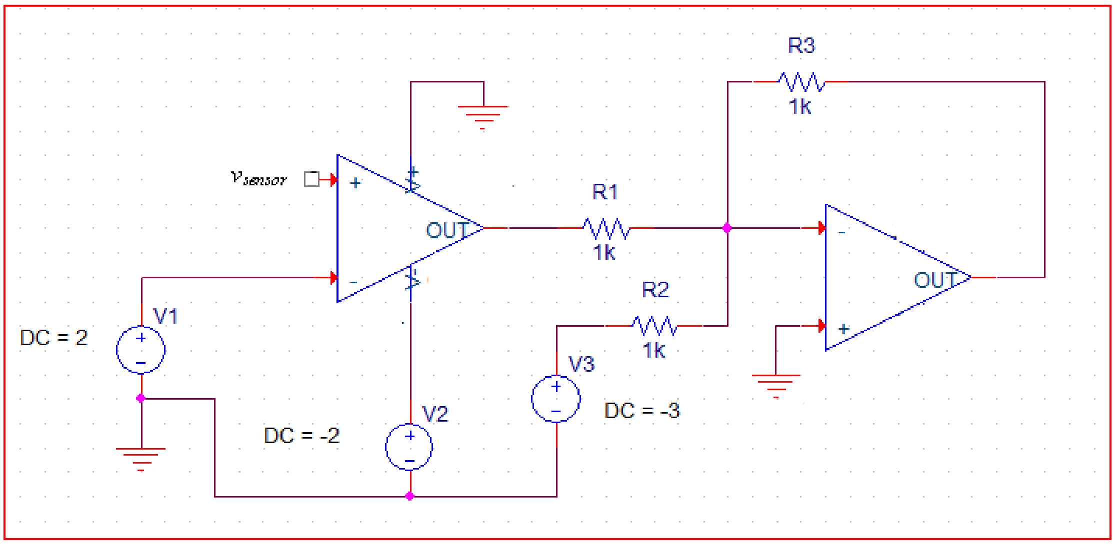
6 candela leads to R_f = 10 kΩ so vout = (1 + 10/140)(1) = 1.07 V and it will activate.

60. <Design> One possible solution:

When the wind velocity exceeds 50 km/h, the fountain height must be ≤ 2 m.

We assume the valve position tracks the applied voltage linearly (e.g. 2.5 V corresponds to 50% open). We also assume the flow rate scales linearly with the valve position.

So, 2 m height corresponds to 5 - (2/5)(5) = 3 V applied to the valve.



Description: When the sensor voltage is above 2 V, corresponding to a wind velocity greater than 50 km/h, the comparator stage outputs 0 V. This is summed with –3 V, the sign of which is inverted by the second stage to yield 3 V, or 2 m height. When the sensor voltage drops below 2 V, the comparator stage output is –2 V. This is summed with –3 V and the sign inverted, or 5 V output, corresponding to the valve fully open, which is presumably 100 l/s flow rate.

61. By nodal analysis,

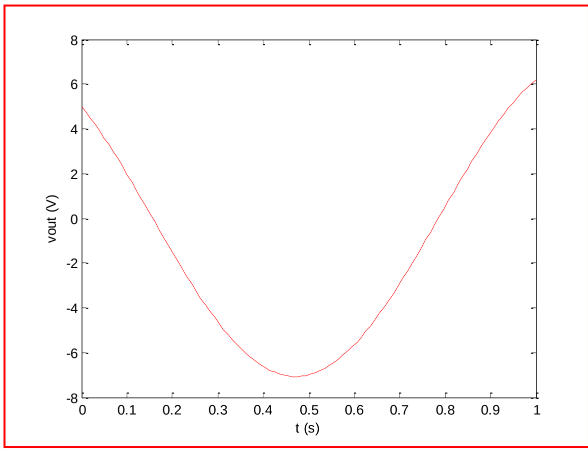
$$0 = \frac{v_m - v_1}{R} + \frac{v_m - v_{out}}{R} \qquad [1]$$

$$v_m = v_2 \frac{R}{2R} = \frac{v_2}{2} \qquad [2]$$

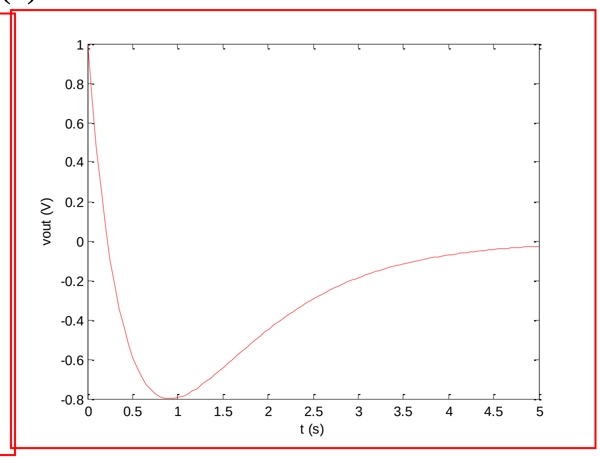
Consequently,

$$0 = \frac{0.5v_2 - v_1}{R} + \frac{0.5v_2 - v_{out}}{R}$$ and $v_{out} = v_2 - v_1$. Thus, the resistor value is unimportant.

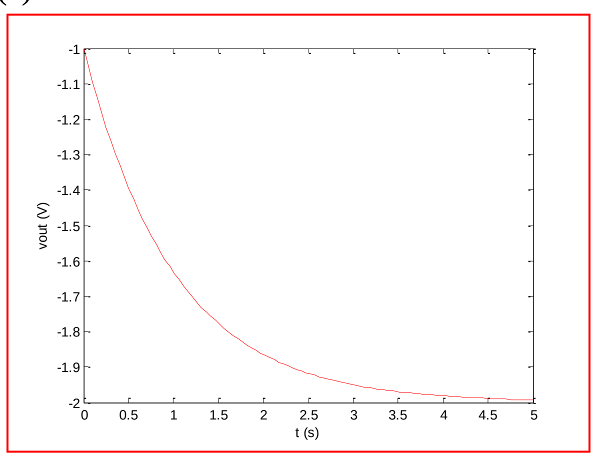
(a)



(b)



(c)



1. Assuming the passive sign convention, $i = C\dfrac{dv}{dt}$.

(a) $\boxed{i = 0}$ (dc)

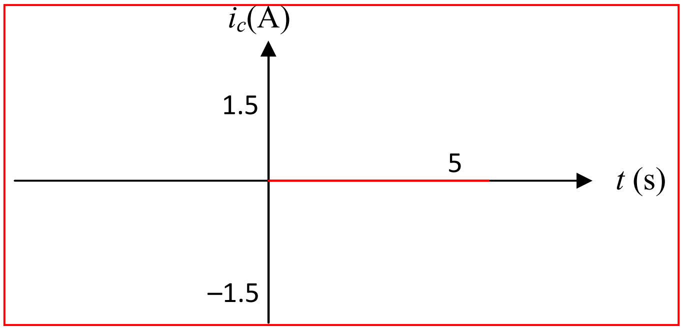
(b) $i = (220)(-9)(16.2)e^{-9t} = \boxed{-32.08e^{-9t}\ \text{A}}$

(c) $i = (220\times10^{-9})(8\times10^{-3})(-0.01)\sin 0.01t = \boxed{-17.6\sin 0.01t\ \text{pA}}$

(d) $i = (220\times10^{-9})(9)(0.08)\cos 0.08t = \boxed{158.4\cos 0.08t\ \text{nA}}$

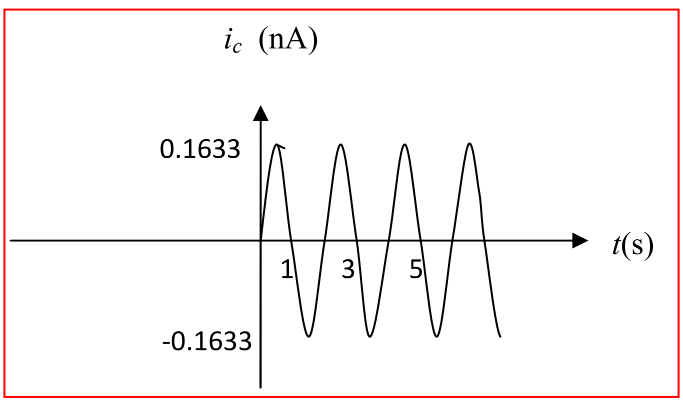
2. (a) C = 13 pF, assume passive sign convention. v =1.5 V for -1 $\leq t \leq$ 5 s

$$i_C = C\frac{dv}{dt} = 0\text{A} \quad \text{for} \quad t \geq 0$$



(b) $v_c = 4\cos\pi t$

$$i_c = C\frac{dv_c}{dt} = (13\times10^{-12})\left(4\times\pi\times\sin\pi t\right) = 0.1633\sin\pi t \text{ nA for t} \geq 0\text{s}$$



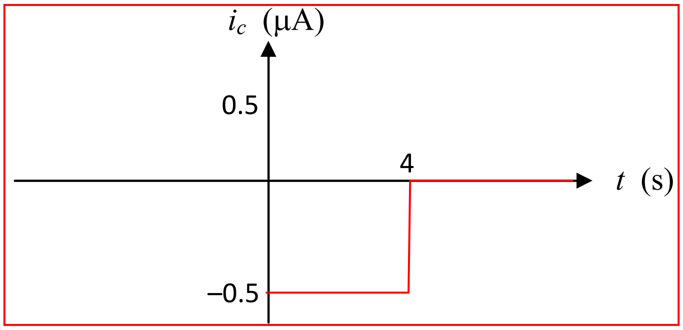
3. (a) C = 1 μF, assume passive sign convention.

For $t > 4$ s, $v = 1$ V therefore $i_C = 0$

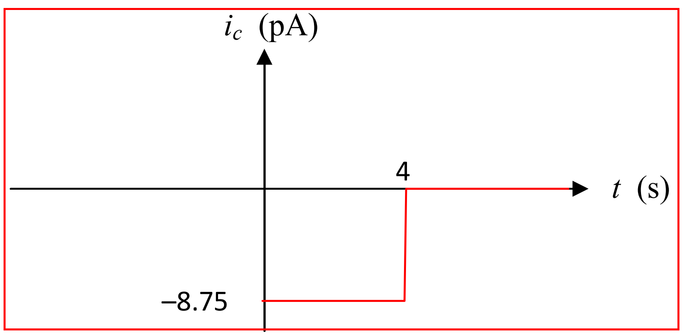
For all other times, $v = -\frac{1}{2}t + 3$

(a) C = 1 μF, assume passive sign convention.

$$i_C = C\frac{dv}{dt} = (10^{-6})\left(-\frac{1}{2}\right) = -0.5\times10^{-6}\text{ A} \quad \text{for } 0 \le t \le 4\text{ s}$$



(b) $i_C = (17.5\times10^{-12})\left(-\frac{1}{2}\right) = -8.75\text{ pA} \quad \text{for } 0 \le t \le 4\text{ s}$



4. (a) $\varepsilon = 1.35\varepsilon_0$

$$A = \left(1\times10^{-3}\right)\times\left(2.5\times10^{-3}\right) = 2.5\times10^{-6}\,m^2$$

$$C = \varepsilon\frac{A}{d} = \frac{1.35\times8.854\times10^{-12}\times2.5\times10^{-6}}{10^{-6}} = \boxed{29.88\,pF}$$

(b) $\varepsilon = 3.5\varepsilon_0$

$$C = \varepsilon\frac{A}{d} = \frac{3.5\times1.35\times8.854\times10^{-12}\times2.5\times10^{-6}}{10^{-6}} = \boxed{77.47\,pF}$$

(c) $\varepsilon = \varepsilon_0;\ d = 3.5\mu m$

$$C = \varepsilon\frac{A}{d} = \frac{8.854\times10^{-12}\times2.5\times10^{-6}}{3.5\times10^{-6}} = \boxed{6.324\,pF}$$

(d) $\varepsilon = \varepsilon_0;\ A = 2A = 5\times10^{-6}\,m^2;\ d = 1\mu m$

$$C = \varepsilon\frac{A}{d} = \frac{8.854\times10^{-12}\times5\times10^{-6}}{1\times10^{-6}} = \boxed{44.27\,pF}$$

5. Gadolinium is a metallic (conducting) element.

$A = (100\times10^{-6})(750\times10^{-6}) = 7.5\times10^{-8}\ \text{m}^2$. $C = \varepsilon \dfrac{A}{d}$.

(a) $\boxed{C = 91.64\ \text{pF}}$

(b) $\boxed{C = 3.321\ \text{nF}}$

(c) $\boxed{C = 3.32\ \text{pF}}$

(d) $\boxed{C = 13.28\ \text{pF}}$

6. One possible solution.

We require a 100nF capacitor constructed from 1μm thick gold foil that fits entirely within the volume of a standard AAA battery with the available dielectric having permittivity $3.1\varepsilon_0$. A standard AAA battery has approximately a length of 4.45cm and the circular diameter of 1.05cm.

$$d = \frac{3.1\varepsilon_0 A}{C} = \frac{3.1\times 8.854\times 10^{-12}}{100\times 10^{-9}} A = 2.744\times 10^{-4}\, Am$$

If we select the area of the plates as 4 cm^2, the gap spacing between the plates is d = 0.1097μm.

7. <Design> One possible solution:

We recall that for a parallel-plate (air separated) capacitor, C = $\dfrac{\varepsilon\varepsilon_0 A}{d}$ where e is the relative permittivity of the spacer material, A = the plate area, d = the plate spacing, and the remaining term is the free-space permittivity. We propose to vary the capacitance value by sliding one conductor past the other.

We being with a flat piece of glass 1 m by 1 m and 2 mm thick (none of these dimensions are critical). Coat the center of the glass with a 0.1 mm thick layer of gold (material and thickness not critical) having dimensions 100 mm by 100 mm. Take a piece of plastic (e = 3) 0.01 mm thick. Coat the top side with a 0.1 mm thick layer of gold measuring 336 mm by 336 mm. We expect the metal to stiffen the plastic sufficiently so sliding will work. Place the plastic on the metal-coated glass slide, both metal layers facing upwards. Make electrical contact off to the side and neglect its contribution to capacitance (assume negligibly thin wire).

When the top metal layer is completely over the bottom metal layer, (the smaller area is the A in our equation), we have C = (3)(8.854e-12)(336e-3)(336e-3)/(0.01e-3) = 300 nF (to three significant figures). We then slide the top metal layer along glass until only one-third its length overlaps the bottom metal layer. This reduces the effective area of the capacitor by one-third, and hence the capacitance becomes 100 nF.

8. <Design> One possible solution:

In the first printing, the capacitance range was stated as 50 nF to 100 nF, but this should be changed to 50 pF to 100 pF to obtain more realistic areas.

We begin by noting that there is no requirement for linear correlation between knob position and capacitance, although that might be desirable.

Take two discs of plastic, each approximately 3 mm thick (so that they are stiff) and radius 85 mm. Coat one side of each disc with gold (thickness unimportant) but over only half the area (i.e. from 0 to 180 degrees). Mount the discs with the metal layers facing each other, separated by a 1 mm thick plastic spacer of very small diameter using a thin nonconducting rod as an axis.

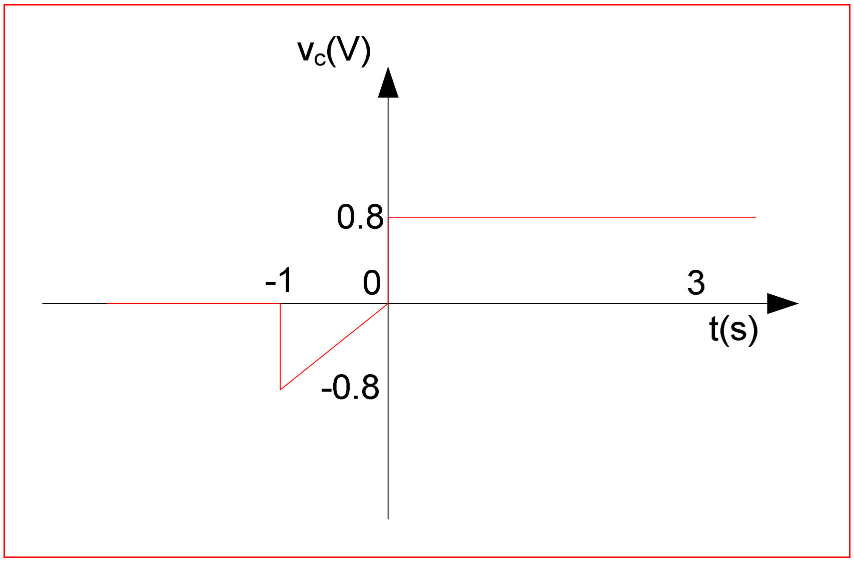
Then, with C = $\frac{\varepsilon\varepsilon_0 A}{d}$ where e is the relative permittivity of the spacer material, A = the plate area, d = the plate spacing, when the two metal halves are aligned over one another the capacitance is 100 pF. Rotating one disc by 90 degrees reduces the effective (overlap) area by one-half, and hence the new capacitance is 50 pF.

9. $$C = \frac{(11.8)(8.854\times10^{-14})(10^{-4}\times10^{-4})}{W}, \quad W \text{ in cm}$$

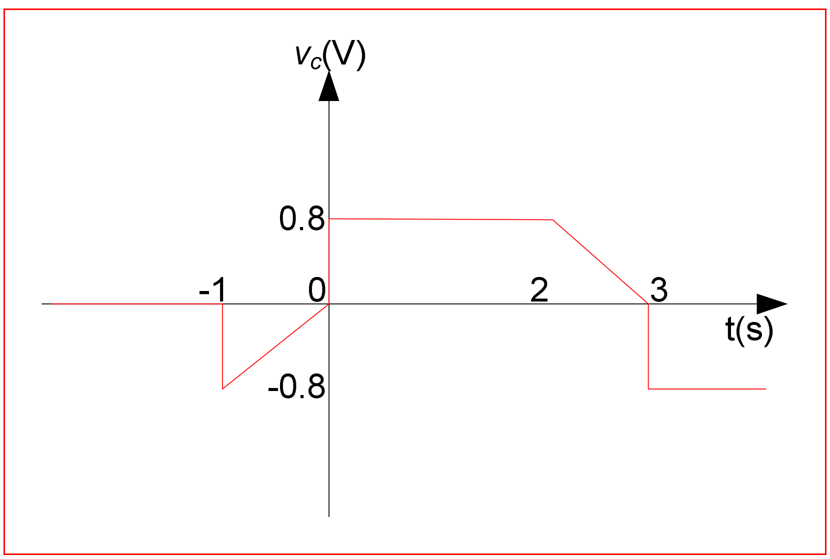
$$W = \left[\frac{(2)(11.8)(8.854\times10^{-14})}{(1.602\times10^{-19})(10^{18})}\left(0.57 - V_A\right)\right]^{\frac{1}{2}}$$

V_A	W(cm)	C
-1 V	4.525×10^{-6}	2.309 fF
-5 V	8.524×10^{-6}	1.226 fF
-10 V	1.174×10^{-5}	890 aF

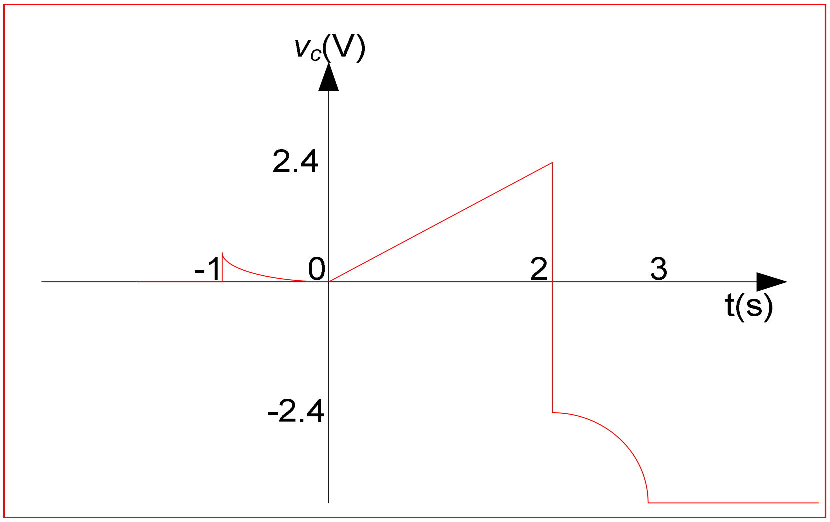
10. (a) Graph



(b) Graph

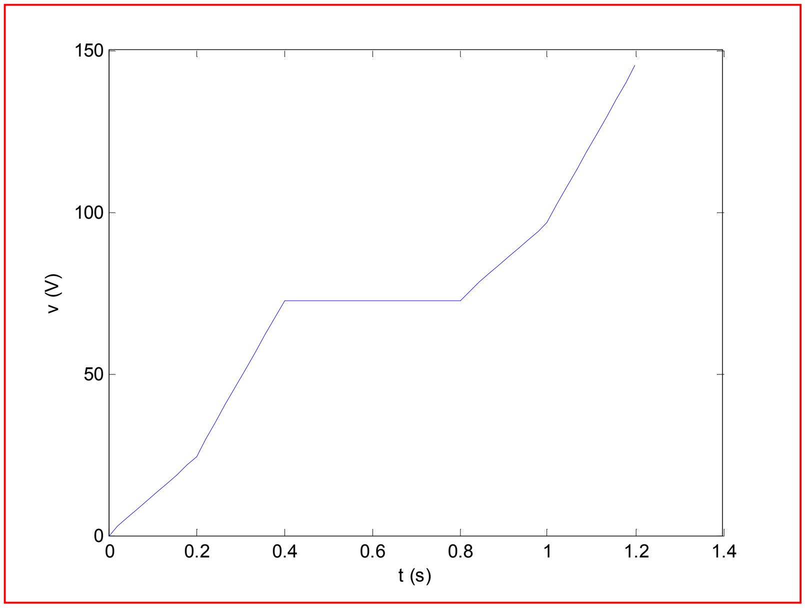


(c) Graph



11. C = 33 mF. $i = C\frac{dv}{dt}$ so $v = \frac{1}{C}\int idt$.

(a) Graph



(b) v(0.3 s) = half-way between 24.24 and 72.72 V = $\boxed{48.48 \text{ V}}$

v(0.6 s) = $\boxed{72.72 \text{ V}}$

v(1.1 s) = 72.72 + 48.48 = $\boxed{121.2 \text{ V}}$

12. (a) $W_c = \frac{1}{2} C {v_c}^2 = 0.5 \times 1.4 \times (8)^2 = \boxed{44.8J}$

(b) $W_c = \frac{1}{2} C {v_c}^2 = 0.5 \times 23.5 \times 10^{-12} \times (0.8)^2 = \boxed{7.52\,pJ}$

(c) $W_c = 295 \times 10^{-9} + \frac{1}{2} C {v_c}^2 = 295 \times 10^{-9} + 0.5 \times 17 \times 10^{-9} \times (12)^2 = \boxed{1.519\,\mu J}$

13. C = 137 pF

$$v_C(t) = \begin{cases} 12 \text{ V}, & t < 0 \\ 12e^{-2t} \text{ V}, & t \geq 0 \end{cases}. \quad \text{Energy} = \frac{1}{2}C\left[v_C\right]^2.$$

(a) t = 0: $\boxed{9.864 \text{ nJ}}$

(b) t = 0.2 s: $\boxed{4.432 \text{ nJ}}$

(c) t = 0.5 s: $\boxed{1.335 \text{ nJ}}$

(d) t = 1 s: $\boxed{180.7 \text{ pJ}}$

14. (a) After being connected to DC source for a very long time, the capacitor act as open circuit.

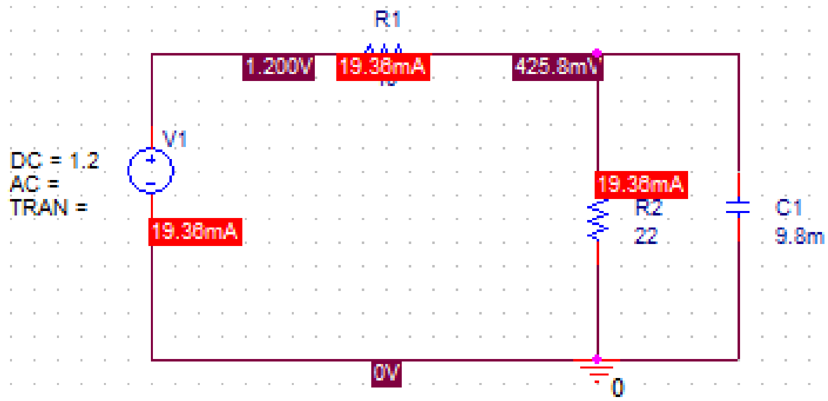
Therefore, current through the circuit is,

$$i = \frac{V}{R} = \frac{1.2}{40+22} = 19.3548mA$$

$$P_{40\Omega} = i^2 R = (19.3548\times10^{-3})^2 \times 40 = \boxed{14.984mW}$$

$$v_C = 0.193548 \times 22 = \boxed{0.4258V}$$

PSpice Verification:



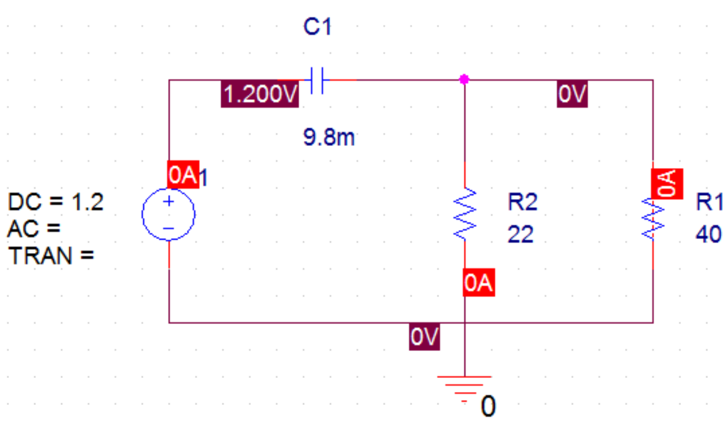
(b) After being connected to DC source for a very long time, the capacitor act as open circuit.

Therefore, current through the circuit is,

$$i = 0A$$

$$P_{40\Omega} = \boxed{0W}$$

$$v_C = \boxed{1.2V}$$



15. (a) no current through 5 ohm resistor so

$$-v_C = (4.5\times10^{-9})(13)\frac{7}{13+10+7} = 13.65\times10^{-9}\text{ V} \quad \text{so } \boxed{v_C = -13.65\text{ nV}}$$

(b) v_C = voltage across current source so

$$(4.5)[10+7\,||\,18] = \boxed{67.68\text{ nV}}$$

16. One possible design solution:

The general equation to calculate inductance is given by:

$$L = \frac{\mu N^2 A}{l}$$

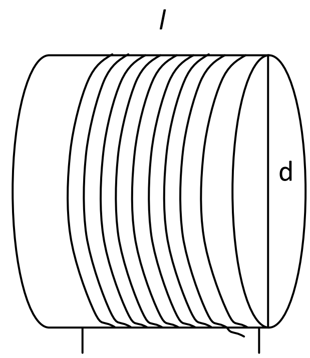
where $\mu = 4\pi \times 10^{-7}\ H/m$

N = Number of turns

A = Area of cross-section = $\pi r^2, m^2$

l = Length of the wire, m

r = outside radius of the coil(form + wire), m



If we take a plastic form and wound 29 AWG copper wire (diameter = 0.286mm) round it, we can construct a 30nH inductor. For that if we choose N = 22 turns, r = 60 mm, we can find the length of the wire.

$$l = \frac{\mu N^2 \pi r^2}{L} = \frac{4\pi \times 10^{-7} \times 22^2 \times \pi \times (30.143 \times 10^{-3})^2}{30 \times 10^{-9}} = 57.87m$$

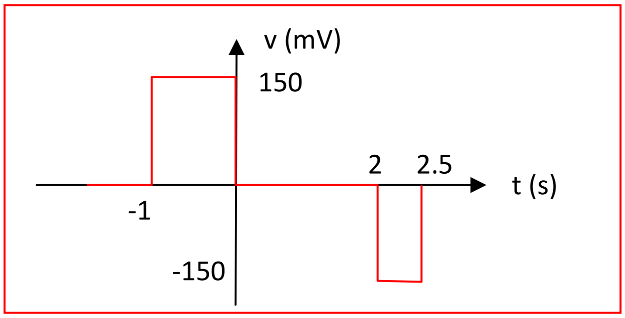
17. L = 75 mH.

$v = L\frac{di}{dt}.$

Voltage can change instantaneously but current cannot (or voltage becomes infinite).

From t = -1 to 0, i = 2(t + 1) so slope = +2. From t = 2 to 2.5, slope = -2.

(a)



(b) t = 1, 2.9 s, 3.1 s

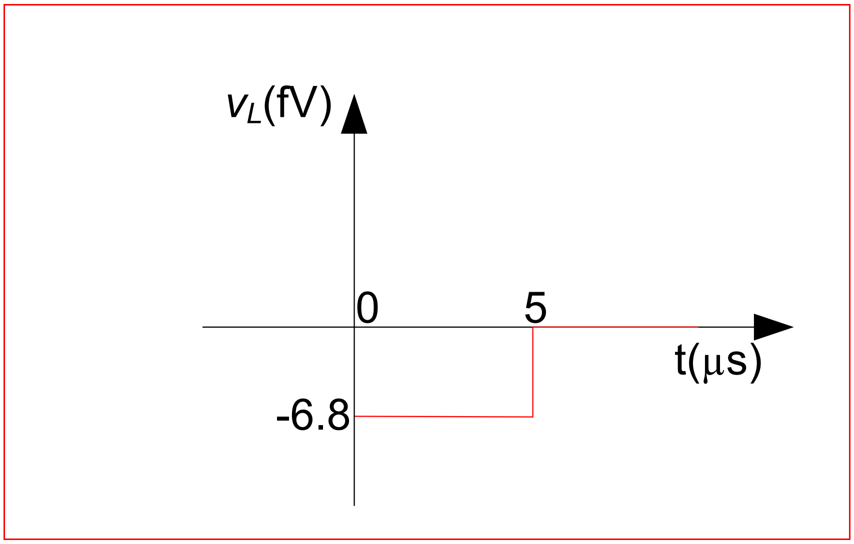
$\boxed{v(1) = 0; \quad v(2.9) = 0; \quad v(3.1) = 0;}$

18. Given, L = 17nH

$$i_L = -\frac{2}{5}\times 10^{-6}\, tA \; for\;\; 0 \le t \le 5\mu s$$

$$i_L = 2A \; for\;\; 5 \le t\mu s$$

$$v_L = L\frac{di_L}{dt} = 17\times 10^{-9}\times -\frac{2}{5}\times 10^{-6} = \boxed{-6.8\, fV}$$



19. $v = L\dfrac{di}{dt} = (4.2\times10^{-3})\dfrac{di}{dt}$

(a) $\boxed{\text{v} = 0}$

(b) $\boxed{75.6 \cos 6\text{t mV}}$

(c) $\boxed{-48.3\sqrt{2}\pi \sin(10\pi t - 9^\circ)\ \text{V}}$

(d) $\boxed{-54.6\text{e}^{-\text{t}}\ \text{pV}}$

(e) $(4.2\times10^{-3})\left[t(-14e^{-14t}) + e^{-14t}\right] = (4.2\times10^{-3})\left[-14t+1\right]e^{-14t}$

$\boxed{= -4.2(1-14t)e^{-14t}\ \text{mV}}$

20. (a) $i_L = 8mA,\ v_L = \boxed{0V}$

(b) $i_L = 800mA,\ v_L = \boxed{0V}$

(c) $i_L = 8A,\ v_L = \boxed{0V}$

(d) $i_L = 4e^{-t}A,\ v_L = L\dfrac{di_L}{dt} = 8\times10^{-12}\times -4e^{-t} = \boxed{-32e^{-t}\,pV}$

(e) $i_L = -3 + te^{-t}A,\ v_L = L\dfrac{di_L}{dt} = 8\times10^{-12}\times\left(e^{-t}\left(1-t\right)\right) = \boxed{-8e^{-t}\left(1-t\right)pV}$

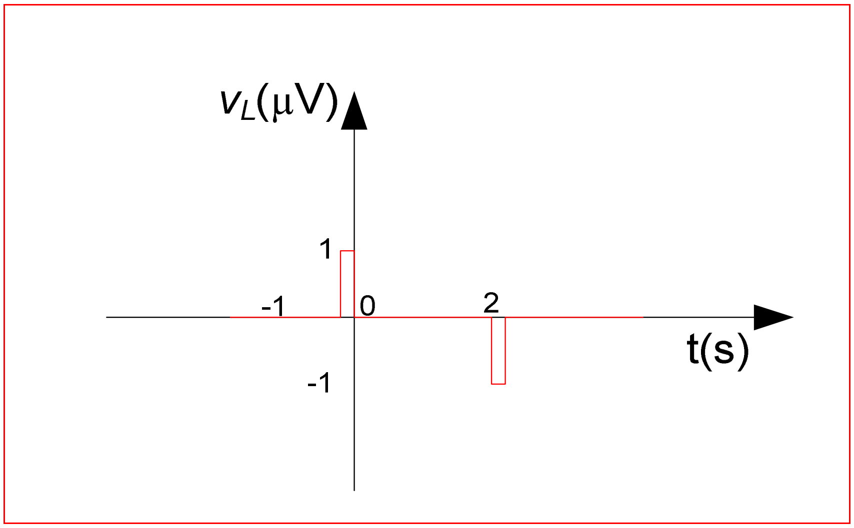
21. is = 1 mA, vs = 2.1 V

(a) $\boxed{vL = 0; \quad iL = is = 1\text{ mA}}$

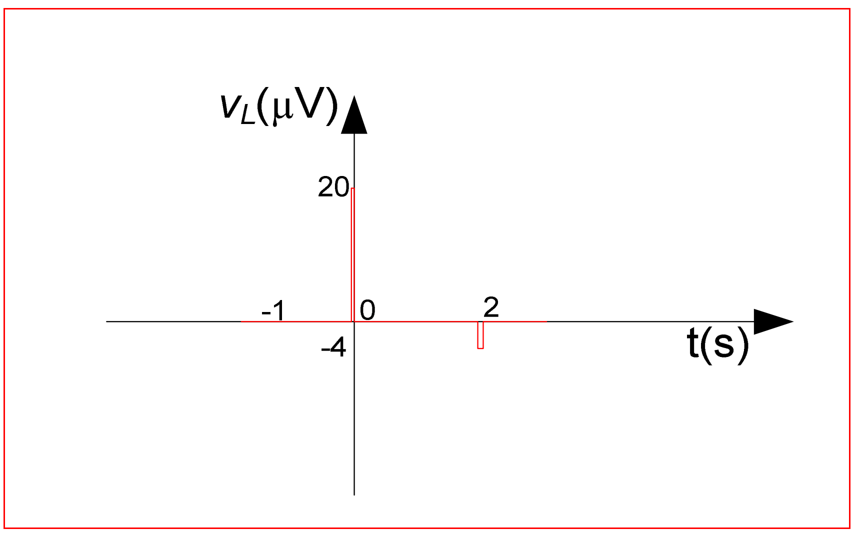
(b) 14 kΩ resistor is irrelevant here. $\boxed{vL = 0; \ iL = is = 1\text{mA}}$

(c) $vL = 0$; $iL = vs/4.7\text{x}10^3 = \boxed{446.8\ \mu\text{A}}$

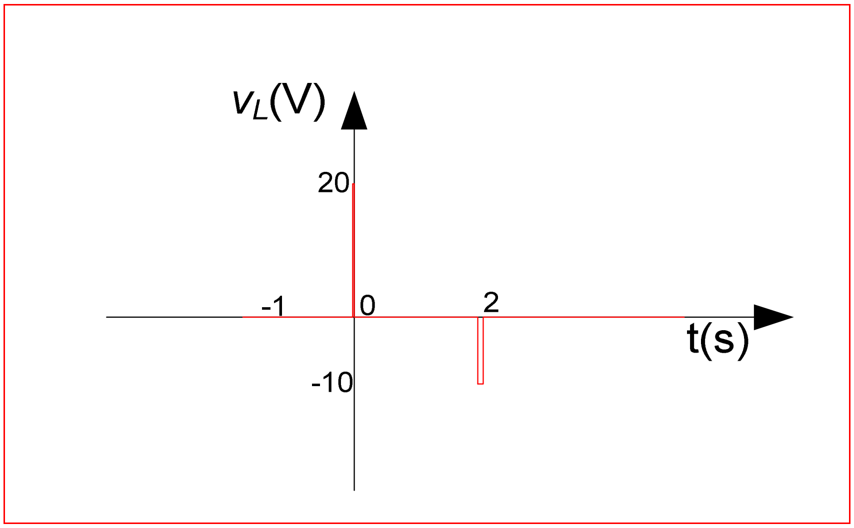
22. (a) Rise time, fall times: 200ms, 200ms



(b) Rise time, fall times: 10ms, 50ms



(c) Rise time, fall times: 10ns, 20ns



23. $v_L = L\dfrac{di_L}{dt}$.

Between -2 and 1 s, slope = (2 – 0)/(-2 – 0) = -1

Between 1 and 3 s, slope = -1/2

For t > 3 s, slope = -2/-3 = 2/3

$\boxed{\begin{array}{l}
\text{(a) } v(-1) = (1)(-1) = -1\text{ V} \\
\text{(b) } v(0) = (1)(-1) = -1\text{ V} \\
\text{(c) } v(1.5) = (1)(-1/2) = -0.5\text{ V} \\
\text{(d) } v(2.5) = (1)(-1/2) = -0.5\text{ V} \\
\text{(e) } v(4) = (1)(2/3) = 0.67\text{ V} \\
\text{(f) } v(5) = 0.67\text{ V}
\end{array}}$

24. (a) $i_L = \frac{1}{L}\int_0^t 5dt' + i_L(0) = \frac{5}{6\times10^{-3}}t = \boxed{833.33tA}$

(b) $i_L = \frac{1}{L}\int_0^t 100\sin 1200\pi t dt' = \frac{100}{6\times10^{-3}}\left[-\frac{\cos 1200\pi t}{1200\pi}\right]_0^t = \boxed{22.619\left(1-\cos 1200\pi t\right)A}$

25. $v_L = 4.3t, \ \ 0 \ \le \mathrm{t} \le 50 \text{ ms}$ and $iL(-0.1) = 100 \ \mu\text{A}$

$$iL = \frac{1}{L}\int_{-0.1}^{t} 4.3t'dt' + iL(-0.1) = \frac{4.3}{4}\left(t'^2 - 0.01\right) + 100 \times 10^{-6}$$

(a) $\boxed{\text{-10.65 mA}}$

(b) $\boxed{\text{-10.65 mA}}$

(c) $\boxed{\text{-8.473 mA}}$

26. (a) $E = \frac{1}{2} L{i_L}^2 = \boxed{0J}$

(b) $E = \frac{1}{2} L{i_L}^2 = 0.5 \times 1 \times 10^{-9} \times \left(10^{-3}\right)^2 = \boxed{5 \times 10^{-16} J}$

(c) $E = \frac{1}{2} L{i_L}^2 = 0.5 \times 1 \times 10^{-9} \times \left(20\right)^2 = \boxed{2 \times 10^{-7} J}$

(d) $E = \frac{1}{2} L{i_L}^2 = 0.5 \times 1 \times 10^{-9} \times \left(5 \sin 6t \times 10^{-3}\right)^2 = \boxed{1.25 \sin^2 6t \times 10^{-14} J}$

27. L = 33 mH, t = 1 ms. $w = 0.5Li^2$

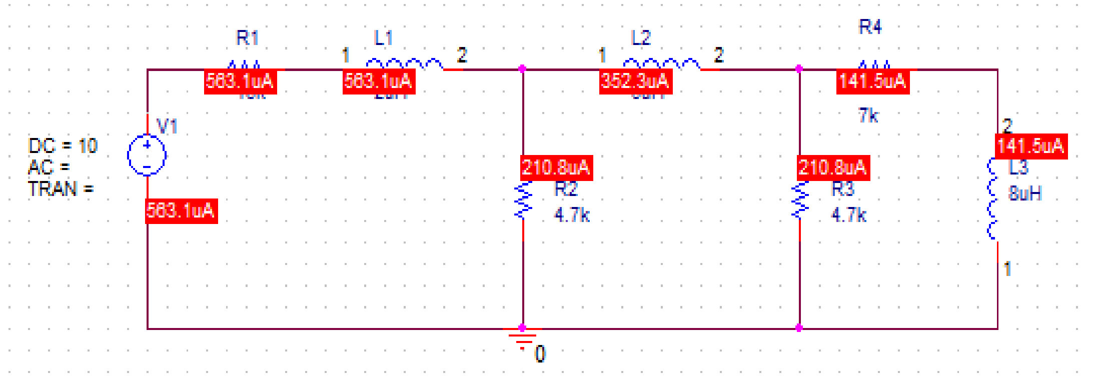
(a) $\boxed{808.5\text{ mJ}}$

(b) $\boxed{1.595\text{ nJ}}$

28. (a) After being connected to DC source for a very long time, the inductors act as short circuits.

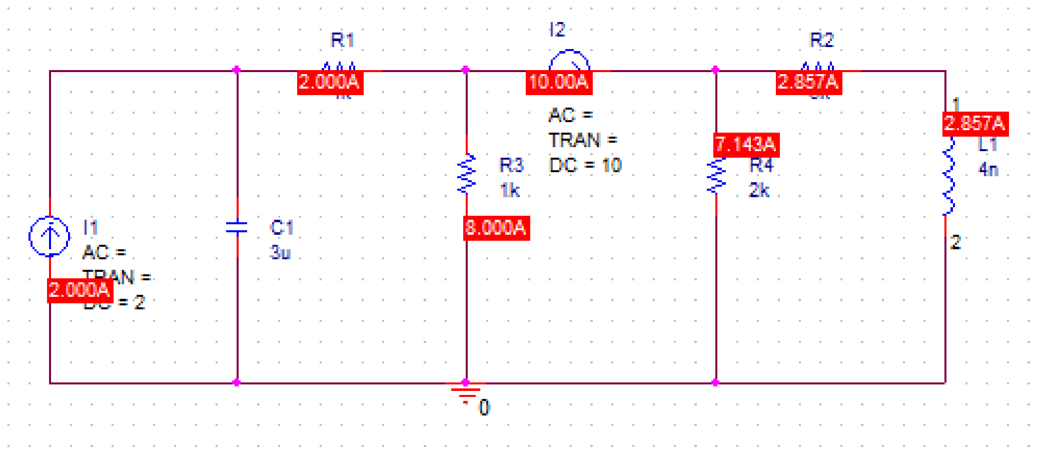
The total current in the given circuit then, $i = \dfrac{10}{\left(\left(\left(\dfrac{4.7}{2}\right)^{-1} + 7^{-1}\right)^{-1} + 16\right) \times 10^3} = 0.563mA$

Therefore, current $i_x = 0.563 \times 10^{-3} \dfrac{\left(\left(\dfrac{4.7}{2}\right)^{-1} + 7^{-1}\right)^{-1} \times 10^3}{7 \times 10^3} = \boxed{141.502\mu A}$



(b) After being connected to DC source for a very long time, the capacitor act as open circuit and inductor act as short circuits.

Therefore, current $i_x = 10 \dfrac{\left(5^{-1} + 2^{-1}\right)^{-1} \times 10^3}{5 \times 10^3} = \boxed{2.857A}$



29. (a) $-5 + V/27 + V/30 + (V-1)/20 = 0$

Solving, V = 41.95 V

By voltage division, Vx = (12/27)(41.95) = $\boxed{18.64\ \text{V}}$

(b) $-5 + V/27 + V/20 + (V-1)/20 = 0$

Solving, V = 36.85 V

By voltage division, Vx = (12/27)(36.85) = $\boxed{16.38\ \text{V}}$

(c) $-5 + V/27 + (V-1)/20 = 0$

V = 58.02 V

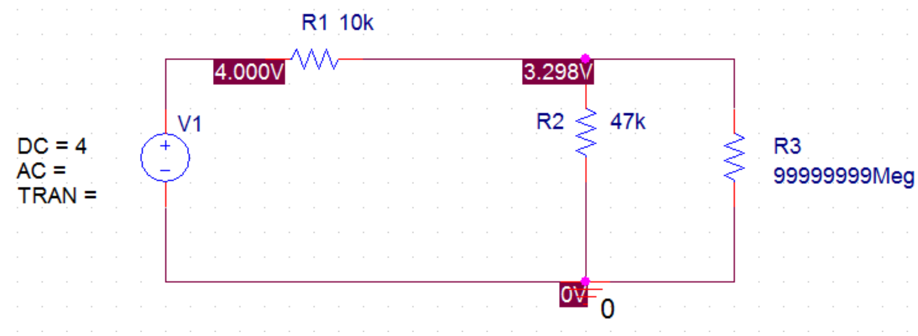
By voltage division, Vx = (12/27)(58.02) = $\boxed{25.79\ \text{V}}$

(d) Same as case (b): Vx = $\boxed{16.38\ \text{V}}$

30. (a) Taking inductor as the load, the thevenin equivalent resistance is given by,

$$R_{eq} = \left(10^{-1} + 47^{-1}\right)^{-1} \times 10^3 = \boxed{8.245k\Omega}$$

The thevenin voltage is given by, $V_T = 4 \times \frac{47 \times 10^3}{(47+10) \times 10^3} = \boxed{3.298V}$



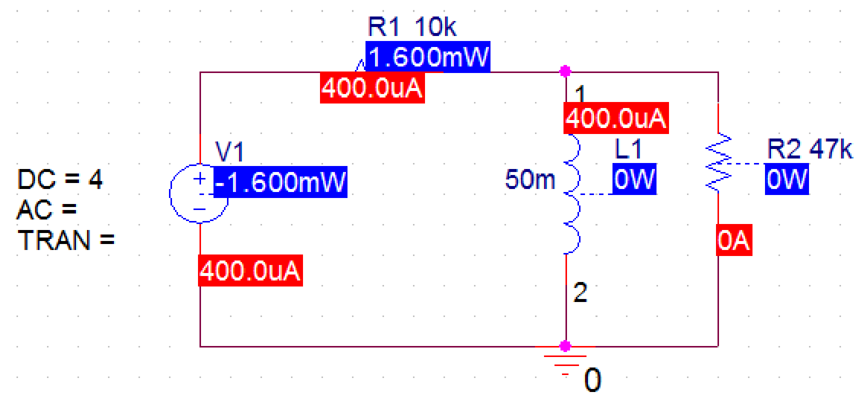
(b) After being connected to DC source for a very long time, the inductor acts as a short circuit.

$$i_L = \frac{4}{10 \times 10^3} = 400\mu A$$

$$P_{10k\Omega} = i_L{}^2 R = \left(0.4 \times 10^{-3}\right)^2 \times 10^4 = \boxed{1.6mW}$$

$$P_{47k\Omega} = \boxed{0W}$$

$$W_L = \frac{1}{2} L i_L{}^2 = 0.5 \times 50 \times 10^{-3} \times \left(0.4 \times 10^{-3}\right)^2 = \boxed{4nJ}$$



31. $C_{eq}^{-1} = \frac{1}{1.5} + \frac{1}{2} + \frac{1}{1.5} = \boxed{545.4 \text{ mF}}$

32. $$L_{equiv} = \left(\frac{1}{\frac{1}{2L} + \frac{1}{L}} \right) + L + \left(\frac{1}{\frac{1}{2L} + \frac{1}{L}} \right) = \boxed{\frac{7}{3} L}$$

33. Create a series strand of 5 1 H inductors (Leq = 5 H).
Place five such strands in parallel.
$(1/5 + 1/5 + 1/5 + 1/5 + 1/5)^{-1} = \boxed{1.25\text{ H}}$

34. Starting from the rightmost end, we have, a series combination of 2F, 12F and 2F, for which the equivalent capacitance is, $C_{eq1} = \dfrac{1}{\dfrac{1}{2}+\dfrac{1}{12}+\dfrac{1}{2}} = \dfrac{12}{13}F$

This is in parallel with the series combination of 8F and 5F. Therefore,

$$C_{eq2} = \frac{1}{\frac{1}{8}+\frac{1}{5}} + \frac{12}{13} = \frac{52}{13}F$$

Now, this is in series with 4F and 1F which yields the new capacitance as,

$$C_{eq3} = \frac{1}{\frac{1}{4}+1+\frac{13}{52}} = \frac{2}{3}F$$

.

This combination is in parallel with 5F and the final equivalent capacitance is,

$$C_{eq2} = \frac{1}{\left(\frac{2}{3}+5\right)^{-1}+7^{-1}} = \boxed{3.1315F}$$

35. $(1/7 + 1/22)^{-1} + 4 = 9.310$ F
$5 + (1/12 + 1/1)^{-1} = 5.923$ F

Thus, $C_{eq} = (1/9.310 + 1/5.923)^{-1} = \boxed{3.62 \text{ F}}$

36. Towards the rightmost end (b terminal) of the given circuit, 12H is in series with 1H, the combination is in parallel with 5H. Therefore, the equivalent inductance is given by,

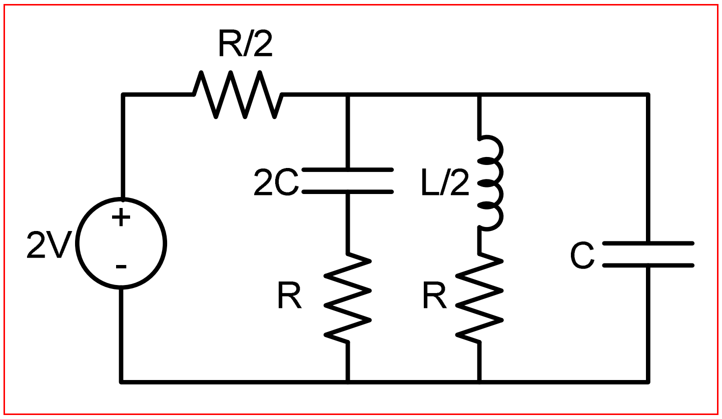
$$L_{eq1} = \frac{1}{\frac{1}{(12+1)} + \frac{1}{5}} = \frac{65}{18} H$$

On the left side (a terminal) 12H is in parallel with 10H, and the combination is in series with 7H and the whole combination is in parallel with 4H. Therefore, the equivalent inductance on this side is given by, $L_{eq2} = \dfrac{1}{\left(\dfrac{1}{\frac{1}{12}+\frac{1}{10}} + 7\right)^{-1} + 4^{-1}} = \dfrac{548}{181} H$

The total equivalent inductance seen from the a b terminal then becomes,

$$L_{eq} = L_{eq1} + L_{eq2} = \frac{65}{18} + \frac{548}{181} = \boxed{6.638 H}$$

37. The circuit can be simplified as:



38. (a) For each element as 10Ω resistor, the equivalent resistance is given as,

$$R_{eq} = \frac{1}{\dfrac{1}{(R+R+R)} + \dfrac{1}{\left(\left(R^{-1}+R^{-1}\right)^{-1} + R + \left(R^{-1}+R^{-1}+R^{-1}\right)^{-1}\right)}}$$

$$= 1.1379R = \boxed{11.379\Omega}$$

(b) For each element as 10H inductor, the equivalent inductance is given as,

$$L_{eq} = \frac{1}{\dfrac{1}{(L+L+L)} + \dfrac{1}{\left(\left(L^{-1}+L^{-1}\right)^{-1} + L + \left(L^{-1}+L^{-1}+L^{-1}\right)^{-1}\right)}}$$

$$= 1.1379L = \boxed{11.379H}$$

(c) For each element as 10F capacitor, the equivalent capacitance is given as,

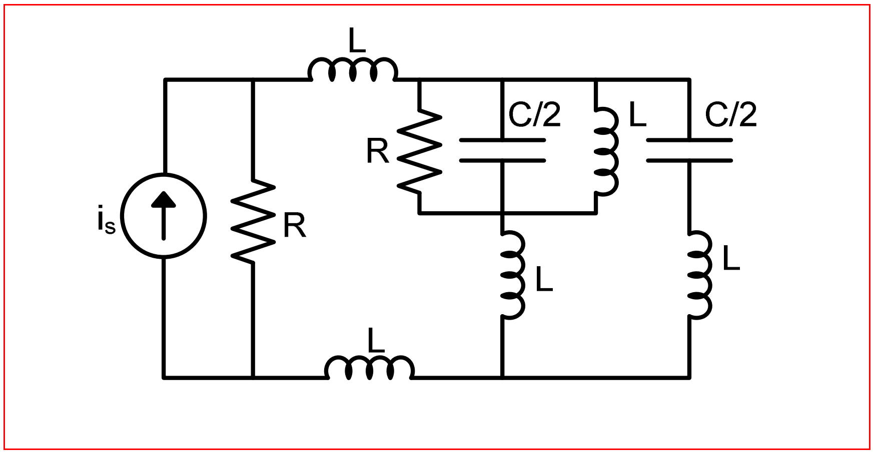
$$C_{eq} = \frac{1}{\left(C^{-1}+C^{-1}+C^{-1}\right)} + \frac{1}{\left((C+C)^{-1} + C^{-1} + \left(C+C+C\right)^{-1}\right)}$$

$$= 0.8787C = \boxed{8.7878F}$$

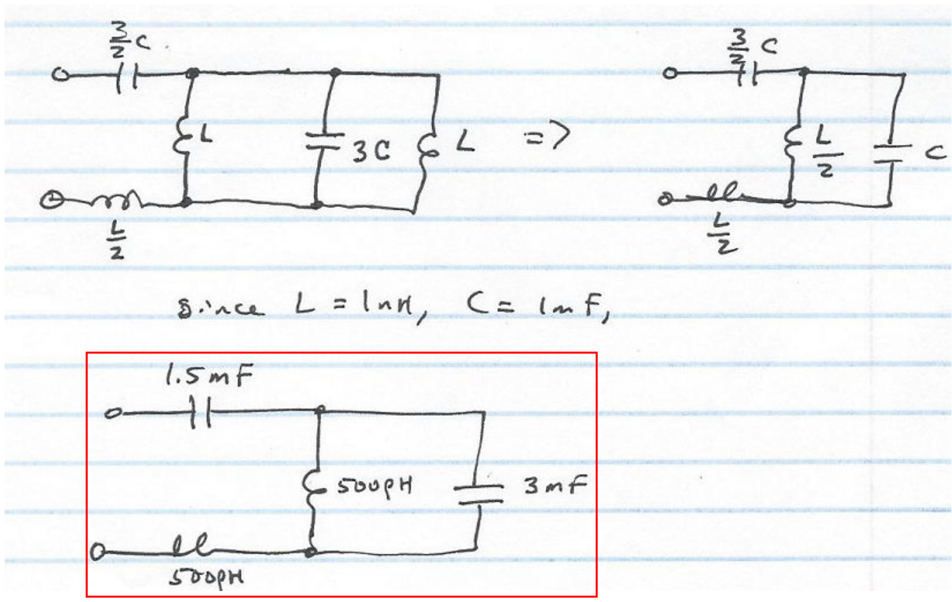
39. $1/L_{eq} = 1/1 + ½ + 1/1 + 1/7 + ½ + ¼$

Thus, $\boxed{L_{eq} = 294.7\ \text{pH}}$

40. The circuit can be simplified as:



41.



42. (a) $L_{equiv} = 1 + \left(\dfrac{1}{\frac{1}{2}+\frac{1}{2}} \right) + \left(\dfrac{1}{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} \right) = \boxed{3H}$

(b) For the given network having 3stages, we can write,

$$L_{equiv} = 1 + \left(\frac{1}{\frac{1}{2}+\frac{1}{2}} \right) + \left(\frac{1}{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} \right) = 1 + \frac{1}{2\left(\frac{1}{2}\right)} + \frac{1}{3\left(\frac{1}{3}\right)}$$

For the general network of this type having N stages, we can write,

$$L_{equiv} = 1 + \left(\frac{1}{\frac{1}{2}+\frac{1}{2}} \right) + \left(\frac{1}{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} \right) + \ldots. + \left(\frac{1}{\frac{1}{N}+\ldots+\frac{1}{N}} \right)$$

$$= 1 + \frac{1}{2\left(\frac{1}{2}\right)} + \frac{1}{3\left(\frac{1}{3}\right)} + \ldots. + \frac{1}{N\left(\frac{1}{N}\right)} = \boxed{N}$$

43. Far right: $\left[(2\,pF)^{-1}+(2\,pF)^{-1}\right]^{-1}=1$ pF

Which is in parlle with 2 pF, for a combined value of 3 pF.

Since $\left[(3\,pF)^{-1}+(2\,pF)^{-1}\right]^{-1}=\dfrac{6}{5}$ pF and this is in parallel with 2 pF,

we obtain a total of 16/5 pF.

This equivalent value is in sries with 2 pF, hence the final value is $\boxed{1.23 \text{ pF.}}$

44. For the rightmost end, 1nH is in series with 1nH, the combination is in parallel with 1nH.

$$L_{eq1} = \frac{1}{\frac{1}{2}+1} = \frac{2}{3}nH$$

This combination is in series with 1nH and the new combination is in parallel with 1nH.

$$L_{eq2} = \frac{1}{\frac{1}{\left(\frac{2}{3}+1\right)}+1} = \frac{5}{8}nH$$

Now, this combination is in series with 1nH. Hence the net equivalent inductance is given by,

$$L_{eq} = \frac{5}{8}+1 = \boxed{1.625nH}$$

45. (a) $0 = C_1 \dfrac{d(v_1 - v_3)}{dt} + C_2 \dfrac{dv_1}{dt} + \dfrac{v_1 - v_2}{R}$

or $\boxed{(C_1 + C_2)\dfrac{dv_1}{dt} + \dfrac{v_1}{R} - \dfrac{v_2}{R} = C_1 \dfrac{dv_3}{dt} \quad [1]}$

Next, $-is = \dfrac{v_2}{R} + \dfrac{v_2 - v_1}{R} + \dfrac{1}{L}\displaystyle\int_{t_0}^{t} (v_2 - v_5)\,dt' - i_L(0^-)$ or

$$\boxed{-\frac{v_1}{R} + 2\frac{v_2}{R} + \frac{1}{L}\int_{t_0}^{t} v_2\,dt' = i_L(0^-) - is + \frac{1}{L}\int_{t_0}^{t} v_s\,dt' \quad [2]}$$

(b)

$$-v_s + \frac{1}{C_1}\int_{t_0}^{t} (i_1 - i_L)\,dt' + \frac{1}{C_2}\int_{t_0}^{t} (i_1 - i_2)\,dt' = 0$$

$$\frac{1}{C_2}\int_{t_0}^{t} (i_2 - i_1)\,dt' + R(i_2 - i_L) + R(i_2 - i_s) = 0$$

$$L\frac{di_L}{dt} + R(i_2 - i_L) + \frac{1}{C_1}\int_{t_0}^{t} (i_L - i_1)\,dt' = 0$$

46. (a)

$$\boxed{\begin{aligned}&\frac{v_C - v_S}{20} - 5\times10^{-6}\times\frac{dv_C}{dt} - \frac{v_C - v_L}{10} = 0\\&\frac{v_C - v_L}{10} - \frac{1}{8\times10^{-3}}\int_0^t v_L\,dt' - 2 = 0\end{aligned}}$$

(b)

$$\boxed{\begin{aligned}&v_S = 20i_{20} + \frac{1}{5\times10^{-6}}\int_0^t (i_{20} - i_L)dt' + 12\\&\frac{1}{5\times10^{-6}}\int_0^t (i_{20} - i_L)dt' + 12 = 10i_L + 8\times10^{-3}\times\frac{di_L}{dt}\end{aligned}}$$

47. is(0) = 60 mA therefore i2 = i3 – i1 = 40 mA

(a) $i_s = \frac{1}{6}\int_0^t v dt' + i_1(0) + \frac{1}{4}\int_0^t v dt' + i_L(0)$

$\frac{di_s}{dt} = \frac{1}{6}v + \frac{1}{4}v \therefore \boxed{v(t) = -5e^{-200t} \text{ V}}$

(b) $i_1(t) = \frac{1}{6}\int_0^t v dt' + i_1(0) = \boxed{4.17e^{-200t} + 20 \text{ mA}}$

(c) $i_2(t) = \frac{1}{4}\int_0^t v dt' + i_2(0) = \boxed{6.25e^{-200t} + 40 \text{ mA}}$

48. (a) $i(t) = C_{eq}\dfrac{dv_S}{dt} = \left(1+4^{-1}\right)^{-1}\times 10^{-6}\times(-80)\times 100e^{-80t} = \boxed{-6.4e^{-80t}\,mA}$

(b) $v_1(t) = v_1(0) + \dfrac{1}{C}\displaystyle\int_0^t i(t)dt'$

$$= 20 + \frac{1}{10^{-6}}(-6.4\times 10^{-3})\int_0^t e^{-80t}dt' = \boxed{80e^{-80t} - 60V}$$

(c) $v_2(t) = v_2(0) + \dfrac{1}{C}\displaystyle\int_0^t i(t)dt'$

$$= 80 + \frac{1}{4\times 10^{-6}}(-6.4\times 10^{-3})\int_0^t e^{-80t}dt' = \boxed{20e^{-80t} + 60V}$$

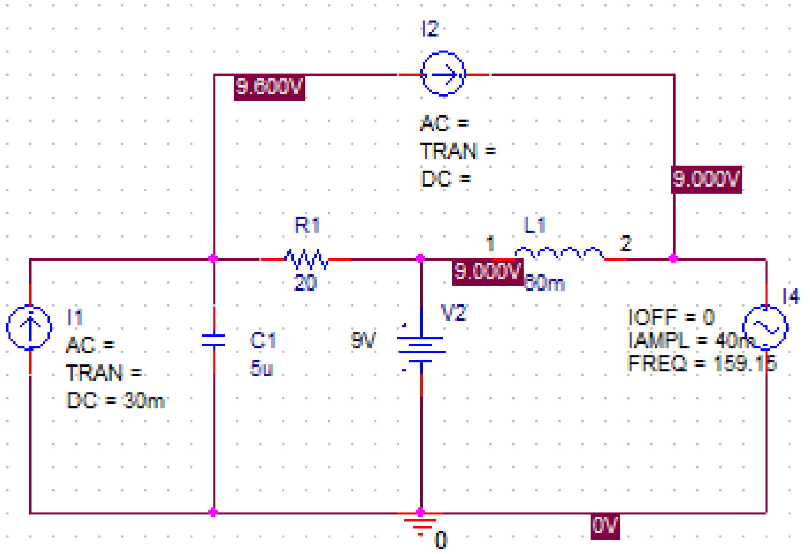
49. It is assumed that all the sources in the given circuit have been connected for a very long time. We can replace the capacitor with an open circuit and the inductor with a short circuit and apply principle of superposition.

By superposition, we get,

$v_C = 0.6 + 9 + 0 + 0 = \boxed{9.6V}$

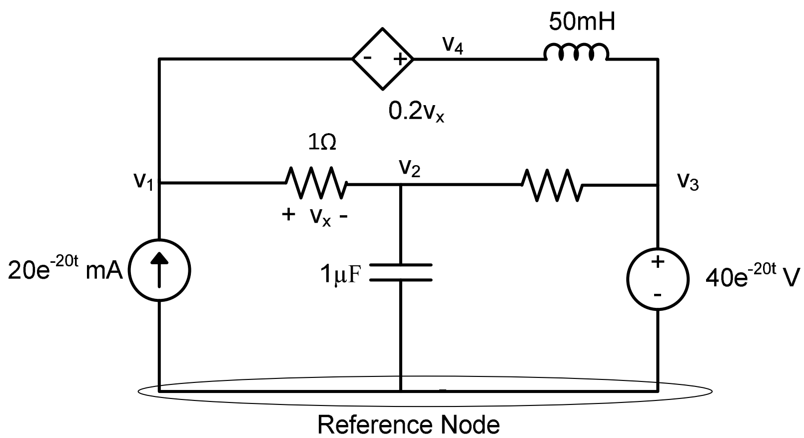
$v_L = 0 = \boxed{0V}$

Pspice verification:



50. Let us assign the node voltages as V_1, V_2, V_3 and V_4 with the bottom node as the reference.

Supernode:



1,4 Supernode: $20\times10^{-3}e^{-20t}+\frac{1}{50\times10^{-3}}\int_0^t(v_3-v_4)dt'=\frac{v_1-v_2}{50}$

and $0.8v_1+0.2v_2-v_4=0$

Node 2: $\frac{v_1-v_2}{50}=10^{-6}\frac{dv_2}{dt}+\frac{v_2-40e^{-20t}}{100}$

51. (a) Assuming an ideal op amp,

$$i_{C_f} = C_f \frac{dv_{C_f}}{dt}$$

$$C_f \frac{dv_s}{dt} = \frac{0 - v_{out}}{R_1}$$

Thus, $\boxed{v_{out} = -R_1 C_f \frac{dv_s}{dt}}$

(b) Assuming a finite A,

$$C_f \frac{dv_s}{dt} = \frac{v_d - Av_d}{R_1}$$

$$C_f \frac{dv_s}{dt} = \frac{\frac{v_{out}}{A} - v_{out}}{R_1}$$

$$R_1 C_f \frac{dv_s}{dt} = \frac{v_{out}}{A} - v_{out}$$

Solving, $\boxed{v_{out} = \left(\frac{A}{1-A}\right) R_1 C_f \frac{dv_s}{dt}}$

52. (a) $A = \infty, R_i = \infty,$ and $R_o = 0;$

$R_1 = 100k\Omega, C_f = 500\mu F, v_s = 20\sin 540t \ mV$

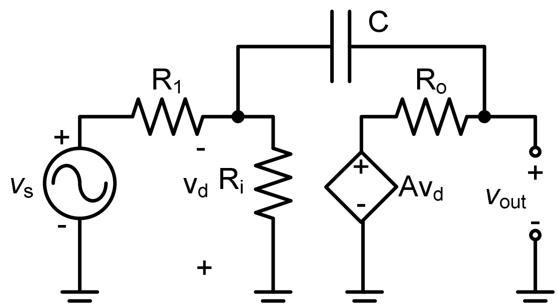
$$v_{out}(t) = -\frac{1}{R_1 C_f}\int_0^t v_s dt' - v_{cf}(0)$$

$$= -\frac{10^{-3}}{100\times10^3\times500\times10^{-6}}\int_0^t 20\sin 540t dt' = \boxed{-0.7407\cos 540t + C\mu V}$$

C being a constant value

(b) $A = 5000, R_i = 1M\Omega,$ and $R_o = 3\Omega;$

$R_1 = 100k\Omega, C_f = 500\mu F, v_s = 20\sin 540t \ mV$



Writing the nodal equations we get,

$$\frac{v_d + v_s}{R_1} + \frac{v_d}{R_i} + C\frac{d}{dt}\left(v_d + v_{out}\right) = 0...............(1)$$

$$C\frac{d}{dt}\left(v_d + v_{out}\right) + \frac{v_{out} - Av_d}{R_o} = 0................(2)$$

On solving these two equations, we get,

$$v_d = 2\times10^{-4}v_{out} - 6\times10^{-9}v_s V$$

$$v_{out} = 6.4787\times10^{-8}\int\cos 540t dt' - 4\times10^{-4}\int\sin 540t dt'$$

$$= \boxed{1.12\times10^{-10}\sin 540t - 7.407\times10^{-7}\cos 540t + C \text{ V}}$$

C being a constant value

53. $i_L = \frac{1}{L}\int_{0^-}^{t} v_s dt' + i_L(0^-)$. Assuming an ideal op amp,

$$\frac{1}{L}\int_{0^-}^{t} v_s dt' + i_L(0^-) = \frac{0 - v_{out}}{R_f}$$

Thus, $\boxed{v_{out} = -\frac{R_f}{L}\int_{0^-}^{t} v_s dt' + R_f i_L(0^-)}$

54. (a) Assuming an ideal op amp,

$$i_{C_f} = C_f \frac{dv_{C_f}}{dt}; i_{R_f} = \frac{0 - v_{out}}{R_f}; v_{C_f} = 0 - v_{out}$$

$$i_{C_f} = C_f \frac{dv_{C_f}}{dt}; i_{R_f} = \frac{0 - v_{out}}{R_f}; v_{C_f} = 0 - v_{out}$$

$$C_f \frac{dv_{out}}{dt} + \frac{v_{out}}{R_f} = -\frac{v_s}{R_1}$$

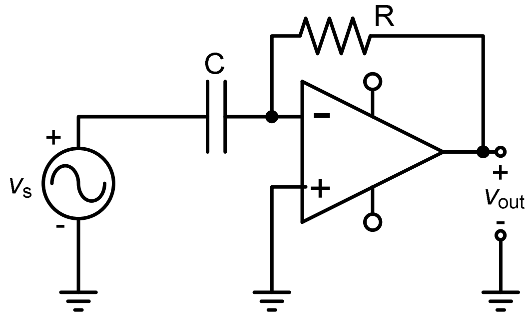
$$\boxed{\frac{dv_{out}}{dt} + \frac{v_{out}}{R_f C_f} = -\frac{v_s}{R_1 C_f}}$$

(b) Equation 17 is:

$$v_{out} = -\frac{1}{R_1 C_f} \int_0^t v_s dt' - v_{C_f}(0)$$

In case of practical integrator circuit, when R_f is very large, the solution of the equation obtained in (a) is equation 17 itself. In case of ideal integrator under DC conditions, the capacitor acts as an open circuit and therefore op-amp gain is very high. As we know that op-amp has input offset voltage, the input offset voltage gets amplified and appears as an error voltage at the output. Due to such error voltage the op-amp can get easily saturated. So in order to reduce the effect of such error voltage, usually a very large resistor is added in parallel with the feedback capacitor in case of practical integrator circuit. Thereby the DC gain is limited to $-R_f/R_1$.

55. One possible solution:



We want, 1V = 1◦C/s

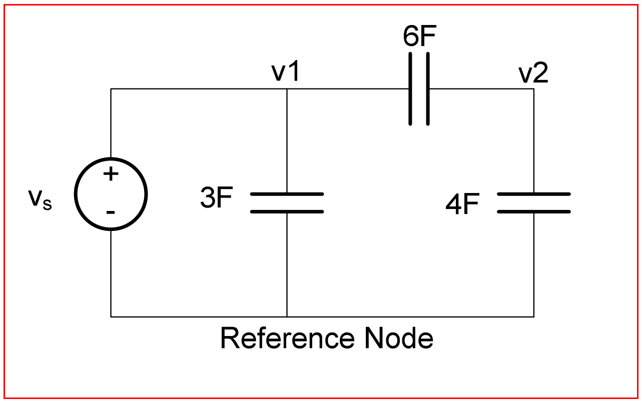
$$v_{out} = -RC\frac{dv_s}{dt}$$

$$|v_{out}| = 1 = RC\left(\frac{10^{-3}V}{s}\right)$$

$$RC = 1000$$

If we arbitarily select $R = 1M\Omega, C = 1000\mu F$

60. (a, b)



(c)

Original circuit, nodal:

$$i_s - \frac{1}{3}\int_{0^-}^{t}(v_1 - v_2)dt' - i_1(0^-) = 0$$

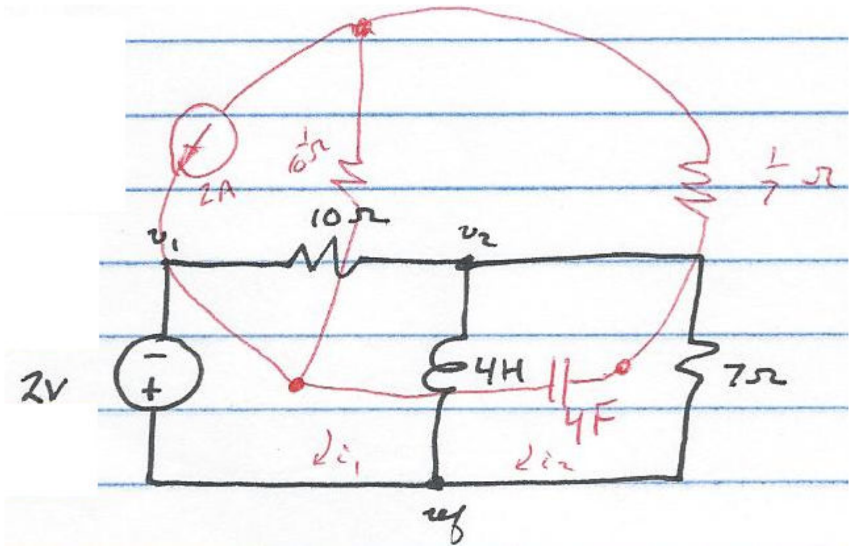
$$\frac{1}{3}\int_{0^-}^{t}(v_1 - v_2)dt' + i_1(0^-) - \frac{1}{6}\int_{0^-}^{t} v_2 dt' - i_1(0^-) - \frac{1}{4}\int_{0^-}^{t} v_2 dt' - i_2(0^-) = 0$$

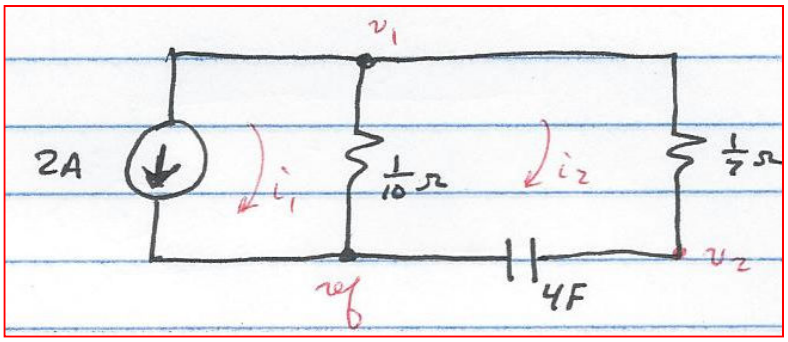
New circuit, nodal:

$$v_1 = v_s$$

$$6\frac{d\left(v_1 - v_2\right)}{dt} - 4\frac{dv_2}{dt} = 0$$

61. (a,b)





(c)

Original circuit, mesh: $2+10i_1+4\dfrac{d}{dt}(i_1-i_2)=0$

$$7i_2+4\frac{d}{dt}(i_2-i_1)=0$$

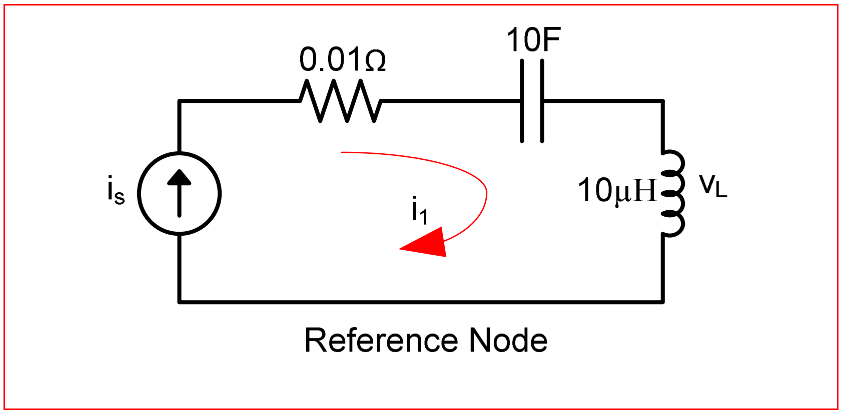
Nodal: v1 = - 2 V

$$0=\frac{v_2-v_1}{10}+\frac{v_2}{7}+\frac{1}{4}\int_{0^-}^{t} v_2\,dt'+i_1(0^-)-i_2(0^-)$$

New, mesh: i1 = -2 A

$$0=\left(\frac{1}{7}+\frac{1}{10}\right)i_2-\frac{1}{10}i_1+\frac{1}{4}\int_{0^-}^{t} i_2\,dt'+v_2(0^-)$$

62. (a, b)



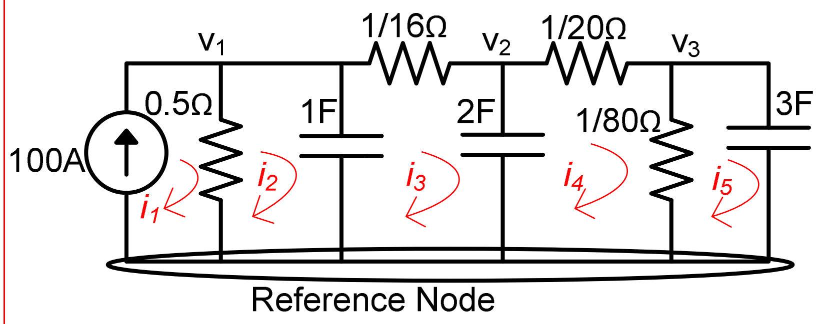
Original circuit, mesh:

$$\boxed{\begin{aligned}&v_s - 100(i_1 - i_2) = 0\\&100(i_1 - i_2) - 10\frac{d(i_2 - i_3)}{dt} = 0\\&10\frac{d(i_2 - i_3)}{dt} - \frac{1}{10\times10^{-6}}\int_{0^-}^{t} i_3 dt' - v_C(0^-) = 0\end{aligned}}$$

New circuit, mesh:

$$\boxed{i_1 = i_s}$$

63. (a)-(b)



(c) Original circuit, mesh:

$$2i_1 + \frac{di_1}{dt} + 16(i_1 - i_2) - 100 = 0$$

$$2\frac{di_1}{dt} + 20(i_2 - i_3) - 16(i_1 - i_2) = 0$$

$$80i_3 + 3\frac{di_3}{dt} - 20(i_2 - i_3) = 0$$

Original circuit, nodal:

$$v_1 = 100V$$

$$\frac{v_1 - v_2}{2} - \int_{0^-}^{t}(v_2 - v_3)\,dt' - i_1(0^-) = 0$$

$$\int_{0^-}^{t}(v_2 - v_3)\,dt' + i_1(0^-) - \frac{v_3}{16} - \frac{1}{2}\int_{0^-}^{t}(v_3 - v_4)\,dt' - i_2(0^-) = 0$$

$$\frac{1}{2}\int_{0^-}^{t}(v_3 - v_4)\,dt' + i_2(0^-) - \frac{v_4}{20} - \frac{(v_4 - v_5)}{80} = 0$$

$$\frac{(v_4 - v_5)}{80} - \frac{1}{3}\int_{0^-}^{t} v_5\,dt' - i_3(0^-) = 0$$

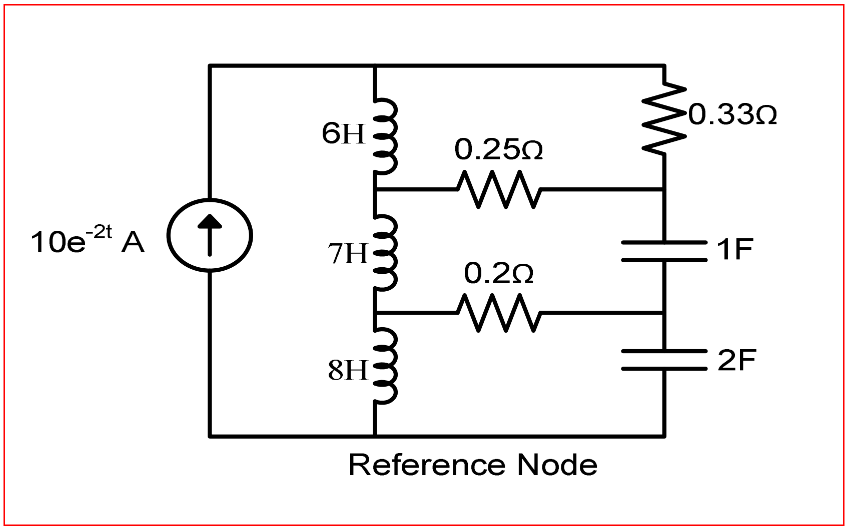
New circuit, mesh:

$$\boxed{\begin{aligned}
&i_1 = 100A\\
&\frac{i_1 - i_2}{2} - \int_{0^-}^{t} (i_2 - i_3)\,dt' - v_1(0^-) = 0\\
&\int_{0^-}^{t} (i_2 - i_3)\,dt' + v_1(0^-) - \frac{i_3}{16} - \frac{1}{2}\int_{0^-}^{t} (i_3 - i_4)\,dt' - v_2(0^-) = 0\\
&\frac{1}{2}\int_{0^-}^{t} (i_3 - i_4)\,dt' + v_2(0^-) - \frac{i_4}{20} - \frac{(i_4 - i_5)}{80} = 0\\
&\frac{(i_4 - i_5)}{80} - \frac{1}{3}\int_{0^-}^{t} i_5\,dt' - v_3(0^-) = 0
\end{aligned}}$$

New circuit, nodal:

$$\boxed{\begin{aligned}
&2v_1 + \frac{dv_1}{dt} + 16(v_1 - v_2) - 100 = 0\\
&2\frac{dv_2}{dt} + 20(v_2 - v_3) - 16(v_1 - v_2) = 0\\
&80v_3 + 3\frac{dv_3}{dt} - 20(v_2 - v_3) = 0
\end{aligned}}$$

64. (a)



65. (a) DC. Thus, $i_L = 7/80\text{x}10^3 = \boxed{87.5\ \mu A}$

(b) $P_{diss} =$ (iL)2(80x10^3) $= \boxed{612.5\ \mu A}$

66. (a) For DC conditions, the capacitor acts as an open circuit. Therefore, the total current I

$$I = \frac{7}{(80+46)\times 10^{3}} = 55.56\mu A$$

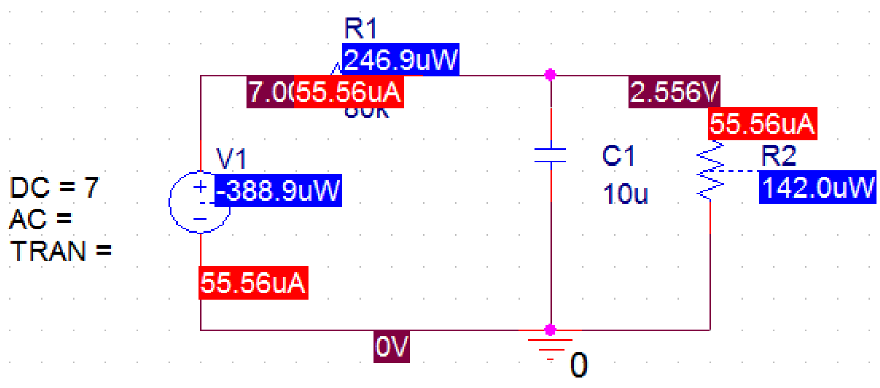
in the circuit is: $P_{80k} = I^2 R = \left(55.56\times 10^{-6}\right)^2 80\times 10^3 = \boxed{246.95\mu W}$

$$P_{46k} = I^2 R = \left(55.56\times 10^{-6}\right)^2 46\times 10^3 = \boxed{141.99\mu W}$$

(b) Voltage across the capacitor: $v_c = I\times 46\times 10^3 = \boxed{2.556V}$

(c) Energy stored in the capacitor: $w_c = \frac{1}{2}Cv_c^{\,2} = 0.5\times 10\times 10^{-6}\times 2.556^2 = \boxed{32.665\mu J}$

(d) Pspice Verification



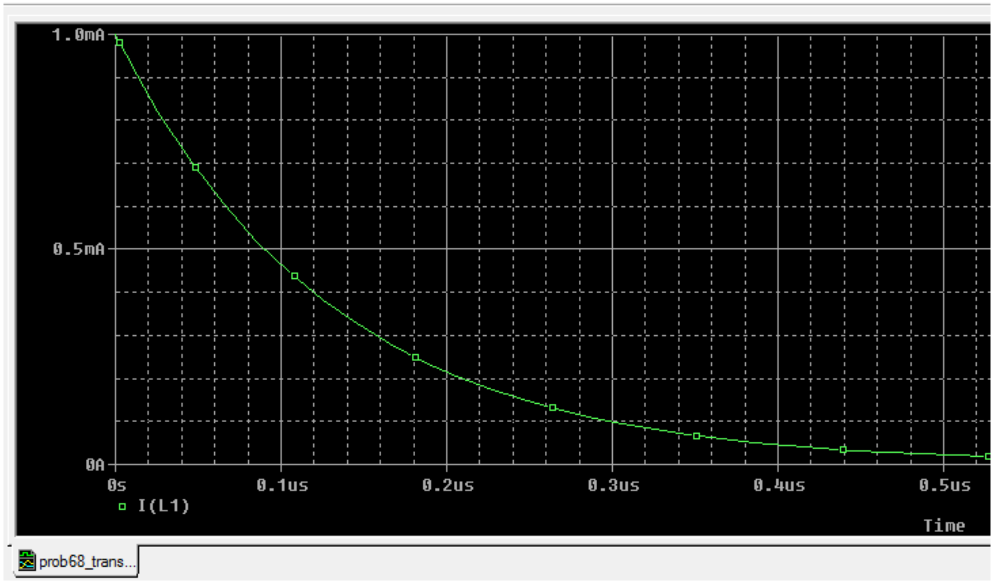
67. (a) By current division, $i_L = -6\times10^{-3}\dfrac{\frac{1}{440}}{\frac{1}{810}+\frac{1}{120}+\frac{1}{440}} = -11.52 \text{ mA}$.

Thus, $v_x = (440(i_L) = \boxed{-5.069 \text{ V}}$

(b) $w_L = \frac{1}{2}Li^2 = \boxed{132.7\ \mu J}$. $w_C = \frac{1}{2}Ci^2 = \boxed{12.85\ \mu J}$

68. (a) $W_L = \frac{1}{2} L i_L^{\,2} = \frac{1}{2} \times 6 \times 10^{-3} \times \left(10^{-3}\right)^2 = \boxed{3nJ}$

(b)



$$i_L(t) = e^{\frac{-t}{L/R}} = e^{\frac{-t}{1.3043\times10^{-7}}} mA$$

$$i_L(0ms) = \boxed{1mA}$$

$$i_L(130ns) = e^{\frac{-130\times10^{-9}}{1.3043\times10^{-7}}} mA = \boxed{0.369mA}$$

$$i_L(260ns) = e^{\frac{-260\times10^{-9}}{1.3043\times10^{-7}}} mA = \boxed{0.1362mA}$$

$$i_L(500ns) = e^{\frac{-500\times10^{-9}}{1.3043\times10^{-7}}} mA = \boxed{0.0216mA}$$

(c) Fraction of energy at selected times:

At, t = 130ns,

$$W_L = \frac{1}{2} L i_L^{\,2} = \frac{1}{2} \times 6 \times 10^{-3} \times \left(0.369 \times 10^{-3}\right)^2 = 0.408nJ = \boxed{13.6\% \text{ of the initial energy stored}}.$$

At, t = 500ns,

$$W_L = \frac{1}{2} L i_L^{\,2} = \frac{1}{2} \times 6 \times 10^{-3} \times \left(0.0216 \times 10^{-3}\right)^2 = 1.399\,pJ = \boxed{0.046\% \text{ of the initial energy stored}}.$$

69. (a) $w = (\frac{1}{2})(10 \times 10^{-6})(81) = \boxed{405\ \mu\text{J}}$

(b) $\boxed{\text{no - the resistor will slowly dissipate the energy stored in the capacitor}}$

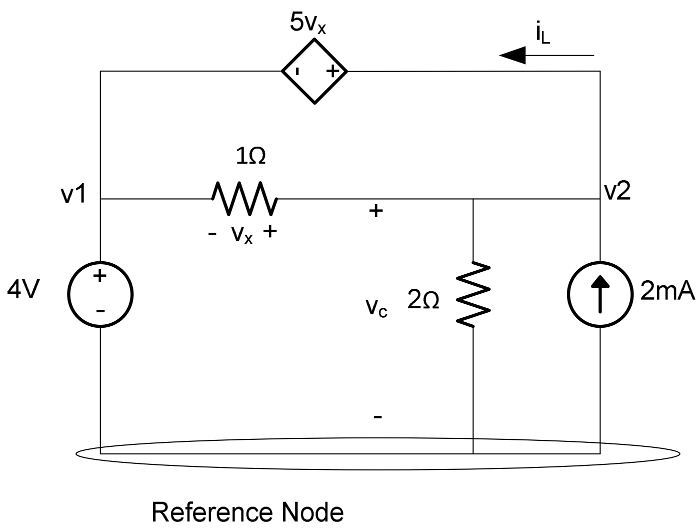
(c) Transient required.

(d) Fraction of energy at selected times:
RC = 0.460 s = 460 ms

So vc(460 ms) = 9e-1 = 3.311 V
Hence w = 54.81 mJ or $\boxed{\text{13.5\% of initial amount stored.}}$

vc(2.3 s) = 0.0606 V
Hence w = 18.39 nJ = $\boxed{\text{0.0045\% of initial amount stored.}}$

70. (a) When we analyze the given circuit with the DC conditions for a long time, the circuit can be redrawn as below.

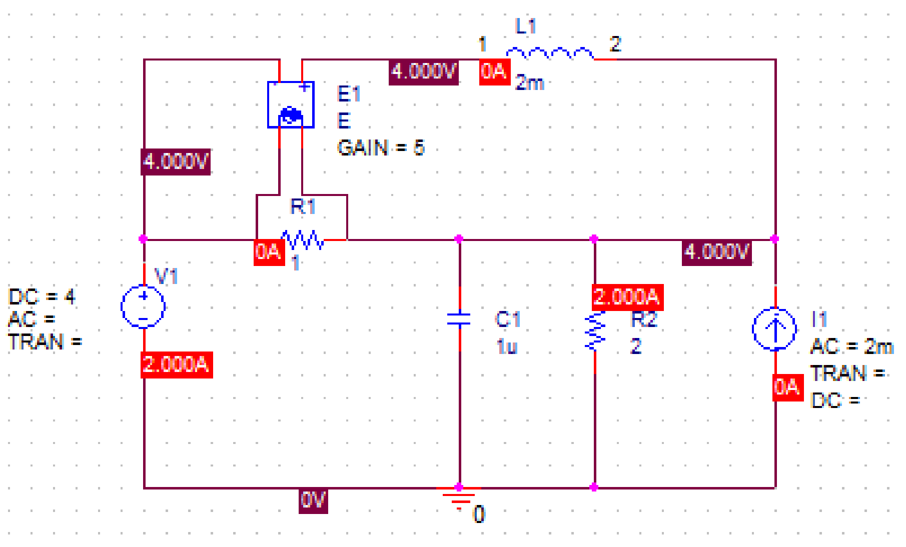


Applying node voltage analysis, we see that v1 = v2. Therefore, $v_c = 4$ V.

Energy stored in the 1μF capacitor is: $W_c = \frac{1}{2}Cv_C^{\,2} = \frac{1}{2}\times 10^{-6}\times 4^2 = \boxed{8\mu J}$

Current, $i_L = 0$ A. Energy stored in the 2mH inductor is: $W_L = \boxed{0J}$

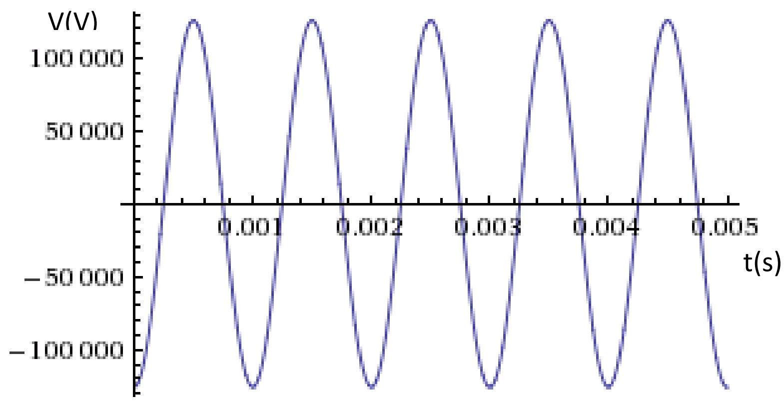
(b) Pspice Verification:



71. $v_{out} = -R_f C_1 \frac{dv_s}{dt} = -10^3 \times 100 \times 10^{-3} \times 2\pi \times 10^3 \times \cos(2\pi \times 10^3 t)$

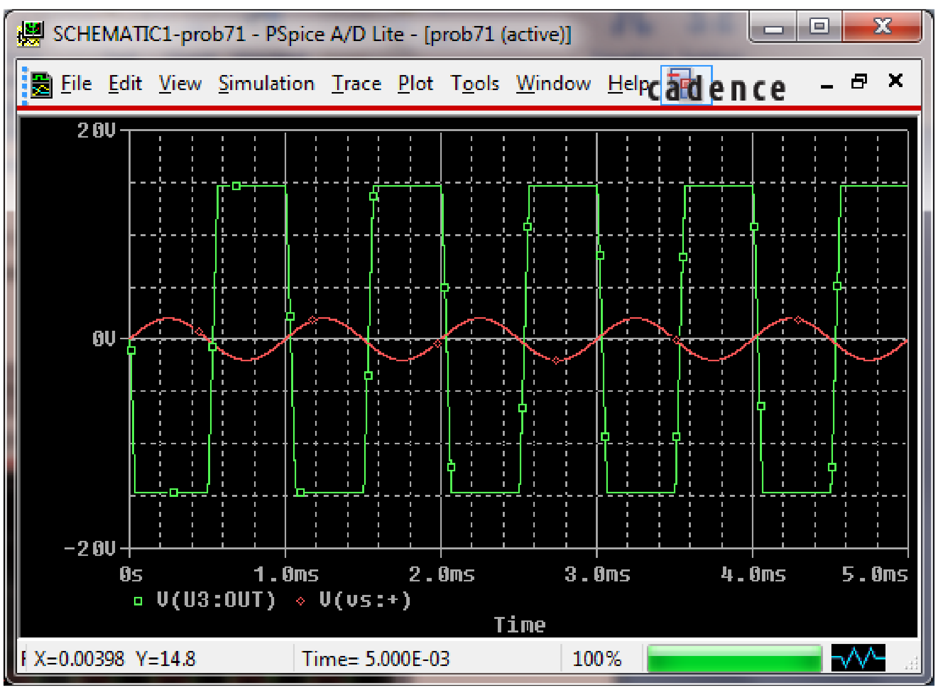
$$= \boxed{-1.25664 \times 10^5 \cos(2\pi \times 10^3 t) V}$$

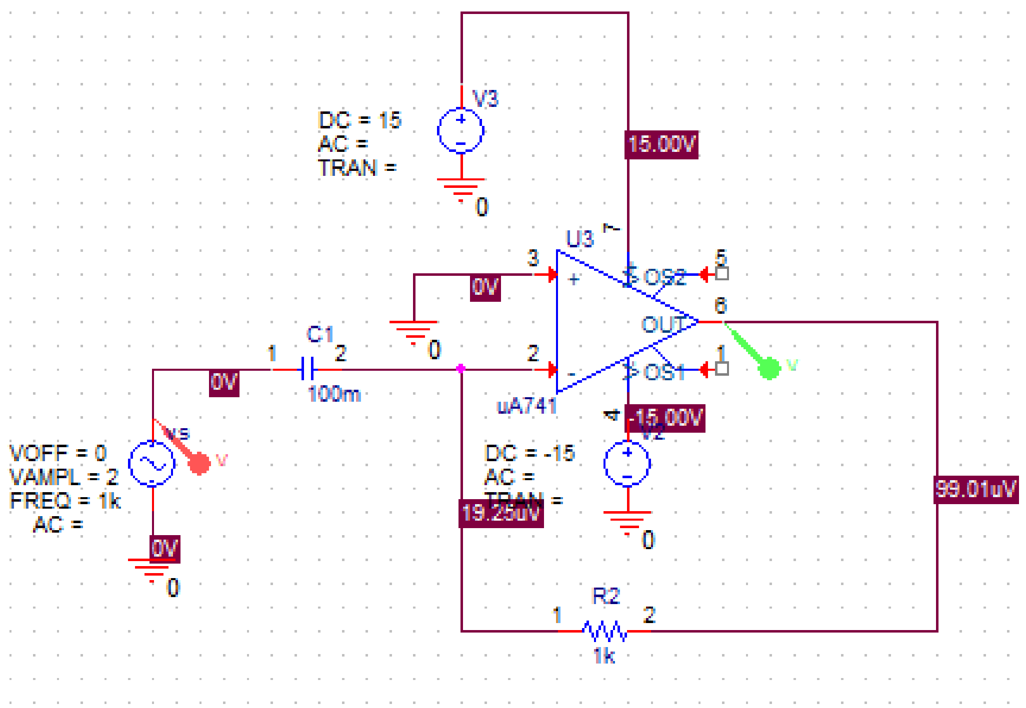
If we plot, v_{out} as a function of time over $0 \le t \le 5$ms, our graph looks as below.



But, since the output for the op-amp is ±15V, we can say that in this case, all the voltage above 15 V will be clipped. This can be easily seen in the output in the Pspice.

(b) Pspice-verification



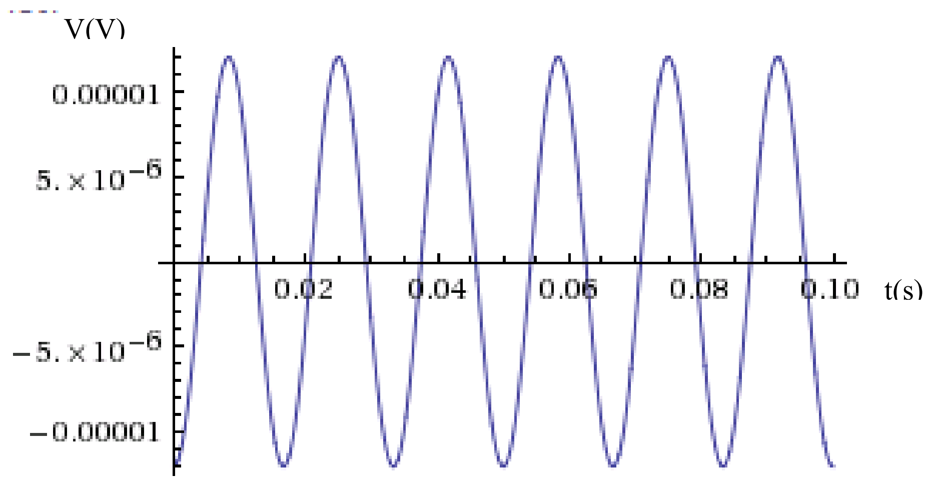


V3
DC = 15
AC =
TRAN =
0
15.00V
U3
3
+
0V
0
OS2
5
OUT
6
C1
1
2
100m
0V
2
-
OS1
uA741
4
-15.00V
V2
DC = -15
AC =
TRAN =
19.25uV
0
99.01uV
VOFF = 0
VAMPL = 2
FREQ = 1k
AC =
Vs
0V
0
R2
1
2
1k

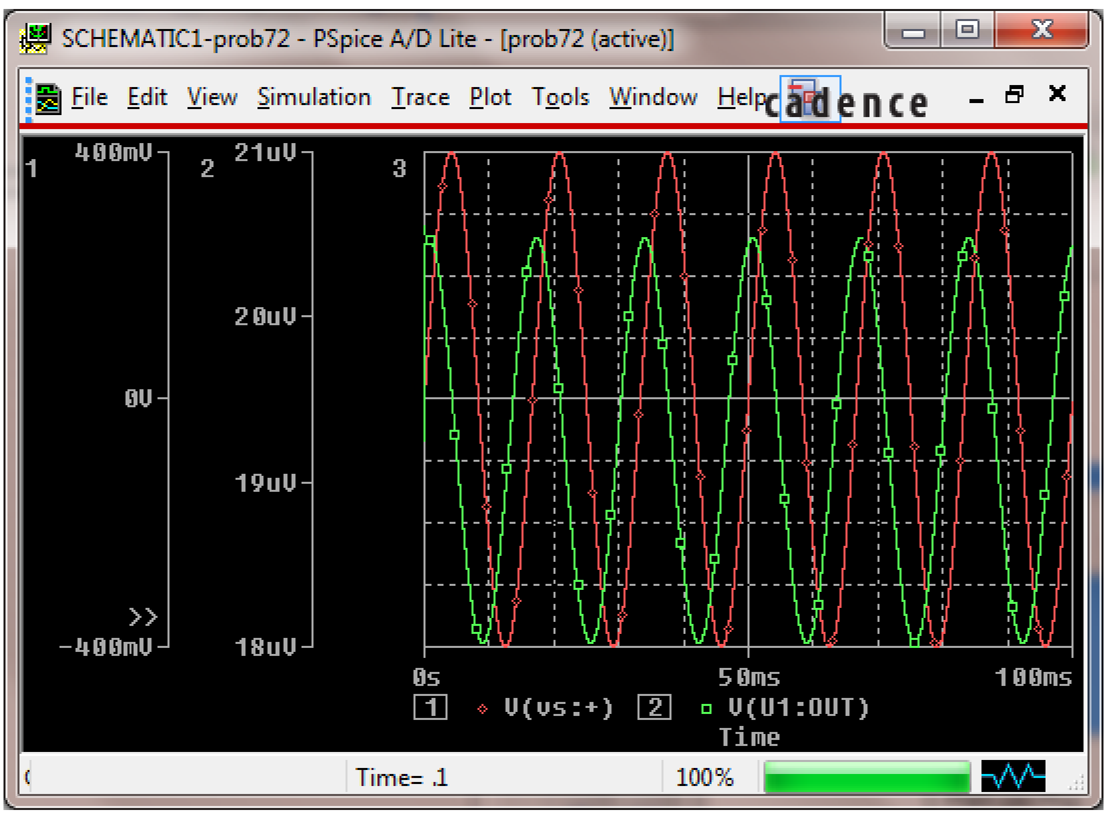
72. (a) $v_{out} = -\frac{L}{R_1}\frac{dv_s}{dt} = -8\times10^{-11}\times150.796\times\cos(120\pi t)$

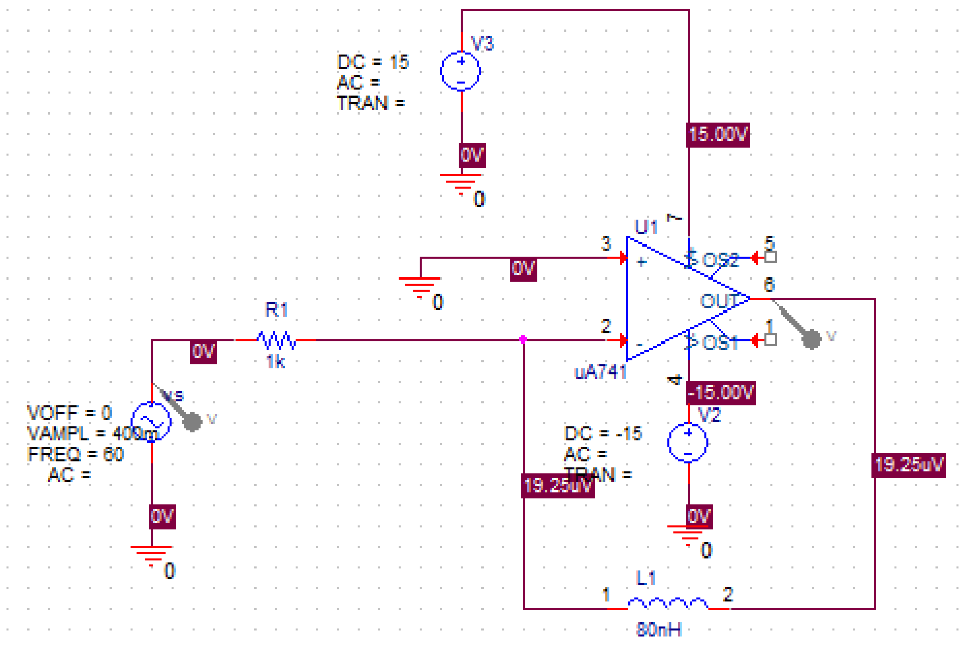
$$= \boxed{-1.2063\times10^{-5}\cos(120\pi t)V}$$

If we plot, v_{out} as a function of time over $0 \le t \le 100$ms, our graph looks as below.



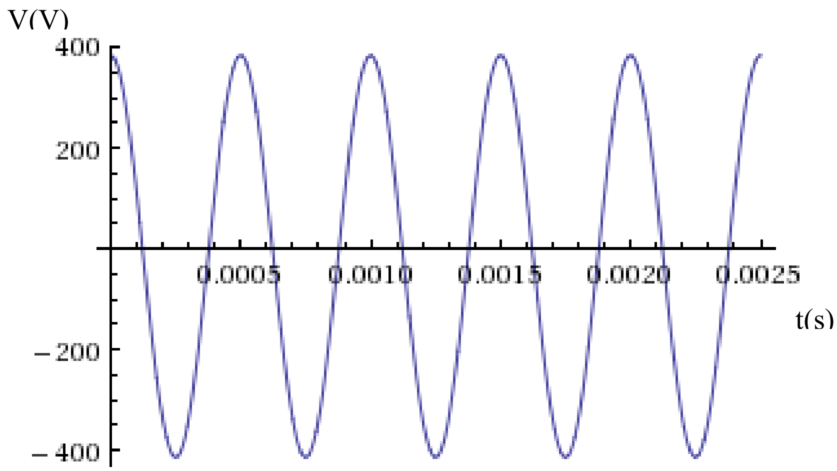
(b) The output voltage graph is different in Pspice simulation.




V3
DC = 15
AC =
TRAN =
15.00V
0V
0
U1
7
3
+
5
OS2
6
OUT
0V
0
R1
1k
0V
2
-
1
OS1
uA741
4
-15.00V
V2
VOFF = 0
VAMPL = 400m
FREQ = 60
AC =
Vs
DC = -15
AC =
TRAN =
19.25uV
19.25uV
0V
0V
0
0
L1
1
2
80nH

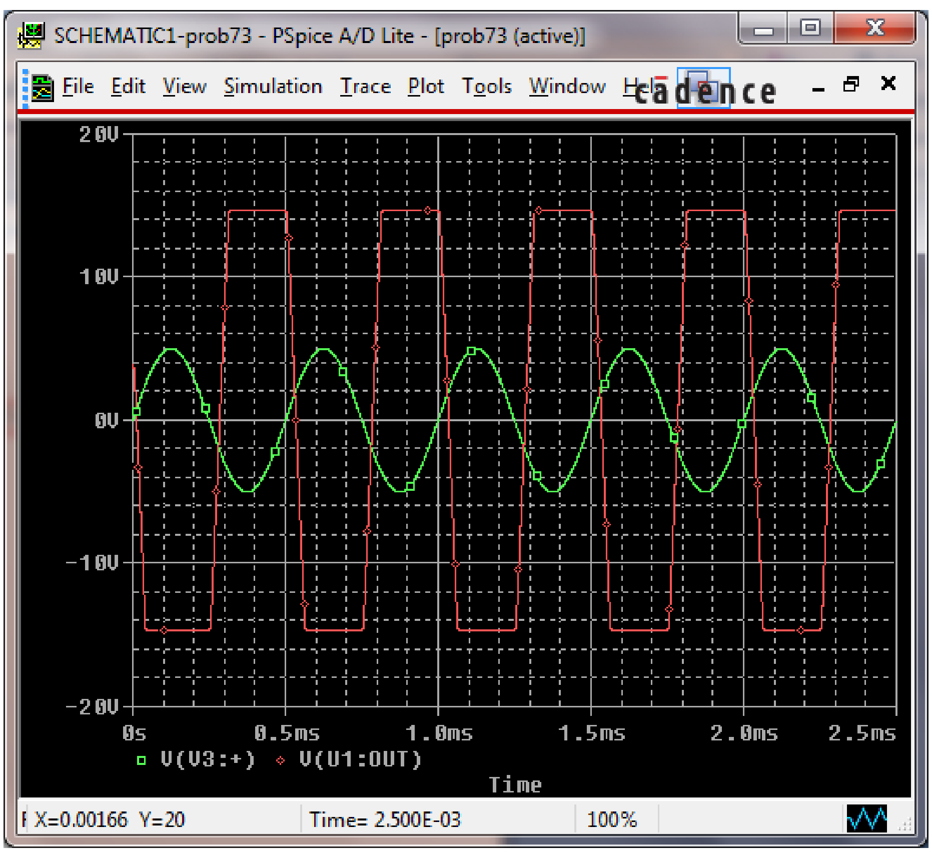
73. a) $v_{out} = -\frac{R_f}{L}\int_0^t v_s dt' = -\frac{100\times10^3}{100\times10^{-3}}\int_0^t 5\sin(4\pi\times10^3 t)dt'$

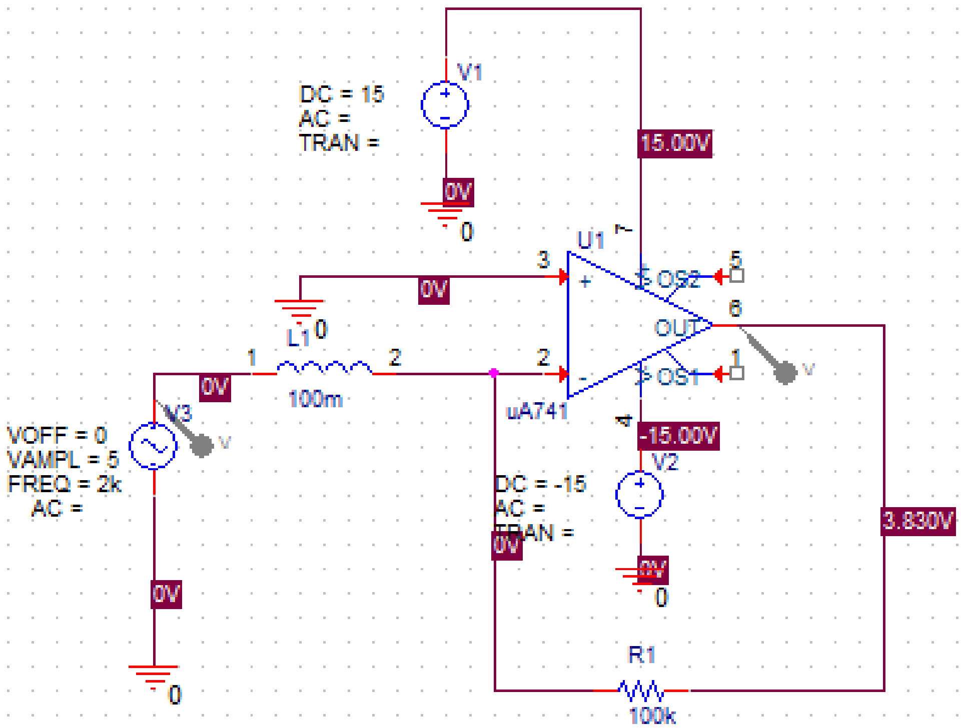
$$= \boxed{397.887\cos(4\pi\times10^3 t) - 5\times10^6 V}$$



(b) Pspice verification

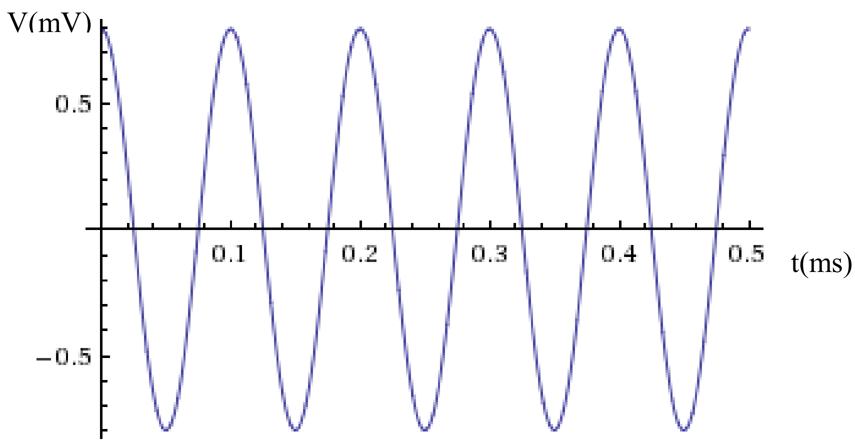
In this case, the op-amp gets saturated around ±15V.



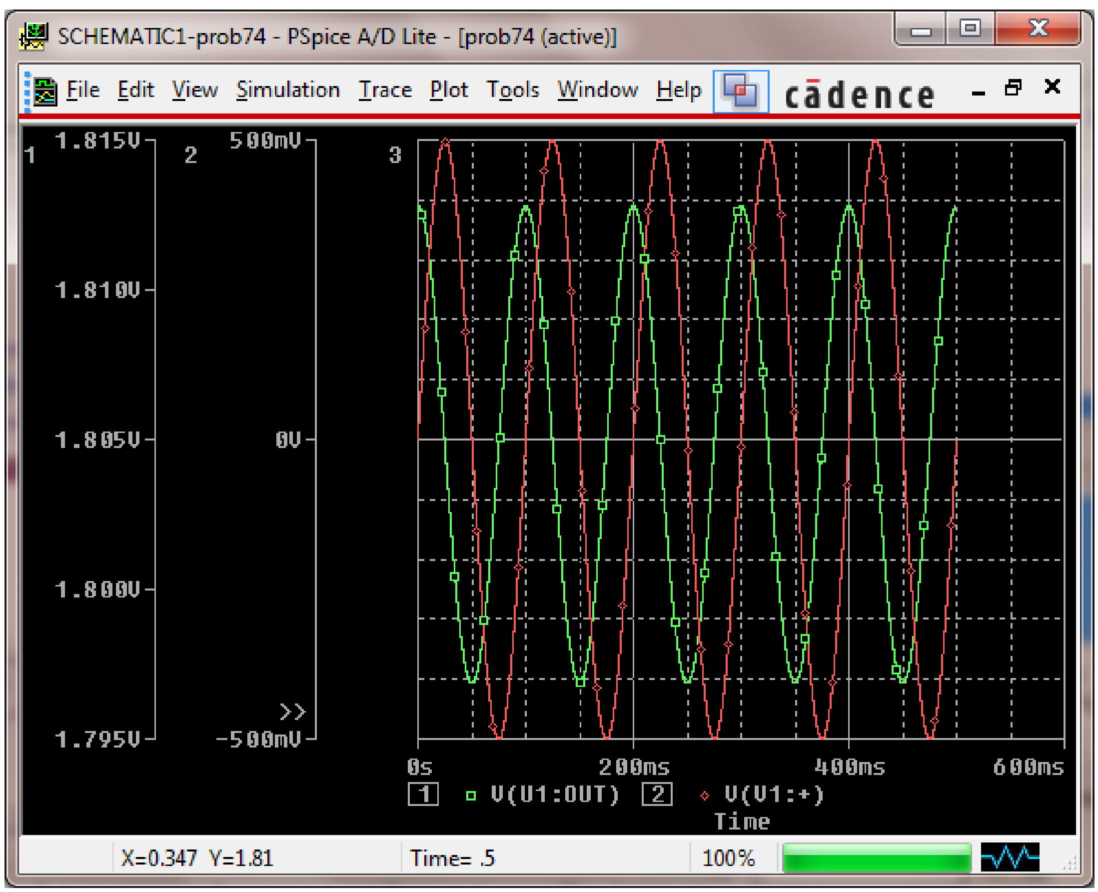

V1
DC = 15
AC =
TRAN =
15.00V
0V
0
U1
3
0V
0
L1
1
2
100m
2
OS2
5
OUT
6
OS1
1
0V
V3
VOFF = 0
VAMPL = 5
FREQ = 2k
AC =
uA741
4
-15.00V
V2
DC = -15
AC =
TRAN =
0V
3.830V
0V
0
0V
R1
100k
0

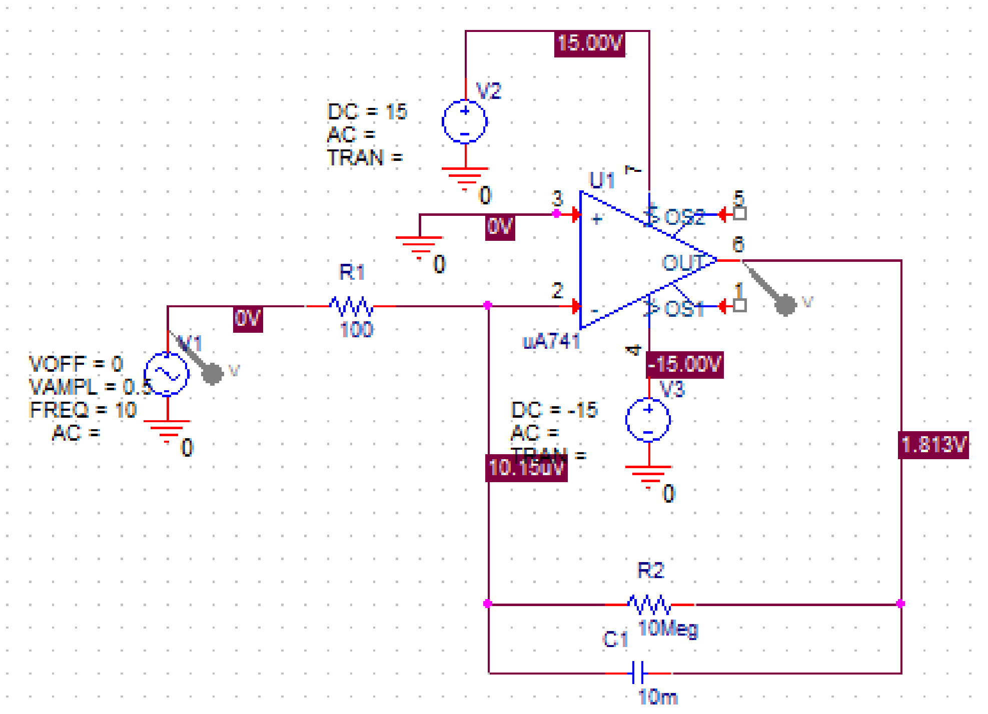
74. (a) $\frac{dv_{out}}{dt} + \frac{v_{out}}{R_f C_f} = -\frac{v_s}{R_1 C_f}$

$v_{out} = \boxed{0.79577\cos(20\pi t)mV}$



(b) Pspice verification:




15.00V
V2
DC = 15
AC =
TRAN =
0
U1
7
3
+
OS2
5
0V
0
6
OUT
R1
2
1
0V
-
OS1
100
uA741
V1
4
-15.00V
VOFF = 0
VAMPL = 0.5
FREQ = 10
AC =
V3
DC = -15
AC =
TRAN =
0
1.813V
10.15uV
0
R2
C1
10Meg
10m

Chapter 8

Ebrahim Forati
09/09/2012

1) a) $i(t) = -3e^{-10^{12}t}$ mA

$i(0) = -3$ mA

$i(1ps) = -1.1$ mA

b) $i(2ps) = -0.4$ mA

$i(5ps) = -0.02$ mA

$W(0) = 4.5\times10^{-15}$ Joul

c) $W(1ps) = 6\times10^{-16}$ Joul

$W(5ps) = 2\times10^{-19}$ Joul

2)) L=5 H

$W(0) = 2.5$ Joul

b) $W(50ms) = 338.6\times10^{-3}$ Joul

$W(100ms) = 45.8\times10^{-3}$ Joul

$W(150ms) = 6.2\times10^{-3}$ Joul

3) L=R=1, I(0)=1

a) $V_R(t) = e^{-t}$ V

$V_R(0) = 1$ V

$V_R(1) = e^{-1}$ V

$V_R(2) = e^{-2}$ V

$V_R(3) = e^{-3}$ V

$V_R(4) = e^{-4}$ V

$V_R(5) = e^{-5}$ V

b)

$P(0) = 1$ Watt

$P(1) = e^{-2} \ Watt$

$P(5) = e^{-10} \ Watt$

C)

0.00454 percent

4) R/L=1000

5) a) 14S+5=0

   b) -9S-18=0
   c) S+18+R/B=0
   d) S^2+8S+2=0

6) a) $\frac{dV}{dt} + 9v = 0, \quad S = -\frac{9}{4}$

   b) $2\frac{dV}{dt} - 4v = 0, \quad S = 2$

   c) $\frac{d^2V}{dt^2} + 7\frac{dV}{dt} + v = 0, \quad S = -6.85, \ S = -0.145$

   d) $5\frac{d^2V}{dt^2} + 8\frac{dV}{dt} + 18v = 0, \quad S = -0.8 + 1.72j, \quad S = S = -0.8 - 1.72j$

7) a) $i_L(0^-) = 4 \ \ mA$
b) $i_L(0^+) = 4 \ \ mA$

c) $i_L(15.8 \ \ \mu s) = 0.7 \ \ mA$

d) $i_L(31.5 \ \mu s) = 0.125 \ \ mA$

$i_L(78.8 \ \ \mu s) = 6.88 \times 10^{-4} \ \ mA$

8) a) $v(0^-) = 0 \ \ V,$ $\qquad w(0^-) = 16 \times 10^{-9} \ \ J$
b) $v(0^+) = 1.2 \ \ V,$ $\qquad w(0^+) = 16 \times 10^{-9} \ \ J$
c) $v(8 \ \ \mu s) = 1.2 \ \ V,$ $\qquad w(8 \ \ \mu s) = 2.72 \times 10^{-9} \ \ J$
d) $v(80 \ \ \mu s) = 1.2 \ \ V,$ $\qquad w(80 \ \ \mu s) = 36 \times 10^{-17} \ \ J$

9) a) $i_L(t) = 0.2e^{-\frac{8500}{4}t}u(t) \qquad A$
$v(t) = 5e^{-\frac{8500}{4}t}u(t) \qquad V$

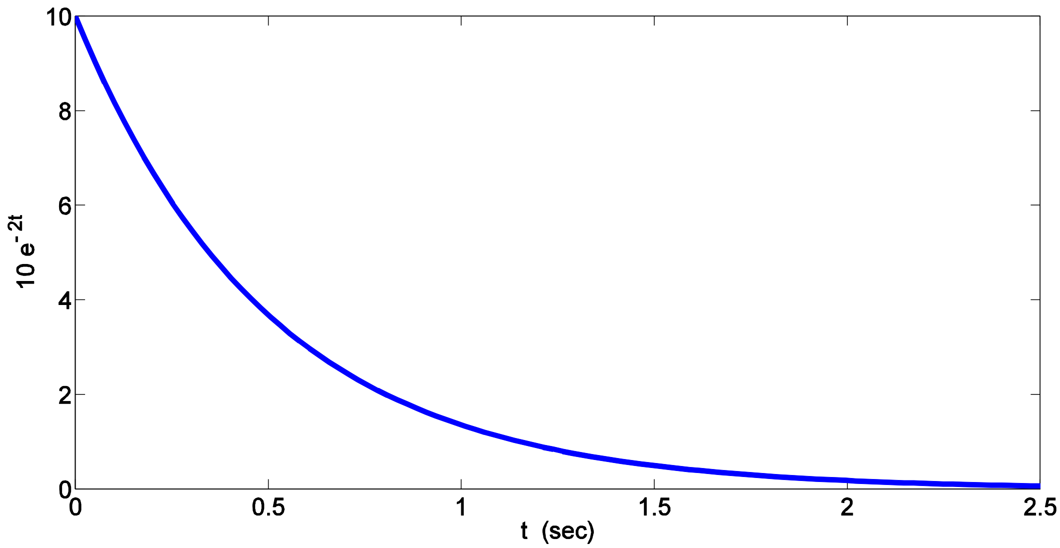
b) $v(0^-) = \frac{250}{35} \ \ V,$ $\qquad i_L(0^-) = 0.2 \ A$

$v(0^+) = 5 \ \ V,$ $\qquad i_L(0^+) = 0.2 \ A$

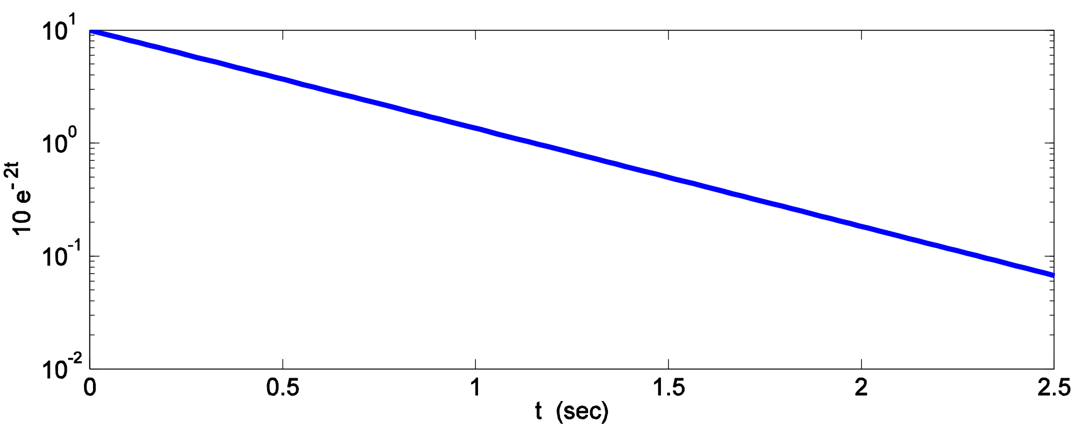
$v(470 \ \ \mu s) = 1.84 \ \ V,$ $\qquad i_L(470 \ \ \mu s) = 73.3 \ \ m\,A$

10) a) $i_w(t) = \frac{1.5}{5}u(t) \quad mA$
b) i(0)=i(1.3 ns)= $\frac{1.5}{5} \quad mA$

11) a)



b)



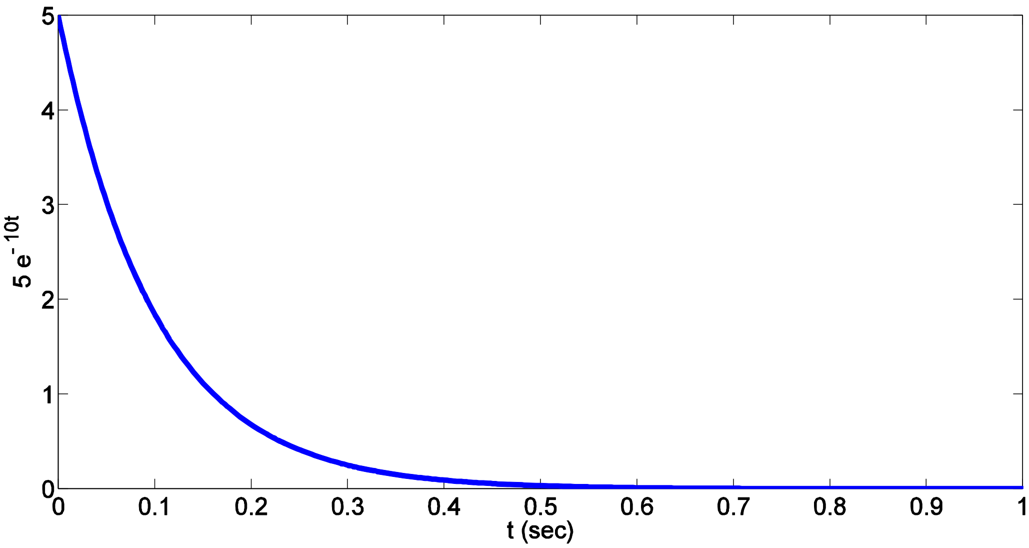
c) 1/sec

d) 0.05 sec, 0.11 sec, 1.15 sec

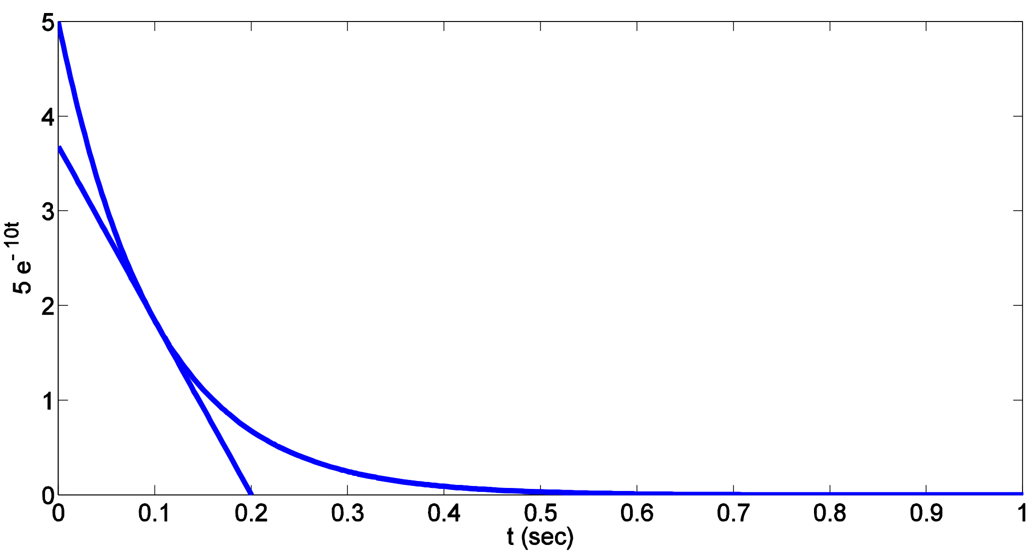
12)

a) t= 0, 69, 230, 690 msec

b)



c)



13)

a) $s\,m^2$

b) $\phi = \phi_0 e^{-\alpha x}$

c) $x = \frac{0.47}{x}\ m$

d) Transmits to the other side of the cell.

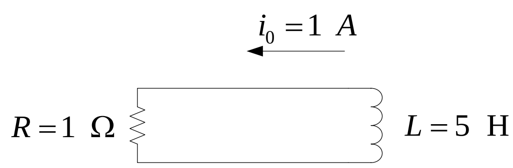
14) a) 1/8 sec

b) 5/41 sec

c) 1/10 sec

d) 4/150 sec

15)

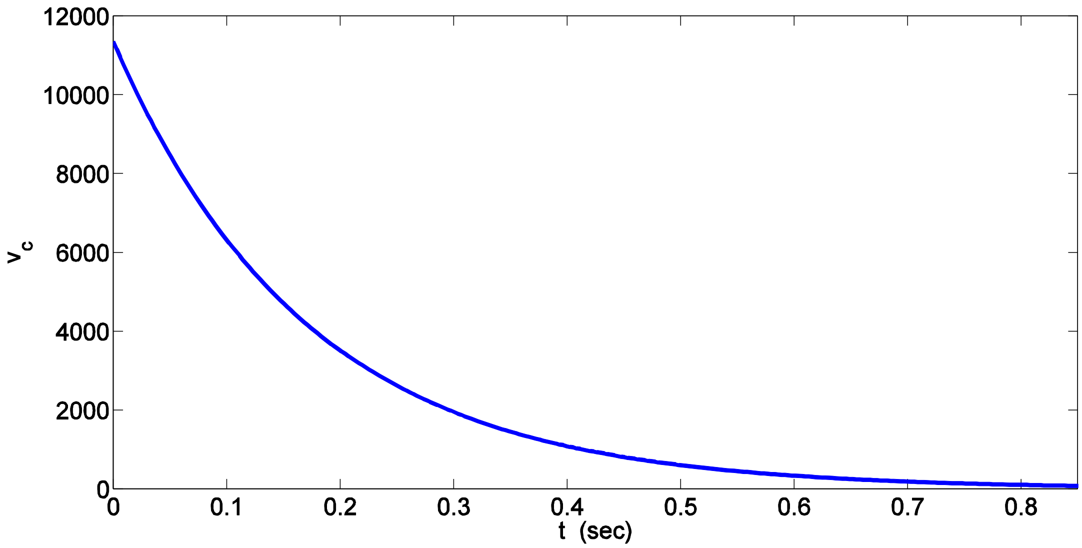


16)

a) v(t) = 11.35*10^3 *exp(-t/ 0.1705) V

b) w = 0.5* 3.1 * 10^-9 *(4187.69)^2 = 0.02718195868 J

c)



17)

a) v(t)=9*exp(-t/0.022)

b)

T (ms)	11 ms	33 ms
W (J)	0.32778058208	0.04436027791

c)

v(t)=9*exp(-t/2200)

T (ms)	11 ms	33 ms
W (J)	0.89099109004	0.890973270

18)

a)0.01 s

b)0.1s

c)1s

d) simulation

19)

A Parallel RC circuit with initial voltage of 9 volts on the capacitor,

a) R=1 ohm , c= 1.98 mF

b)R=1 ohm , c=216.6 pF

20)

a)9*10^-7 s

b)

T	tau	2 *tau	5*tau
V(t) -volt	1.47152	0.541341	0.02695

21)

a) 1.05 s

b)i1(t)=0.380952 * exp(-t/1.05)

c)0.671905 w

22)

a) tau = 169.15*10^-3 s

b) v(t)= 0.2955*exp(-t/tau) volt

c) w =2.92483 * 10 ^ -8 j

23)

a)

T	0	984	1236
v(t) volt	20	7.35759	5.7301

b)

w =1.95857 j

24)

a) 3*10^-3 (s)

b)20 volt

c)20* exp(-t/15*10^4)

d)2.70671 volt

25)

a) $i(0^-) = 2\ mA$

b) i((0^+)=0

c) $P_{parallel\ resistor} = 0\ W$

$P_{series\ resistor} = 20e^{-\frac{2\times10^3 t}{1.5}} \quad mW$

26)

a) v(t)=(R4*R3/(R4+R3)iL(0)*exp(-1.97143*10^6))

b) 0.000191917 volt

27)

Ix(0-)	Ix(0+)	iL(0-)	iL(0+)	vL(t<0)	vL(t>0)
0.2105	0.1263	0.1263	0.1263	0	10.637 *exp(-t/0.011875)

28)

T	0+	0-	1 mu s	10 mu s
iL	3.6/5	3.6/5	0.6514829	0.26487319
vL	0.00001296	0	0.00001296*0.9048 = 0.0000117262	0.00001296*0.3678= 0.00000476668
vR	1.44	1.442*10^3	2*10^3*0.6514829=1302.9658	2*10^3*0.26487319=529.74638

29)

I1(t) = -3 A t>0

I1(t) = 2 A t<0

iL(t)= 3*exp(-t/tau), tau=0.7333 s

30)

a) T = Ln(0.5) *(L/R)=0.00770163533 s

b) T= Ln(0.1)* (L/R)= 0.02558427881 s

31)
I1=36/5*10^-3*exp(-4/5*10^3*t) -36/5*exp(-4/5*10^3*t)
IL= 9/5*10^-3*exp(-4/5*10^3*t)
I2=-i1-iL
I2=-(36/5*10^-3*exp(-4/5*10^3*t) -36/5*exp(-4/5*10^3*t)+ 9/5*10^-3*exp(-4/5*10^3*t))

T	1ms	3ms
I1(A)	-3.2319	-0.6525
I2(A)	3.2311	0.6524

32)
vx=(0.75)*(1+exp(-100*t))
t=0.005s
vx=1.2049 v

33)

34)

35)
a) R1=40 ohms
Ro= 60 ohms

36)
Vc(0-) = Vc(0+) = 2.5 v
C=21/22 F

37)

a) $v_c(0^-) = 0.75\ V$

$v_0(0^-) = \frac{2}{7}\ V$

b) $v_c(0^+) = 0.75\ V$

$v_0(0^+) = \frac{3}{16}\ V$

c) $v_c(10\ ms) = 7.7\ m\,V$

$v_0(10\ ms) = \frac{7.7}{4}\ mV$

d)) $v_c(12\ ms) = 3.1\ mV$

$v_0(12\ ms) = \frac{3.1}{4}\ mV$

38)
a)Vc(0-) = 80/23 v
b) Vc(0+) = 80/23 v
c) RC= -11.5/24 ms
d) Vc = 80/23 * exp(15/16 t)

39)
a) v1(0-) = 100
v2(0-) = 0

vR(0-)=0

b) v1(0+) = 100
v2(0+) = 0
vR(0+)=100

c)
0.08 s

d)

$$v_R(t) = 100e^{-\frac{t}{0.08}}u(t) \quad V$$

e)

$$i(t) = 5e^{-\frac{t}{0.08}}u(t) \quad mA$$

f)

$$v_1(t) = 80 + 20e^{-\frac{t}{0.08}} \quad V$$

$$v_2(t) = 80 - 80e^{-\frac{t}{0.08}} \quad V$$

g)

$$W_1(\infty) = 64 \ mJ$$

$$W_2(\infty) = 16 \ mJ$$

$$W_R(0 \to \infty) = \int_0^{\infty} 20 \times 25e^{-\frac{t}{0.04}}dt = 20 \ mJ$$

$$W_1(0) = 100 \ mJ$$

$$W_2(0) = 0$$

40)
iL(0+)=iL(0-)=1.5 mA
iL(t)=1.5*10^-3*exp(-21/24*10^3*t)
a) t=0, P=54 nJ
b) t=1 ms, iL= 6.2529e-004, p= 9.3837e-009 J
c) t=5 ms, iL=1.8882e-005, p= 8.5567e-012 J

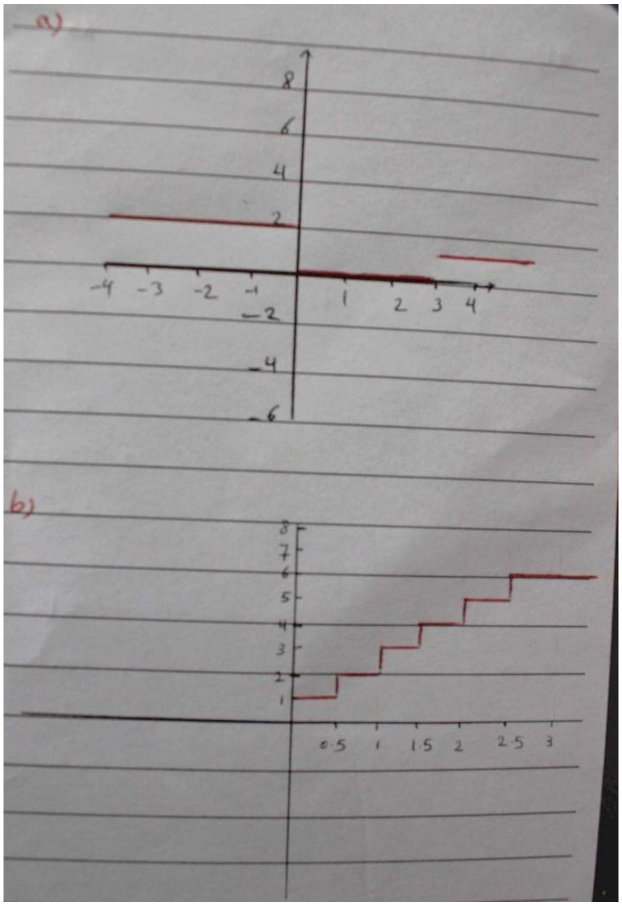
41)

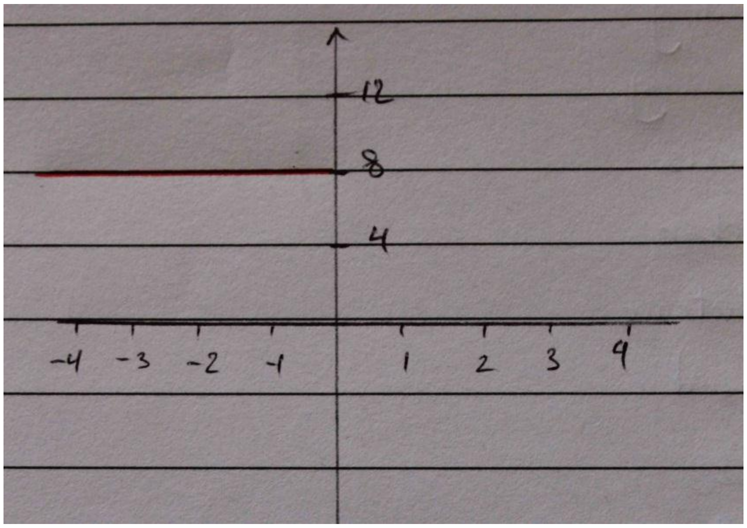
T	-2	0	2
F(t)	0	3	3
G(t)	8	8	0
H(t)	0	0	0
Z(t)	11	11	4

42)

T	-1	0	3
F(t)	-1	0	0
G(t)	10	10	8
H(t)	2	2	1
Z(t)	2	2	3

43)





44)

F(t)=u(t-1)-u(t-2)+u(1-t)

45)

v(t)=3[u(t-2)-u(t-3)]-2[u(t-3)-(t-5)]-4[u(t-1)-u(t-2)]-2[u(1-t)]

46)

a) i=0
b) i=0

c) i=0

d) i=0;

e) $i(2\ ms) = 2.85\ mA$

47)

a) $i_L(0^-) = i_L(0^+) = v_L(0^-) = 0$

$v_L(0^+) = 40\ mV$

b) $i_L(150\ ns) = 1.26\ mA$

C) Spice simulation

48)

a)

$$i_L(t) = \frac{100}{3}\left(1 - e^{-\frac{3}{40}\times 10^6 t}\right) u(t) \quad mA$$

b) $i_L(10\ \ \mu s) = 17.58\ \ mA$

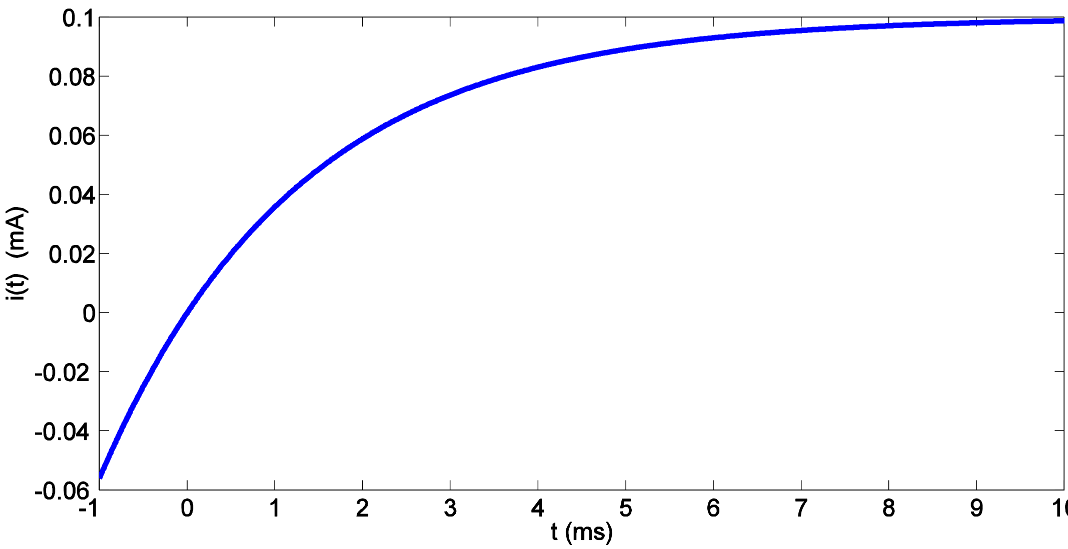
$$i_L(20\ \ \mu s) = 25.9\ \ mA$$

$$i_L(50\ \ \mu s) = 32.5\ mA$$

49) a)

$$i(t) = 0.1\left(1 - e^{-\frac{20}{45}\times 10^3 t}\right)u(t)\quad mA$$

b)



50)

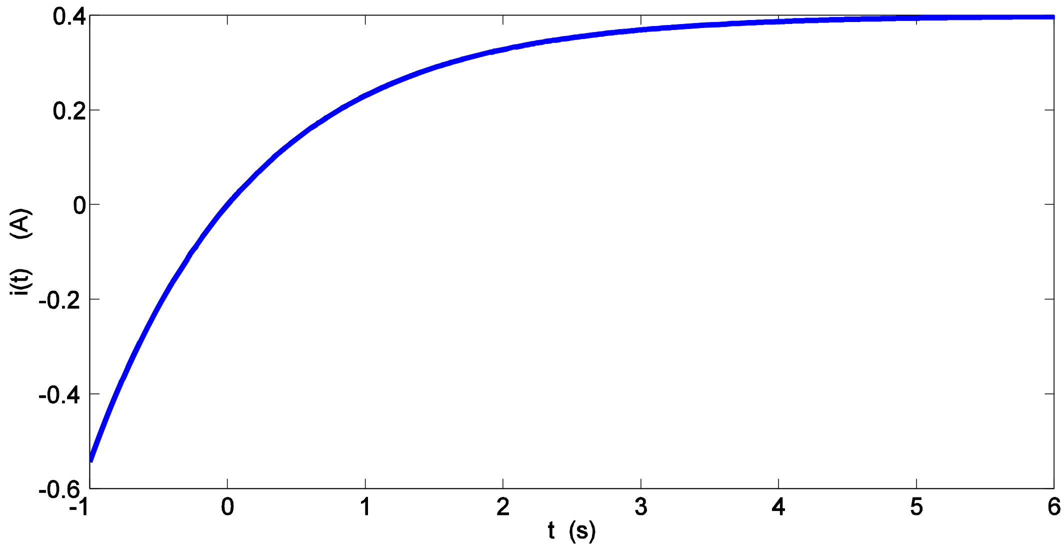
a)

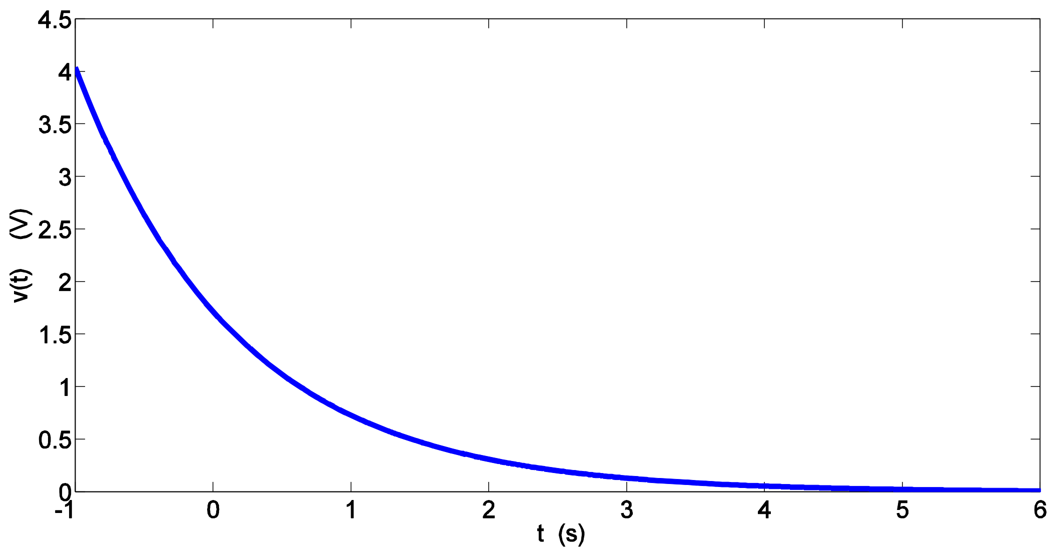
$$i(t) = 0.4\left(1 - e^{-\frac{6}{7}t}\right)u(t)\quad A$$

b)

$$v(t) = \frac{12}{7}e^{-\frac{6}{7}t}u(t)\quad V$$

c)





51)

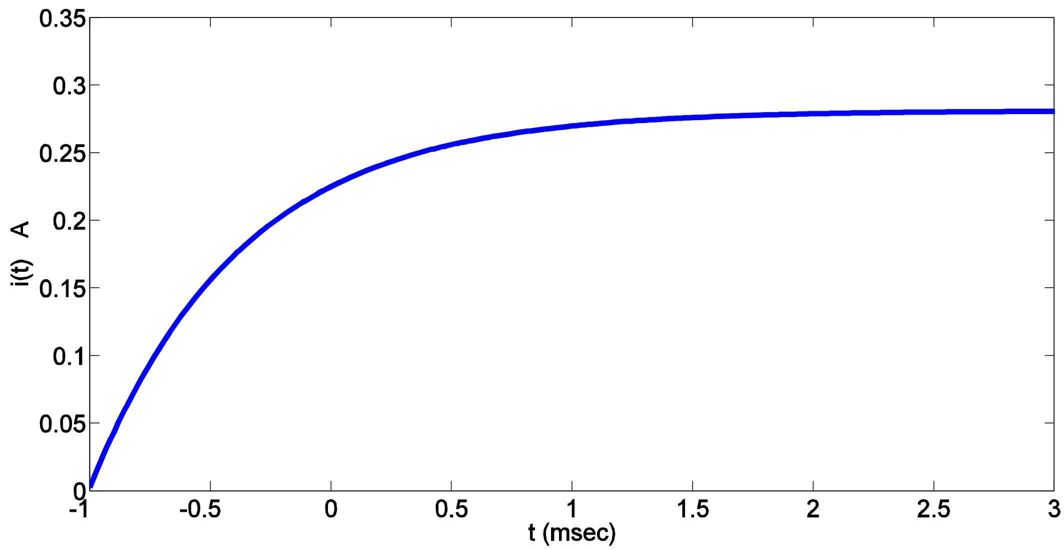
a) $i(t) = 0.05 + 0.06\left(1 - e^{-16t}\right)u(t)\ A$

b) $v(t) = \frac{24}{5}e^{-16t}u(t)\ V$

c) t=58.6 msec

52) a) $i(t) = \frac{4.5}{20} + \frac{4.5}{80}\left(1 - e^{-1600t}\right)u(t)\ A$

b)



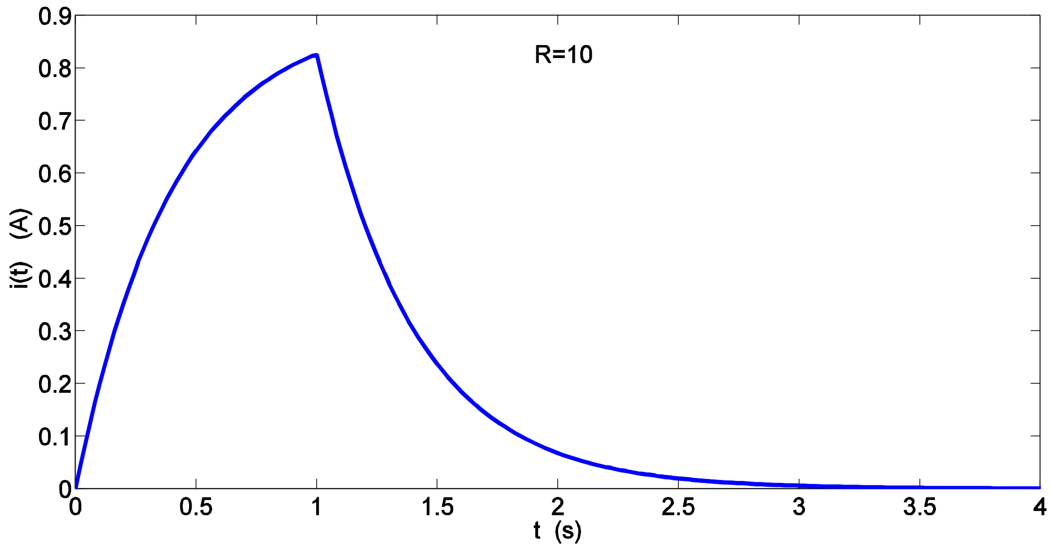
53)

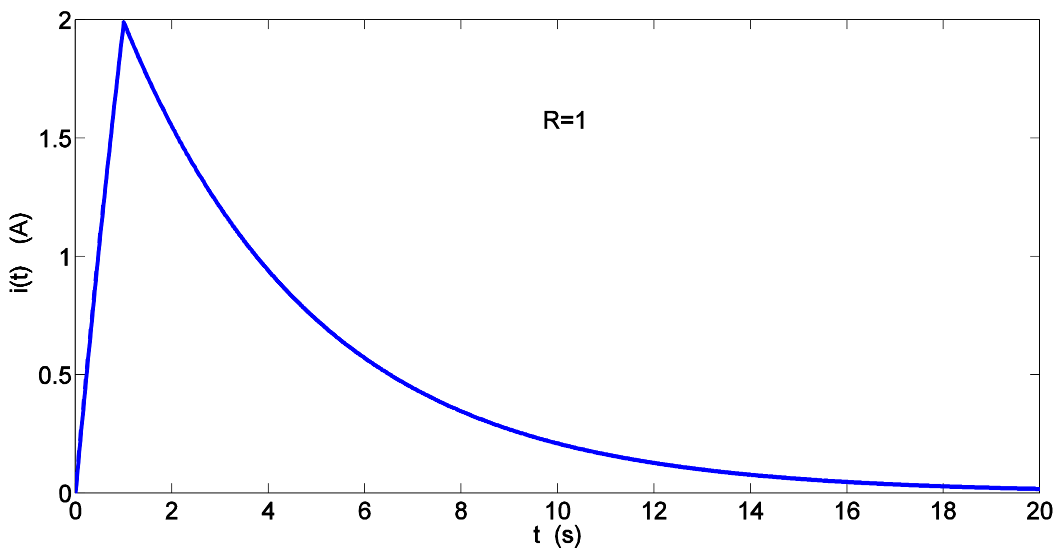
$$i(t) = \frac{0.3}{7} e^{-\frac{4000}{7}t} u(t)\ A$$

P(2.5 ms)= 4.2 mW

54) $i(t) = 0.4 - 0.25e^{-\frac{10^9}{26}t}\ \ A$

55)



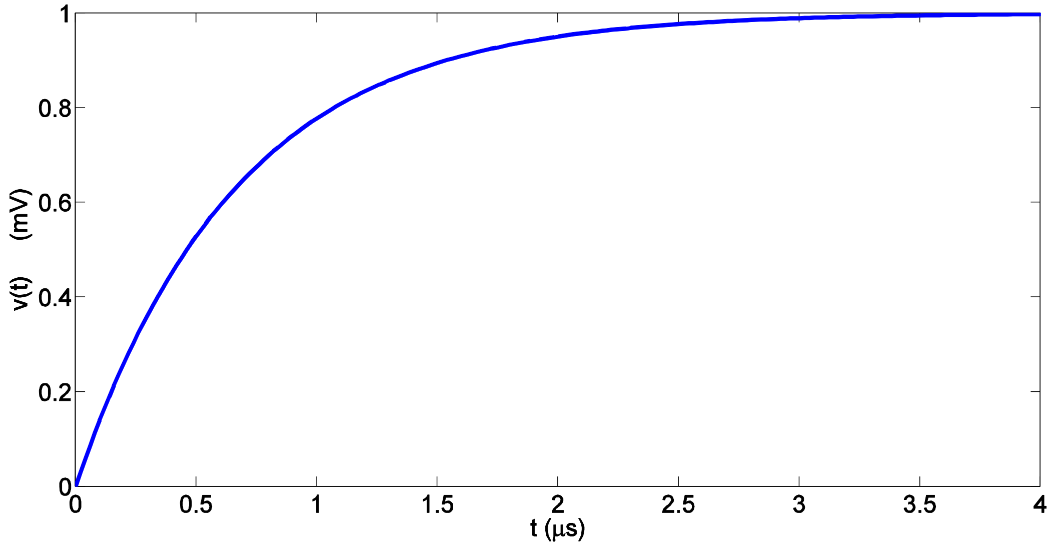


In R=1 case, the inductor stores more energy at t=1 sec. Because, the current peak is bigger.

56) a)

$$v(t) = \left(1 - e^{-\frac{3}{2}\times 10^{6} t}\right) u(t) \quad mV$$

b)



57)

$$\boldsymbol{i(t) = 2.5e^{-\frac{5}{6}\times 10^3 t} u(t) \quad mA}$$

58)

a) $i(70\ ms) = 85.8 \quad \mu A$

b) Spice verification

59)

a) $i(70\ ms) = -71 \quad \mu A$

b) Spice verification

60)

a)

$$v(t) = 8 - 3\left(1 - e^{-0.6\times 10^6 t}\right)u(t) \quad V$$
$$i(t) = 0.1 + 0.06e^{-0.6\times 10^6 t} \quad A$$

b) $P = 25 \ \mu W$

61)

a)p = 0.4036 w

b)p = 17.9072 w

62)

Vc(t)=0.09*25(1-exp(-t/25*10^6))

T	0+	25	150
Vc(t)	0	1.4223	2.25

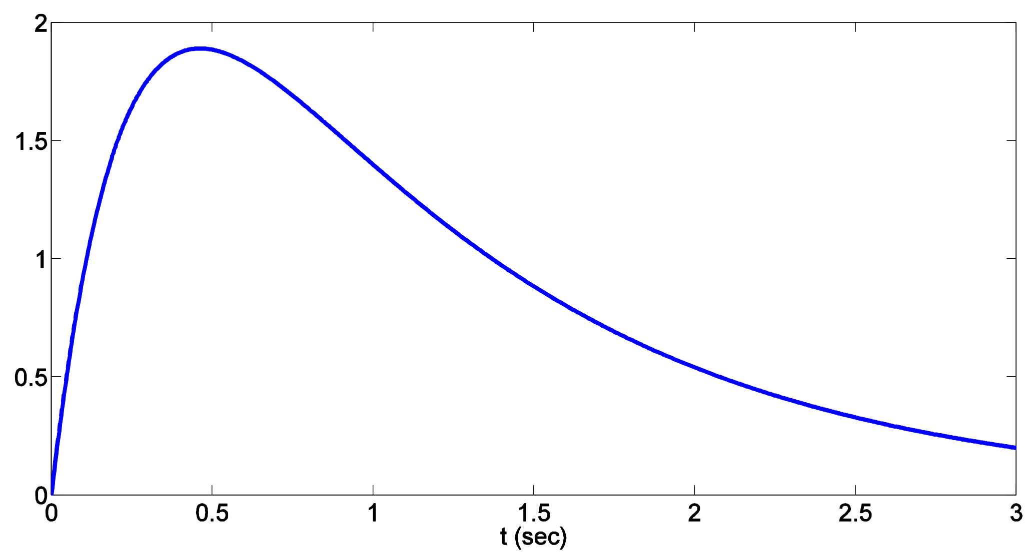
63)

Vc(t)=5-5*exp(-t/25*10^6)

T=25 mu s --------→ p=2w

64) a) v(t)= 4(exp(-t)- exp(-4t))

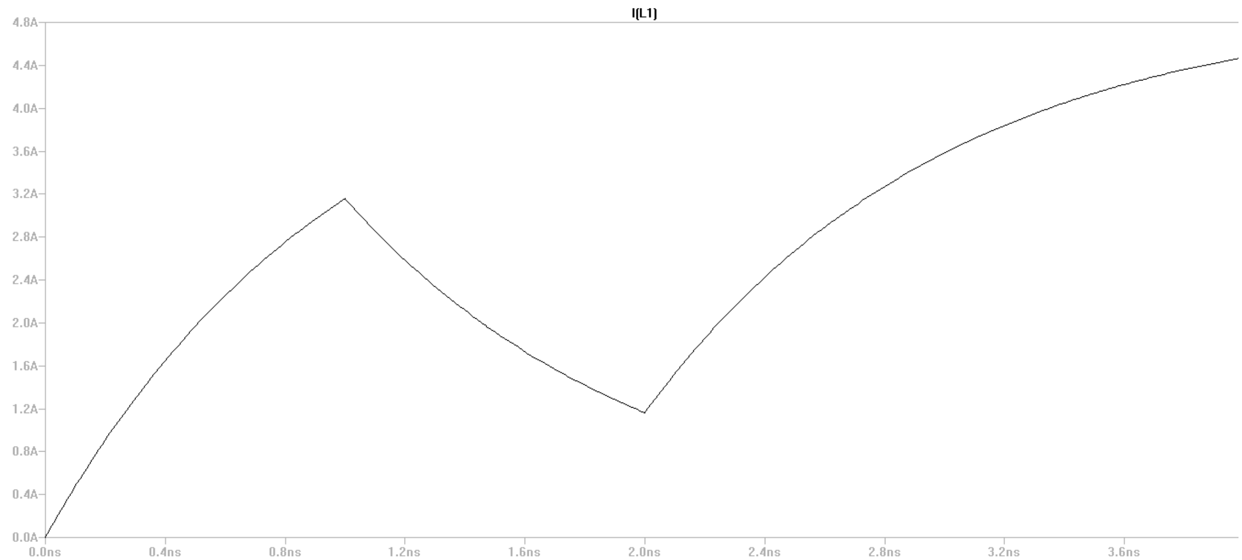
b)

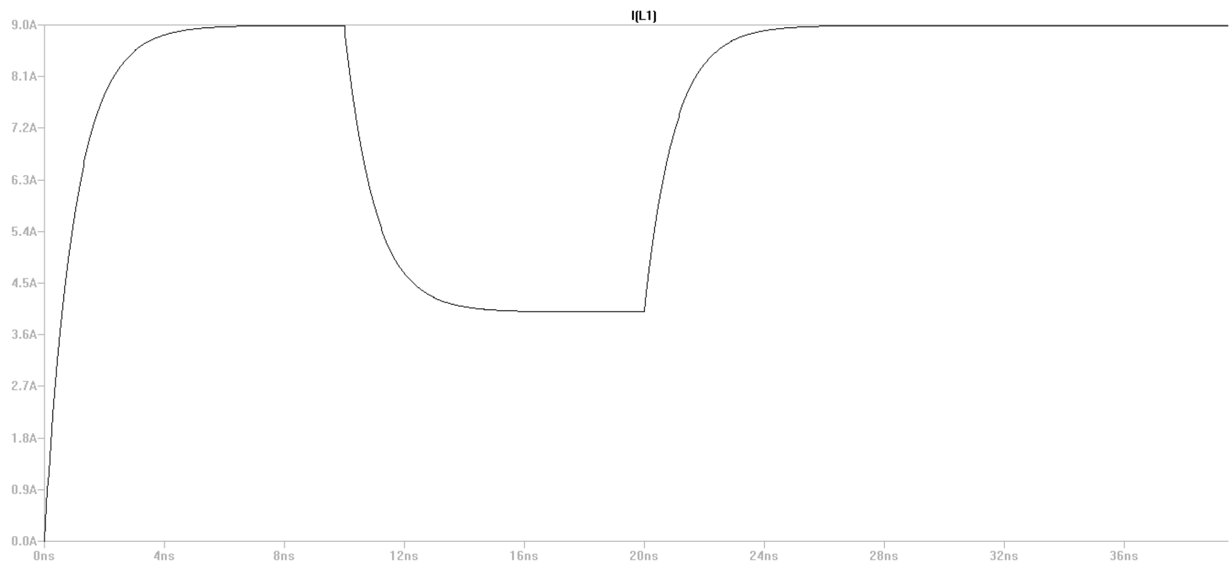


65)

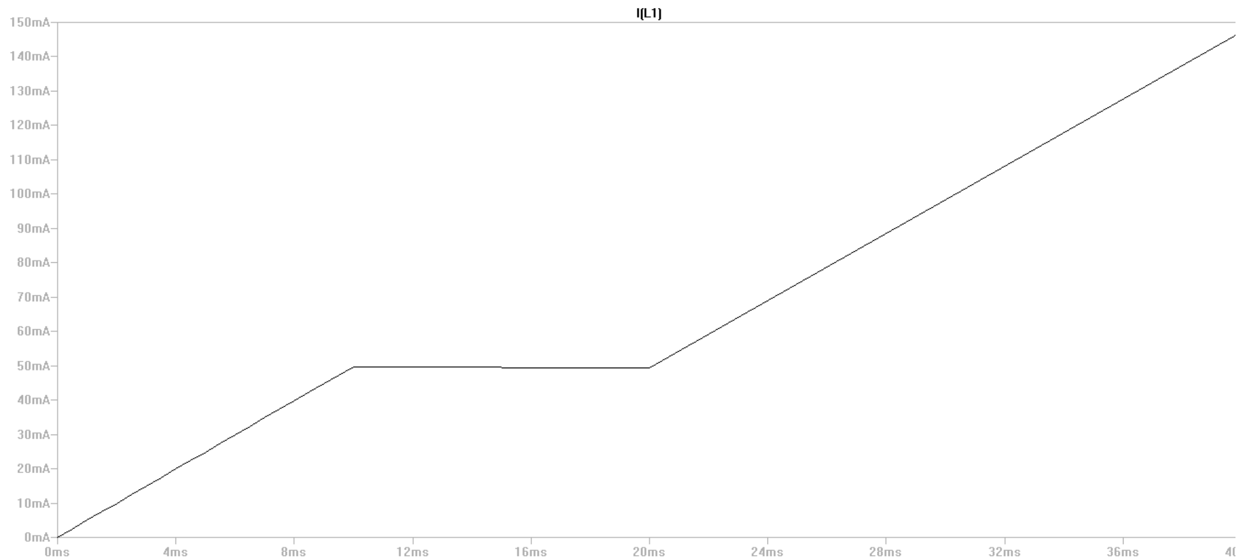
V(t)= 9/16*10^3*(exp(-t/16*10^3)-exp(-t/8*10^3))

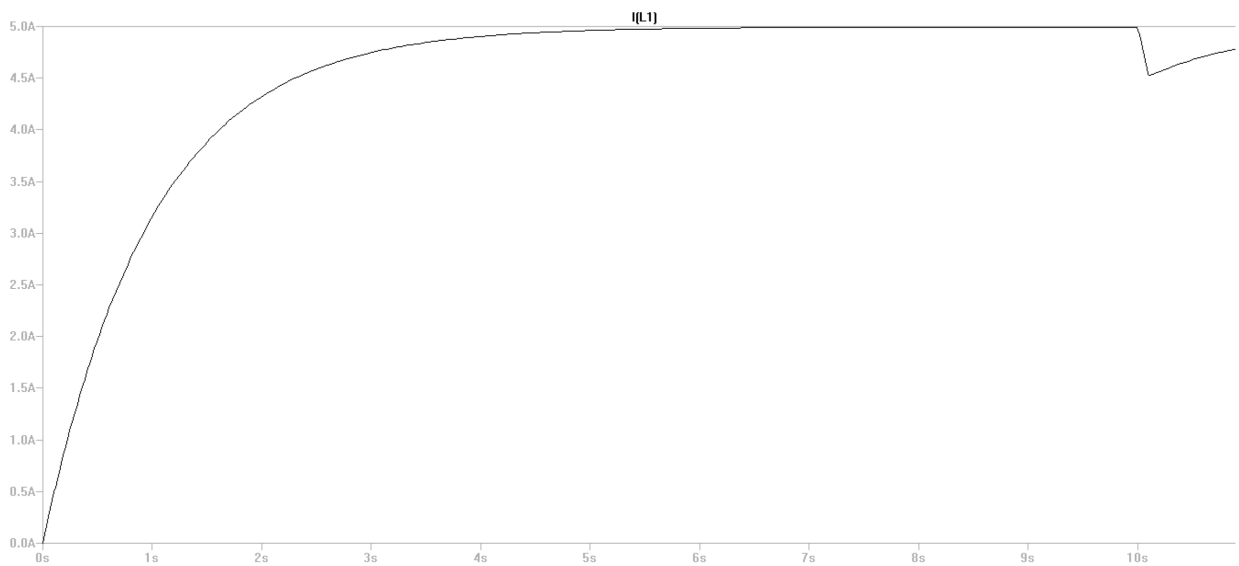
66)


I(L1)
4.8A
4.4A
4.0A
3.6A
3.2A
2.8A
2.4A
2.0A
1.6A
1.2A
0.8A
0.4A
0.0A
0.0ns
0.4ns
0.8ns
1.2ns
1.6ns
2.0ns
2.4ns
2.8ns
3.2ns
3.6ns


I(L1)
9.0A
8.1A
7.2A
6.3A
5.4A
4.5A
3.6A
2.7A
1.8A
0.9A
0.0A
0ns
4ns
8ns
12ns
16ns
20ns
24ns
28ns
32ns
36ns

67)





68) this is not a pulse! (pulse width is the same as the period)

69) this is not a pulse! (pulse width is the same as the period)

70) a) $v(0^-) = 21\ \ V$

b) $v(0^+) = 21\ \ V$

c) $v(40\,ms^-) = 15.6\ \ V$

d) $v(40\,ms^+) = 15.6\ \ V$

e) $v(50\,ms^-) = 16.31\ \ V$

71) a) $w(0^-) = 2.205\ \ Joul$

b) $w(0^+) = 2.205\ \ Joul$

c) $w(200\,ms) = 0.763\,Joul$

d) $w(400^-\ ms) = 0.756\ \ Joul$

e) $w(400^+\ ms) = 0.756\ \ Joul$

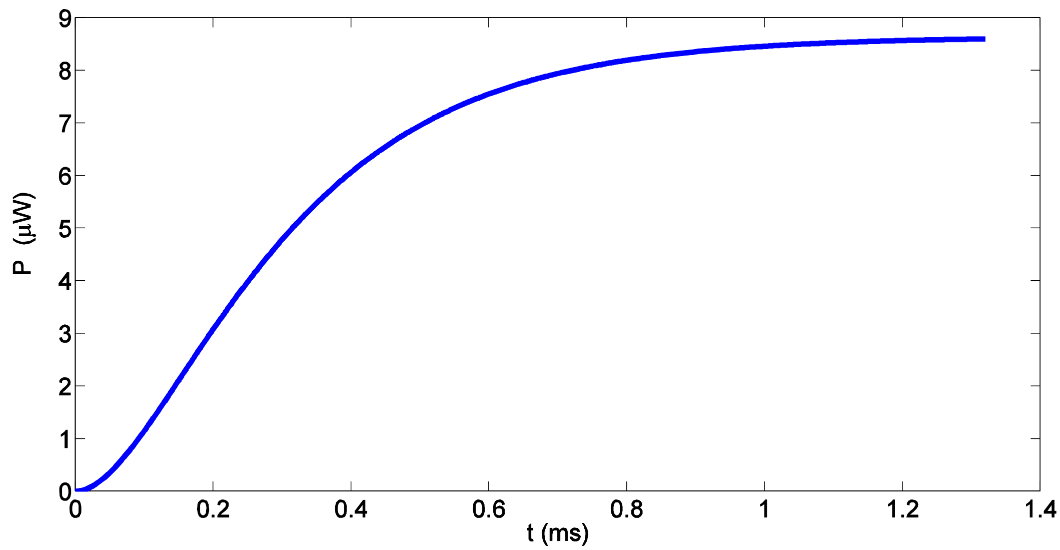
f) $w(700\,ms) = 2.18\ \ Joul$

72)

a) $\tau = 0.22\,msec$

b) $v_x = 5.88\left(1 - e^{-\frac{t\times 10^3}{0.22}}\right)\quad V$

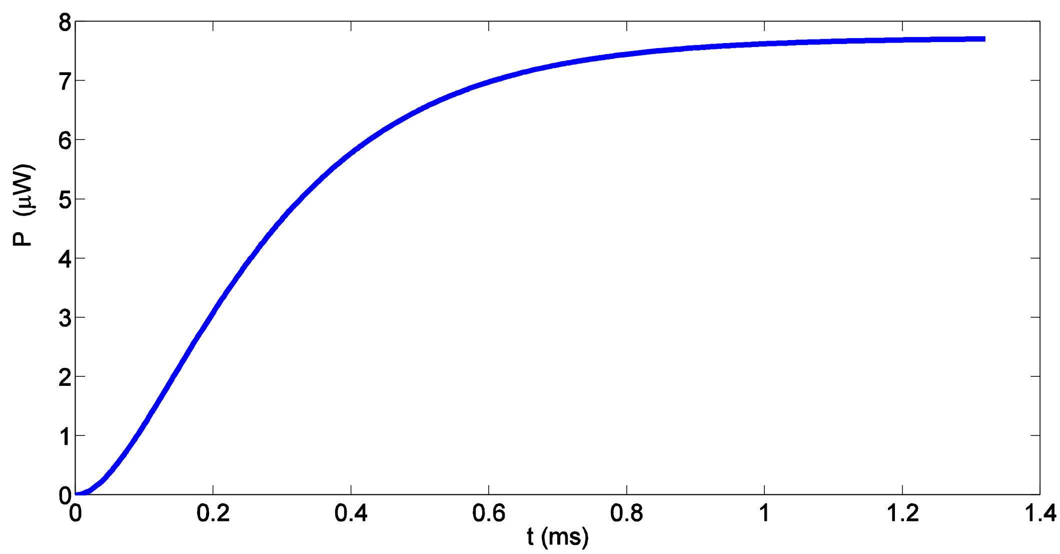
c)



d)

$$\tau = 0.2 \ ms$$

$$v_x = 5.56\left(1 - e^{-\frac{t\times 10^3}{0.2}}\right) \quad V$$



e) Yes, because as time goes to infinity, voltage remains limited.

73)

a) Without dielectric resistance, $\tau = 40.8\ ms$

With dielectric resistance, $\tau = 40.7446\ ms$

0.14 %

b) v(200 ms)= 19.85 mV

No, it doesn't. Because, the time constant remains almost the same.

74)

a) $v_o(t) = -6 - \frac{4}{3} \times 10^6 t \quad V$

b) $v_o(t) = -16e^{-130000t} \quad V$

1. R = 1KΩ, C = 3 μF, and L is such that the circuit response is overdamped.

   (a) For Source-free parallel RLC circuit:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\alpha = \frac{1}{2RC}$$

For overdamped $\alpha > \omega_0$ or $L > 4R^2C$

$$\boxed{L > 12\text{H}}$$

   (b) $v(0^-) = 9V, \frac{dv}{dt}\Big|_{t=0^+} = 2V/s$

Choose L = 13 H

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times1000\times3\times10^{-6}} = 166.67s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{13\times3\times10^{-6}}} = 160.128rad/s$$

$$\omega_0^{\ 2} = \frac{1}{LC} = \frac{1}{13\times3\times10^{-6}} = 25641$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^{\ 2}} = -120.43$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^{\ 2}} = -166.67 - 46.24 = -212.91$$

$$v(t) = A_1e^{s_1t} + A_2e^{s_2t}$$

$$v_R(t) = A_1e^{-120.43t} + A_2e^{-212.91t}$$

$$v(0^-) = 9\text{V}$$

$$9 = \text{A}_1 + A_2$$

$$\frac{dv}{dt}\Big|_{t=0^+} = A_1(-120.43) + A_2(-212.91) = 2$$

$$A_1 = 5.75, A_2 = 3.25$$

$$\boxed{v_R(t) = 5.75e^{-120.43t} + 3.25e^{-212.91t}}$$

2. L = 2 nH, C = 10 mF, the circuit source-free parallel RLC circuit.

(a) R so that the circuit is just barely overdamped.

For Source-free parallel RLC circuit:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\alpha = \frac{1}{2RC}$$

For overdamped $\alpha > \omega_0$ or $R < \frac{1}{2}\sqrt{\frac{L}{C}}$

R<0.0002236Ω

Choose $\boxed{R = 0.00021\ \Omega}$

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 0.00021\times 10\times 10^{-3}} = 238095.2s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times 10^{-9}\times 10\times 10^{-3}}} = 223606.8rad/s$$

$$\alpha > \omega_0$$

$$(b) i_R(0^+) = 13PA, di_E/dt\big|_{t=0^+} = 1nA/s$$

$$\omega_0 = 223606.8rad/s$$

$$\alpha = 227272.7s^{-1}$$

$$\omega_0{}^2 = 5\times 10^{10}$$

$$\alpha^2 = 5.165\times 10^{10}$$

$$\sqrt{\alpha^2 - \omega_0{}^2} = 40620.2$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -227272.7 + 40620.2 = -186652.5s^{-1}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -227272.7 - 40620.2 = -267892.9s^{-1}$$

$$i_R(t) = A_1e^{s_1t} + A_2e^{s_2t}$$

$$i_R(0^+) = 13pA$$

$$A_1 + A_2 = 13pA$$

$$\frac{di_E}{dt}\Big|_{t=0+} = 1nA$$

$$\frac{di_R}{dt}\Big|_{t=0^+} = A_1s_1 + A_2s_2 = 1nA/s$$

$$A_1 = 4.28\times 10^{-11}, A_2 = -2.98\times 10^{-11}$$

$$\boxed{i_R(t) = 4.28\times 10^{-11}e^{-186652.5t} - 2.98\times 10^{-11}e^{-267892.9t}}$$

3. C = 16 mF, L = 1 mH, choose R such that the circuit is

(a) Just barely overdamped;

$$\alpha = \frac{1}{2RC}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times10^{-3}\times16\times10^{-3}}} = 250 rad/s$$
$$R < \frac{1}{2}\sqrt{\frac{L}{C}}$$
$$R < 0.125$$
$$\boxed{R = 0.124\Omega}$$
$$\alpha = 252 s^{-1}$$
$$\alpha > \omega_0$$

(b) Just barely underdamped;

$$\alpha < \omega_0$$
$$R > 0.125$$
$$\boxed{R = 0.126\Omega}$$
$$\alpha = 248 s^{-1}$$
$$\alpha < \omega_0$$

(c) Critically damped.

$$\alpha = \omega_0$$
$$\boxed{R = 0.125\Omega}$$

(d) Does your answer for part (a) change if the resistor tolerance is 1%? 10%?

$$R = 0.124 \pm 1\%$$
$$R = 0.124 + 0.01(0.124) = 0.12524\Omega$$
$$\alpha = 249.52$$
$$\alpha < \omega_0 \Rightarrow underdamped$$
$$R = 0.124 - 1\%(0.124) = 0.12276$$
$$\alpha = 254.56 > \omega_0 \Rightarrow overdamped$$
$$R = 0.124 \pm 10\%$$
$$R = 0.124 + 0.0124 = 0.1364\Omega \Rightarrow \alpha = 229.1 < \omega_0 underdamped$$
$$R = 0.124 - 0.0124 = 0.1116\Omega \Rightarrow \alpha = 280 > \omega_0 overdamped$$

So, yes the answer for part (a) will change if the resistor tolerance is 1% or 10%

(e) Increase the exponential damping coefficient for part (c) by 20%. Is the circuit now underdamped, overdamped, or still critically damped? *Explain*.

$$\zeta = \frac{\alpha}{\omega_0}$$

$$\zeta_{old} = 1$$

$$\zeta_{new} = 1.2$$

$$1.2 = \frac{\alpha}{\omega_0} = \frac{\alpha}{250} \Rightarrow \alpha = 300s^{-1}$$

$$\alpha > \omega_0$$

Since $\alpha > \omega_0$ the circuit now is overdamped.

4. Calculate α, $\omega 0$, $s1$, and $s2$ for a source-free parallel *RLC* circuit if
   (a) $R = 4\ \Omega$, $L = 2.22$ H, and $C = 12.5$ mF;
   α = 10
   $\omega_0 = 6$
   $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2}$
   $s_1 = -2$
   $s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2}$
   $s_2 = -18$

(b) $L = 1$ nH, $C = 1$ pF, and R is 1% of the value required to make the circuit underdamped.

$$\omega_0 = \frac{1}{\sqrt{LC}} = 3.16\times 10^{10}\, rad/s$$

$$underdamped \Rightarrow \alpha < \omega_0$$

$$R > 15.82$$

$$1\%R = 0.1582$$

$$\alpha = \frac{1}{2RC} = 3.16\times 10^{12}\, s^{-1}$$

$$\alpha > \omega_0 \Rightarrow overdamped$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -158\times 10^6$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6.3198\times 10^{12}$$

(*c*) Calculate the damping ratio for the circuits of parts (*a*) and (*b*)

$$\zeta_a = \frac{\alpha}{\omega_0} = \frac{10}{6} = 1.667$$

$$\zeta_b = \frac{\alpha}{\omega_0} = \frac{3.16\times 10^{12}}{3.16\times 10^{10}} = 100$$

5. No 1 k_ resistors, C = 3 µF, L>12H, 1 meter long piece of 24 AWG soft solid copper wire.

Replace the resistor with 1 meters of 24 AWG copper wire. From Table 2.4, 24 AWG soft solid copper wire has a resistance of 25.7 Ω/1000ft. Thus, the wire has a resistance of

$$R = (1m)\left(\frac{100cm}{1m}\right)\left(\frac{1in}{2.54cm}\right)\left(\frac{1\,ft}{12in}\right)\left(\frac{25.7\Omega}{1000\,ft}\right) = 0.0843\Omega$$

$$\alpha = \frac{1}{2RC} = 1977066s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{13\times 3\times 10^{-6}}} = 160rad/s$$

$$\alpha > \omega_0 \Rightarrow overdamped$$

$$\alpha^2 = 3.909\times 10^{12}, \omega_0^{\,2} = 25641$$

$$\sqrt{\alpha^2 - \omega_0^{\,2}} = 1977119$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^{\,2}} = -1977066 + 1977119 = 53$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -1977066 - 1977119 = -3954185$$

6. source-free parallel RLC circuit having $\alpha = 10^8 s^{-1}, \omega_0 = 10^3 rad/s, \omega_0 L = 5\Omega.$

(a)

$$1H = \frac{V \cdot s}{A} = \Omega \cdot s$$
$$\omega_0 \Rightarrow 1/s$$
$$\omega_0 L = \Omega \cdot s \times \frac{1}{s} = \Omega$$

(b)

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -10^8 + \sqrt{10^{16} - 10^6} = -683.77 \times 10^6$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -10^8 - \sqrt{10^{16} - 10^6} = -131.62 \times 10^7$$

(c)

$$v_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$v_c(t) = A_1 e^{-683.77 \times 10^6 t} + A_2 e^{-131.62 \times 10^7 t}$$

(d) Given L and C each initially store 1mJ of energy

$$w_c = \frac{1}{2} C v^2(t_0) = 1 \times 10^{-3} J$$
$$w_L = \frac{1}{2} L i^2(t_0) = 1 \times 10^{-3} J$$
$$\frac{v}{R} + \frac{1}{L}\int_{t_0}^{t} v dt' - i(t_0) + C\frac{dv}{dt} = 0$$

7. R = 500 Ω, C = 10 μF, and L such that it is critically damped.

(a) Determine L.

$$L = 4R^2C$$

$$L = 4\times(500)^2\times 10\times 10^{-6} = 10H$$

Yes, this value s large for a printed-circuit board mounted component. For example, 11H inductor measuring 10cm (tall) ×8 cm (wide) ×8 cm (deep).

(b) Add a resistor in parallel to the existing components such that the damping ratio is equal to 10.

$$\zeta = 10 \Rightarrow \zeta = \frac{\alpha}{\omega_0}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 500\times 10\times 10^{-6}} = 100$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 100$$

$$\zeta = 1$$

$$10 = \frac{\alpha}{\omega_0} \Rightarrow \alpha = 1000$$

$$1000 = \frac{1}{2\times R\times 10\times 10^{-6}} \Rightarrow R = 50\Omega$$

$$50 = 500 \,//\, R_{new} \Rightarrow 50 = \frac{R_{new}\times 500}{R_{new}+500}$$

$$R_{new} = 55.6\Omega$$

(c) Increasing the damping ratio further lead to an overdamped circuit since

$\zeta = \dfrac{\alpha}{\omega_0}$ When increasing the damping ratio we are increasing α

$$\alpha > \omega_0$$

8. R=1KΩ, L= 7mH, C= 1nF. L initially discharged and C strong 7.2mJ

(a)

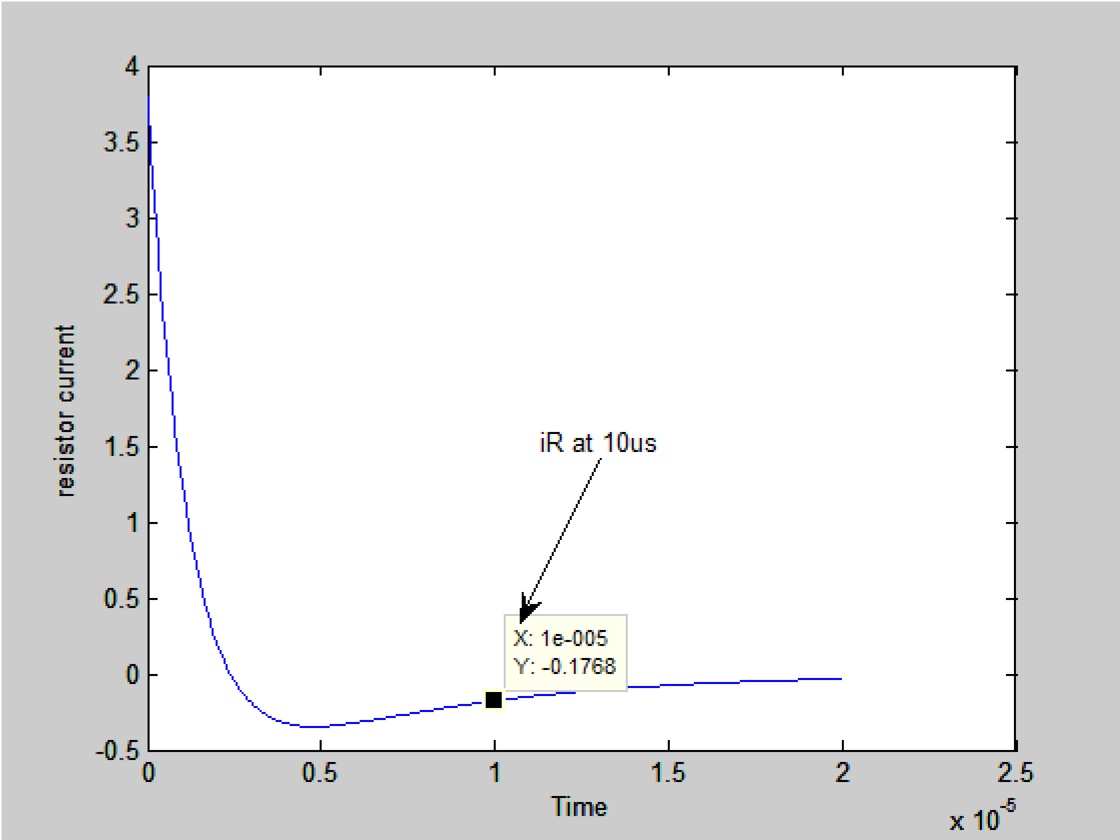
$$\alpha = \frac{1}{2RC} = 5\times10^{5}\,s^{-1}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{7\times10^{-3}\times1\times10^{-9}}} = 377964\,rad/\sec$$
$$\alpha > \omega_0 \Rightarrow overdamped$$
$$\alpha^2 = 2.5\times10^{11}, \omega_0{}^2 = 1.429\times10^{11}$$
$$\sqrt{\alpha^2 - \omega_0^2} = 327261.4$$
$$s_1 = -172738.6$$
$$s_2 = -827261.4$$

(b) i(t) through the resistor for t>0

$$i_R(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$i_L(0^-) = i_L(0^+) = 0$$
$$w_C = \frac{1}{2} C v_C{}^2(0) = 7.2\times10^{-3} \Rightarrow v_C(0^-) = \sqrt{\frac{2\times7.2\times10^{-3}}{1\times10^{-9}}} = 3.795\times10^3 V = v_C(0^+)$$
$$v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} = A_1 e^{-172738.6t} + A_2 e^{-827261.4t}$$
$$v_C(0^+) = 3.795\times10^3 V = A_1 + A_2$$
$$\frac{dv_C}{dt}\Big|_{t=0^+} = -172738.6A_1 - 827261.4A_2 = \frac{i_C(0^+)}{C} = \frac{-i_R(0^+) - i_L(0^+)}{C} = \frac{-3.795}{1\times10^{-9}} = -3.79510^9$$
$$A_1 = -1.0009\times10^3, A_2 = 4.7956\times10^3$$
$$v_C(t) = -1.0009\times10^3 e^{-172738.6t} + 4.7956\times10^3 e^{-827261.4t}$$
$$i_R(t) = \frac{v_C(t)}{R} = -1.0009e^{-172738.6t} + 4.7956e^{-827261.4t}$$

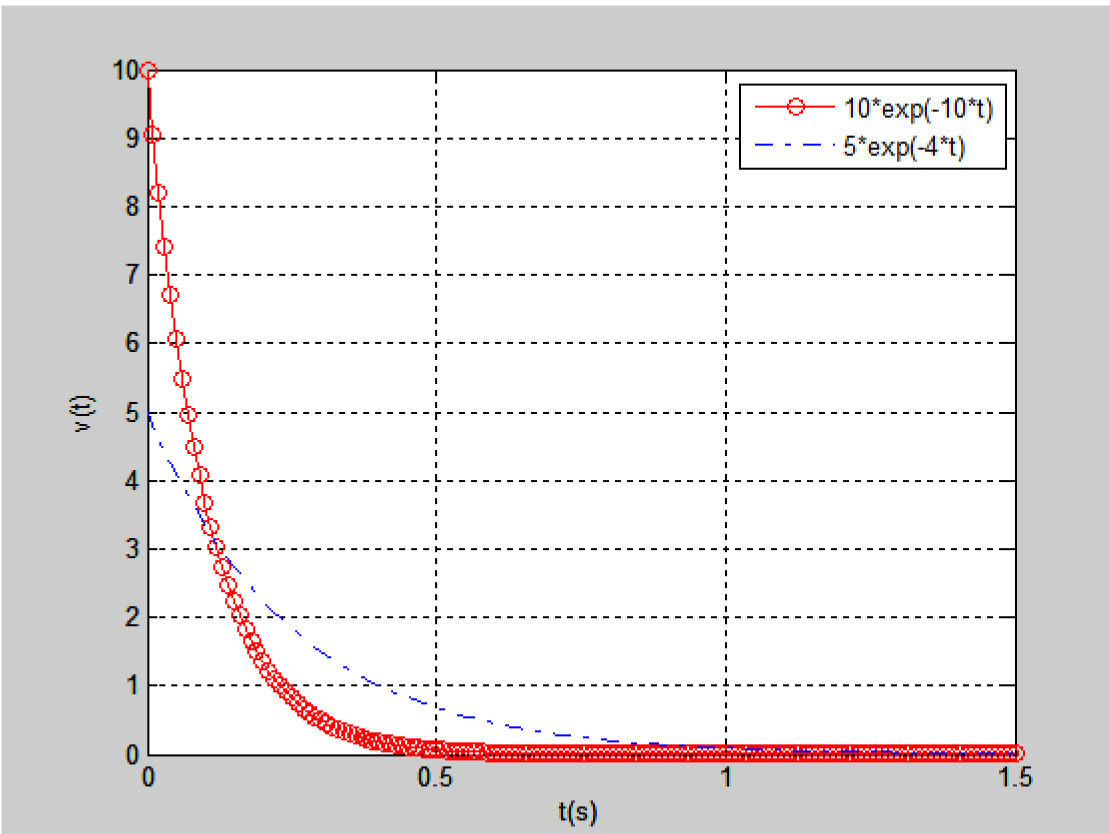
(c)

$i_R(10\mu s) = \text{-}0.1768\text{A}$

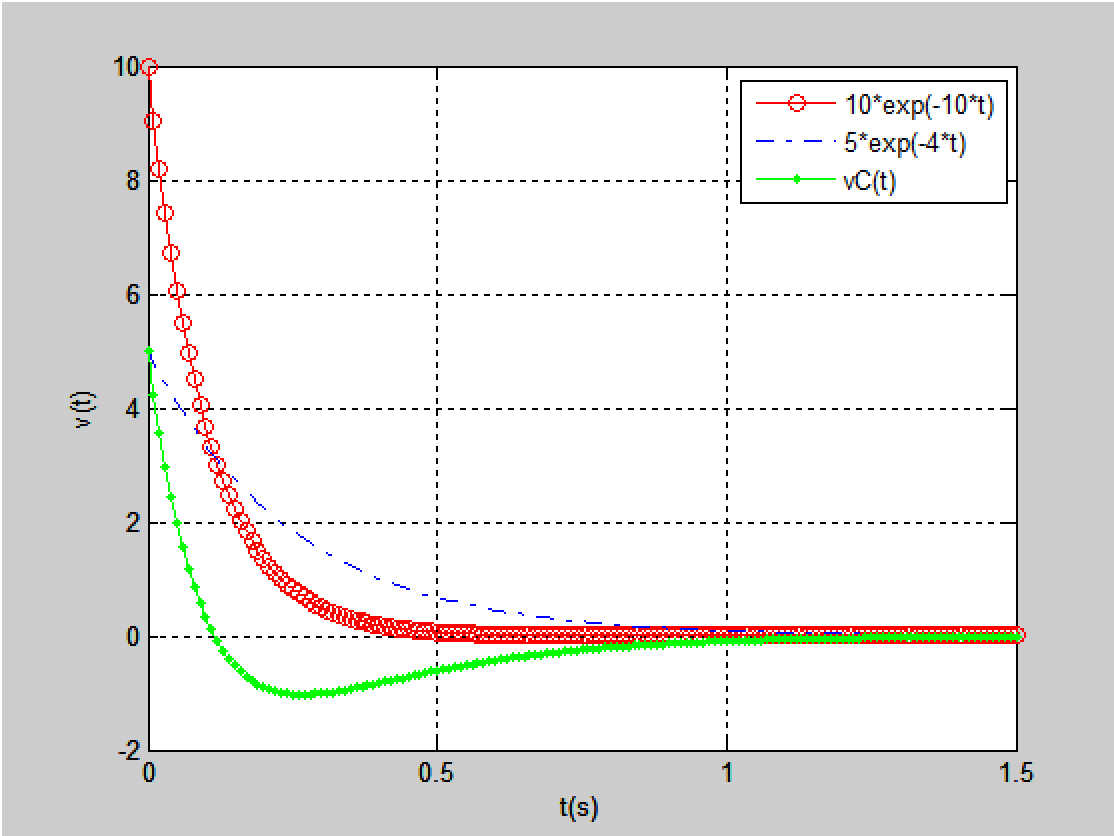


9. $v_C(t) = 10e^{-10t} - 5e^{-4t}V$

(a)
```
clear all;
close all;
clc;
t=[0:0.01:1.5];
vc1=10*exp(-10*t);
vc2=5*exp(-4*t);
vc=vc1-vc2;
plot(t,vc1,'-ro')
hold on
plot(t,vc2,'-.b')
hold on
plot(t,vc,'.-g')
hleg1 = legend('10*exp(-10*t)','5*exp(-4*t)','vC(t)');
grid on
xlabel('t(s)')
ylabel('v(t)')
```



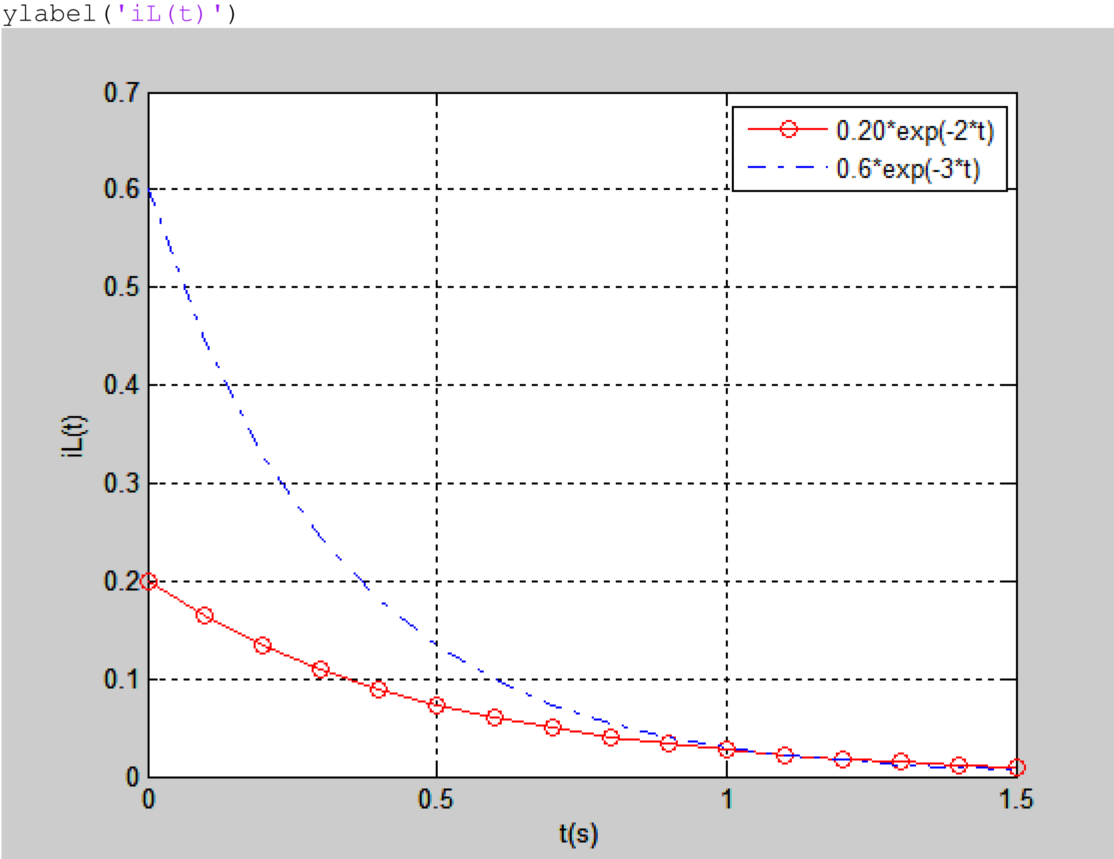
(b)



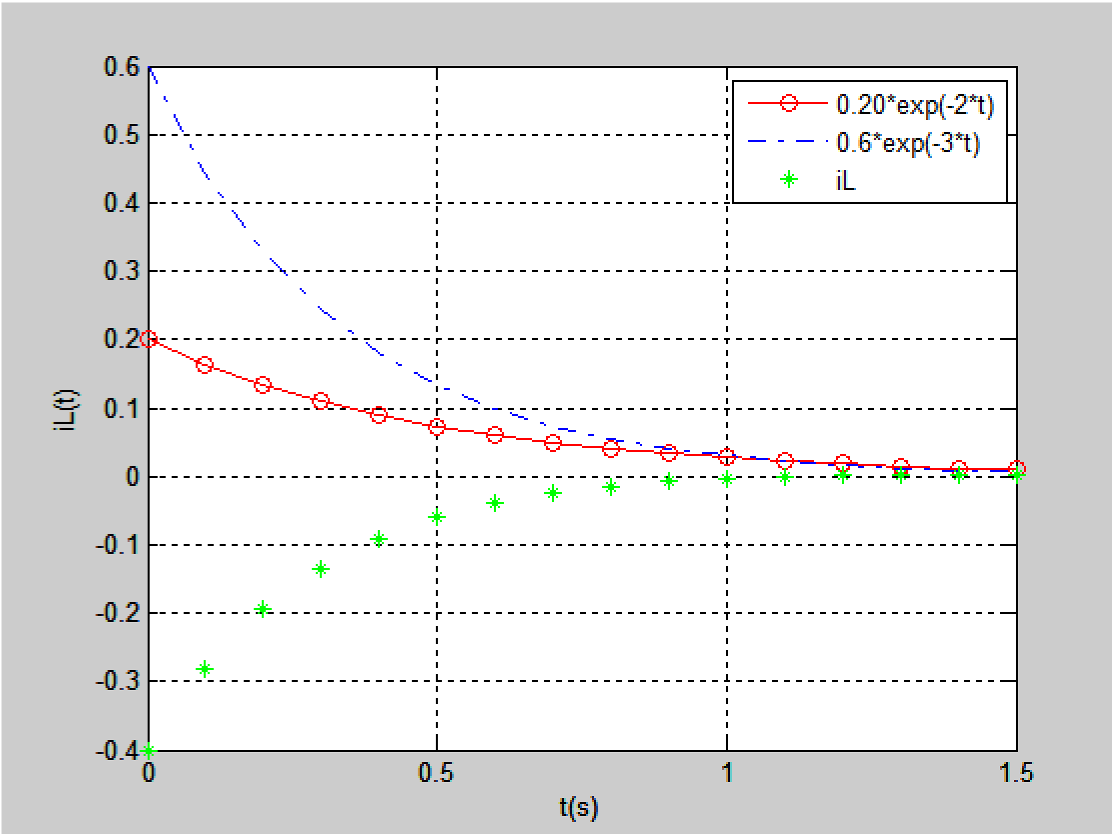
10. $i_L(t) = 0.20e^{-2t} - 6e^{-3t} A$

(a)

```
clear all;
close all;
clc;
t=[0:0.1:1.5];
il1=0.20*exp(-2*t);
il2=0.6*exp(-3*t);
il=il1-il2;
plot(t,il1,'-ro')
hold on
plot(t,il2,'-.b')
hleg1 = legend('0.20*exp(-2*t)','0.6*exp(-3*t)');
grid on
xlabel('t(s)')
ylabel('iL(t)')
```



(b)



(c) $w_L(t) = \frac{1}{2} L i_L^2$

$$s_1 = -2 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2}$$

$$s_2 = -3 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} \Rightarrow s_1 + s_2 = -5 = -2\alpha \Rightarrow \alpha = 2.5 s^{-1}$$

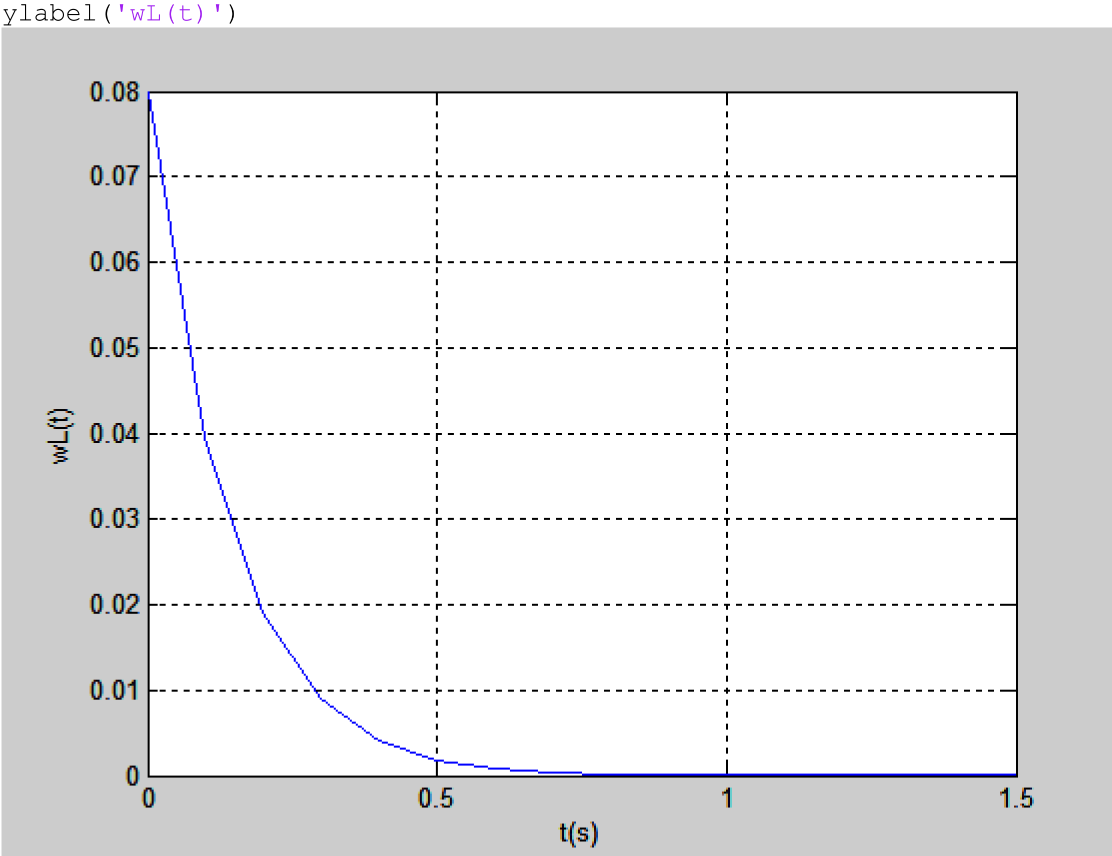
$$\omega_0 = \sqrt{6} rad / \sec$$

$$let R = 1\Omega \Rightarrow \alpha = \frac{1}{2RC} \Rightarrow 2.5 = \frac{1}{2(1)C} \Rightarrow C = 0.2F$$

$$overdamped \Rightarrow \alpha > \omega_0 \Rightarrow L > \frac{1}{\omega_0{}^2 C} \Rightarrow L > 0.8H \Rightarrow L = 1H$$

$$w_L = \frac{1}{2} L i_L{}^2 = \frac{1}{2}[0.20e^{-2t} - 6e^{-3t}]^2$$

```
t=[0:0.1:1.5];
il1=0.20*exp(-2*t);
il2=0.6*exp(-3*t);
il=il1-il2;
plot(t,il1,'-ro')
hold on
plot(t,il2,'-.b')
il=il1-il2;
hold on
plot(t,il,'*g')
ylabel('iL(t)')
hleg1 = legend('0.20*exp(-2*t)','0.6*exp(-3*t)','iL');
figure
wl=(0.5)*(il).^2;
plot(t,wl)
grid on
xlabel('t(s)')
ylabel('wL(t)')
```



11.

(a)

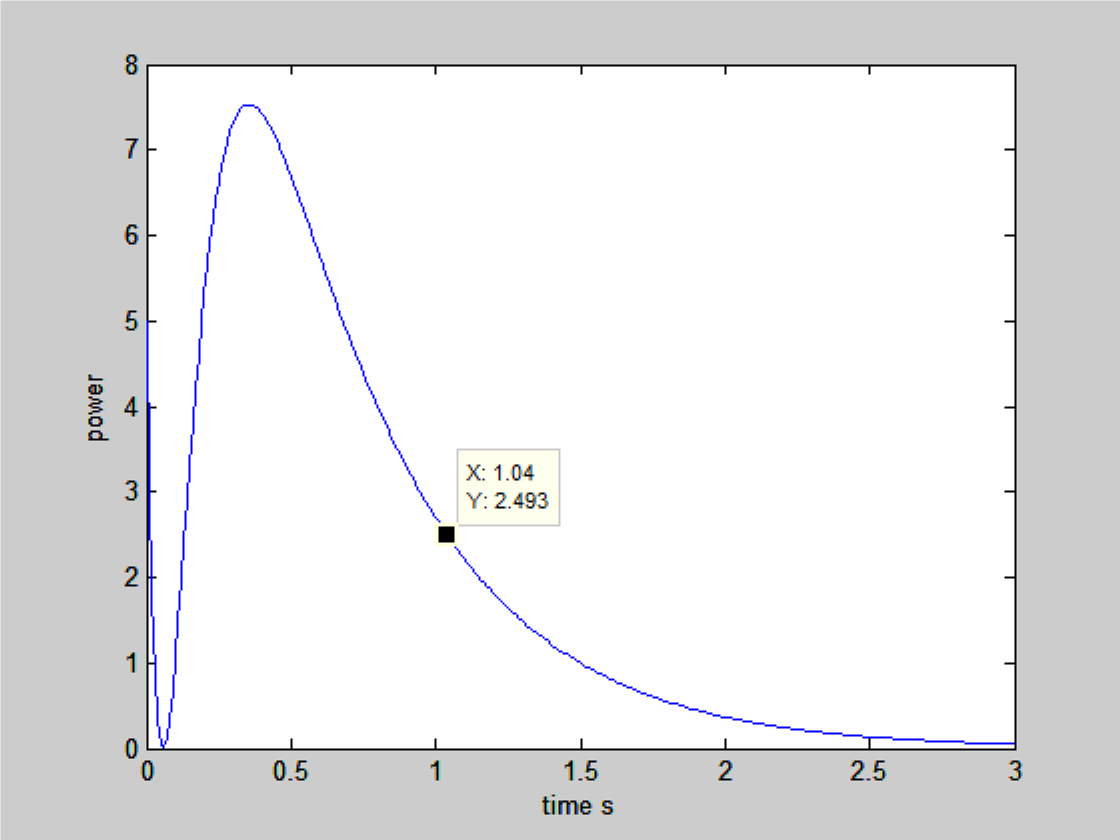
$$i_R(t) = 2e^{-t} - 3e^{-8t}, t > 0$$
$$\frac{di_R}{dt} = -2e^{-t} + 3\times 8e^{-8t} = 0$$
$$-2e^{-t} + 24e^{-8t} = 0$$
$$12 = e^{7t_m} \Rightarrow \ln(7) = 7t_m \Rightarrow t_m = 1.714s$$
$$i_R(t_m) = 2e^{(-1.714)} - 3e^{-8\times(1.714)}$$
$$i_R(t_m) = i_{R\max} = 0.3603$$

(b)

$$\left|i_{R\max}\right| = 0.3603$$
$$1\%\left|i_{R\max}\right| = 0.003603$$
$$0.003603 = 2e^{-t_s} - 3e^{-8t_s}$$
$$t_s = 6.319s$$

(c) The time corresponding to the resistor absorbing 2.5 W

$$P = i_R{}^2 R = 5\left(2e^{-t} - 3e^{-8t}\right)^2$$
$$2.5 = 5\left(2e^{-t} - 3e^{-8t}\right)^2$$
$$\sqrt{0.5} = 2e^{-t} - 3e^{-8t} \Rightarrow \ln(\sqrt{0.5}) = \ln(2e^{-t} - 3e^{-8t}) \Rightarrow t = 1.04\sec$$



X: 1.04
Y: 2.493
power
time s
0
0.5
1
1.5
2
2.5
3
1
2
3
4
5
6
7
8

12.

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 0.1\times 250\times 10^{-3}} = 20s^{-1}$$
$$\alpha^2 = 400$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{2}{13}\times 250\times 10^{-3}}} = 5.1 rad/s$$
$$\omega_0{}^2 = 26.1$$
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -20 + \sqrt{400 - 26.1} = -0.66$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -39.34$$
$$\alpha > \omega_0 \Rightarrow overdamped$$

At $t = 0^-$ replace the inductor with short crcuit and the capacitor with open circuit. The voltage power supply will be on.

$$i_L(0^-) = -\frac{6}{20k\Omega + 0.1} = -2.99\times 10^{-4} A$$
$$v_C(0^-) = 6\cdot\frac{0.1}{20k\Omega + 0.1} = 2.99\times 10^{-5} V$$

At $t = 0^+$

$$v_C(0^+) = v_C(0^-) = 2.99\times 10^{-5} V$$
$$i_L(0^+) = i_L(0^-) = -2.99\times 10^{-4} A$$
$$v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$v_C(t) = A_1 e^{-0.66t} + A_2 e^{-39.34t}$$
$$v_C(0) = 2.99\times 10^{-5} = A_1 + A_2$$
$$\frac{dv_C}{dt} = -0.66A_1 e^{-0.66t} - 39.34A_2 e^{-39.34t}$$
$$i_C = C\frac{dv_C}{dt} \Rightarrow i_C(0^+) = -i_L(0^+) - i_R(0^+) = -(-2.99\times 10^{-4}) - \left[\frac{v_C(0^+)}{0.1}\right] = 0$$
$$i_C(0) = C\frac{dv_C}{dt} \Rightarrow \frac{dv_C}{dt}\Big|_{t=0} = \frac{i_C(0)}{C} = 0$$
$$0.66A_1 + 39.34A_2 = 0$$
$$A_1 = 0.3\times 10^{-4}, A_2 = -5.1\times 10^{-7}$$
$$v_C(t) = 0.3\times 10^{-4} e^{-0.66t} - 5.1\times 10^{-7} e^{-39.34t}$$

13.

(a)

$$\alpha = 20, \omega_0 = 5.1 rad/s, s_1 = -0.66, s_2 = -39.34$$
$$i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t > 0 \Rightarrow i_L(t) = A_1 e^{-0.66t} + A_2 e^{-39.34t}$$
$$i_L(0) = -2.99 \times 10^{-4} A = i_L(0^+)$$
$$i_L(0) = A_1 + A_2 = -2.99 \times 10^{-4}$$
$$\frac{di_L}{dt} = -0.66 A_1 e^{-0.66t} - 39.34 A_2 e^{-39.34t}$$
$$v_L(t) = L\frac{di_L}{dt}, v_L(0^+) = v_C(0+) = 2.99 \times 10^{-5} V$$
$$v_L(0) = \frac{2}{13}(-0.66A_1 - 39.34A_2) = 2.99 \times 10^{-5} \Rightarrow 0.66A_1 + 39.34A_2 = -1.94 \times 10^{-4}$$
$$A_1 = -2.99 \times 10^{-4}, A_2 = 7.7 \times 10^{-8}$$
$$i_L(t) = -2.99 \times 10^{-4} e^{-0.66t} + 7.7 \times 10^{-8} e^{-39.34t}$$

(b)

$$i_R(t), t > 0$$
$$i_R(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow i_R(t) = A_1 e^{-0.66t} + A_2 e^{-39.34t}$$
$$t = 0^- \Rightarrow i_R(0^-) = i_L(0^-) = -2.99 \times 10^{-4} A, v_C(0^-) = 2.99 \times 10^{-5} V$$
$$t = 0^+ \Rightarrow i_L(0^+) = i_L(0^-) = -2.99 \times 10^{-4} A, v_C(0^+) = v_C(0^-) = 2.99 \times 10^{-5} V$$
$$i_R(0^+) = \frac{v_C(0^+)}{R} = 2.99 \times 10^{-4}$$
$$i_R(0) = A_1 + A_2 = 2.99 \times 10^{-4}$$
$$v_C(t) = Ri_R(t) = 0.1 \times \left[A_1 e^{-0.66t} + A_2 e^{-39.34t}\right]$$
$$i_C(t) = C\frac{dv_C}{dt} = 250 \times 10^{-3} \times 0.1 \times \left[-0.66 A_1 e^{-0.66t} - 39.34 A_2 e^{-39.34t}\right]$$
$$i_C(0^+) = -i_L(0^+) - i_R(0^+) = 0$$
$$0.66A_1 + 39.34A_2 = 0 \Rightarrow A_1 = 3.04 \times 10^{-4}, A_2 = -5.1 \times 10^{-6}$$
$$i_R(t) = 3.04 \times 10^{-4} e^{-0.66t} - 5.1 \times 10^{-6} e^{-39.34t}$$

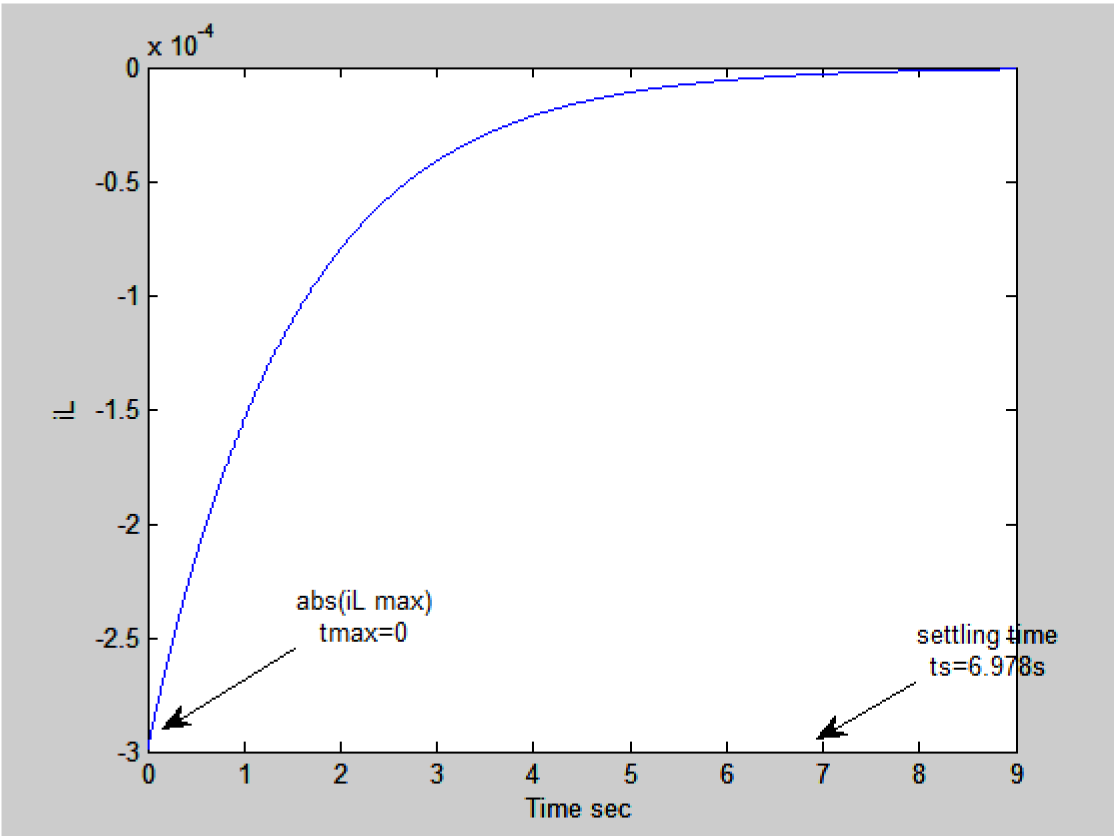
(c)

$$\left|i_{L\max}\right| \Rightarrow \frac{di_L}{dt} = 1.7\times10^{-4}e^{-0.66t} - 3.03\times10^{-6}e^{-39.34t} = 0 \Rightarrow t_{\max} = 0$$

$$\left|i_{L\max}\right| = 2.989\times10^{-4}\,A$$

$$1\%\left|i_{L\max}\right| = 2.989\times10^{-6}\,A$$

$$t_s = 6.978s$$



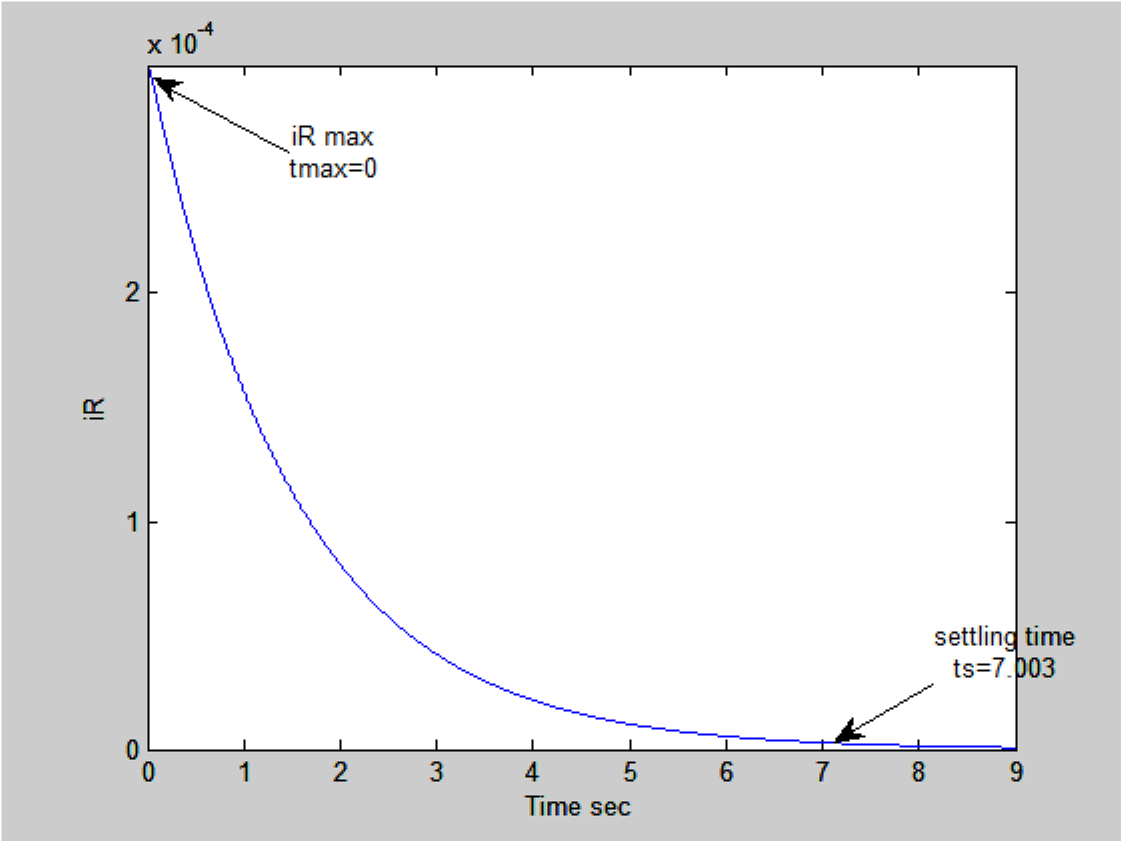
$$i_R(t) = 3.04\times10^{-4}e^{-0.66t} - 5.1\times10^{-6}e^{-39.34t}$$

$$\left|i_{R\max}\right| \Rightarrow \frac{di_R}{dt} = -2\times10^{-4}e^{-0.66t} + 2\times10^{-4}e^{-39.34t} = 0 \Rightarrow t_{\max} = 0$$

$$\left|i_{R\max}\right| = 2.989\times10^{-4}\,A$$

$$1\%\left|i_{L\max}\right| = 2.989\times10^{-6}\,A$$

$$t_s = 7.003s$$


x 10^-4
iR max
tmax=0
settling time
ts=7.003
iR
Time sec
0
1
2
0
1
2
3
4
5
6
7
8
9

14. at t=0 both the 10 A source and the 48 Ω are removed leaving the 2mF, 250 mH, and 1Ω resistor in parallel.

(a) $i_C(0^-) = 0$

(b) $i_L(0^-) = \dfrac{0.009796}{48} = 2.041\times10^{-4}\,A$

(c) $i_R(0^-) = \dfrac{0.009796}{1} = 9.79\times10^{-3}\,A$

(d) $v_C(0^-) = (48\Omega \,//\, 1\Omega)(10\times10^{-3}\,A) = 0.9796\times10\times10^{-3} = 0.009796V$

(e) $i_C(0^+) = -i_L(0^+) - i_R(0^+) = -2.041\times10^{-4} - 9.796\times10^{-3} = -0.01A$

(f) $i_L(0^+) = i_L(0^-) = 2.041\times10^{-4}\,A$

(g) $i_R(0^+) = \dfrac{v_C(0^+)}{R} = \dfrac{0.009696}{1} = 0.009796A$

(h) $v_C(0^+) = v_C(0^-) = 0.009796V$

15. $v_R(t)$ for t>0

(a)

$$\alpha=\frac{1}{2RC}=\frac{1}{2\times1\times2\times10^{-3}}=250s^{-1}$$

$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{250\times10^{-3}\times2\times10^{-3}}}=44.7rad/s$$

$$\alpha>\omega_0\Rightarrow overdamped$$

$$v_R(t)=A_1e^{s_1t}+A_2e^{s_2t}$$

$$s_1=-\alpha+\sqrt{\alpha^2-\omega_0{}^2}=-250+\sqrt{(250)^2-(44.7)^2}$$

$$s_1=-4$$

$$s_2=-\alpha-\sqrt{\alpha^2-\omega_0{}^2}=-250-\sqrt{(250)^2-(44.7)^2}$$

$$s_2=-495.97$$

$$v_R(t)=A_1e^{-4t}+A_2e^{-495.97t}$$

$$v_R(0^-)=v_C(0^-)=9.796\times10^{-3}$$

$$v_R(0^+)=v_C(0^+)=9.796\times10^{-3}$$

$$v_R(0)=9.796\times10^{-3}V$$

$$A_1+A_2=9.796\times10^{-3}$$

$$v_C(t)=A_1e^{-4t}+A_2e^{-495.97t}\Rightarrow\frac{dv_C}{dt}=-4A_1e^{-4t}-495.97A_2e^{-495.97t}$$

$$\frac{dv_C}{dt}\Big|_{t=0^+}=-4A_1-495.97A_2\Rightarrow i_C(0^+)=C\frac{dv_C}{dt}\Big|_{t=0^+}=-i_L(0^+)-i_R(0^+)=-0.01A$$

$$-4A_1-495.97A_2=-5\Rightarrow A_1=-2.04\times10^{-4},A_2=0.01$$

$$\therefore v_R(t)=-2.04\times10^{-4}e^{-4t}+0.01e^{-495.97t}$$

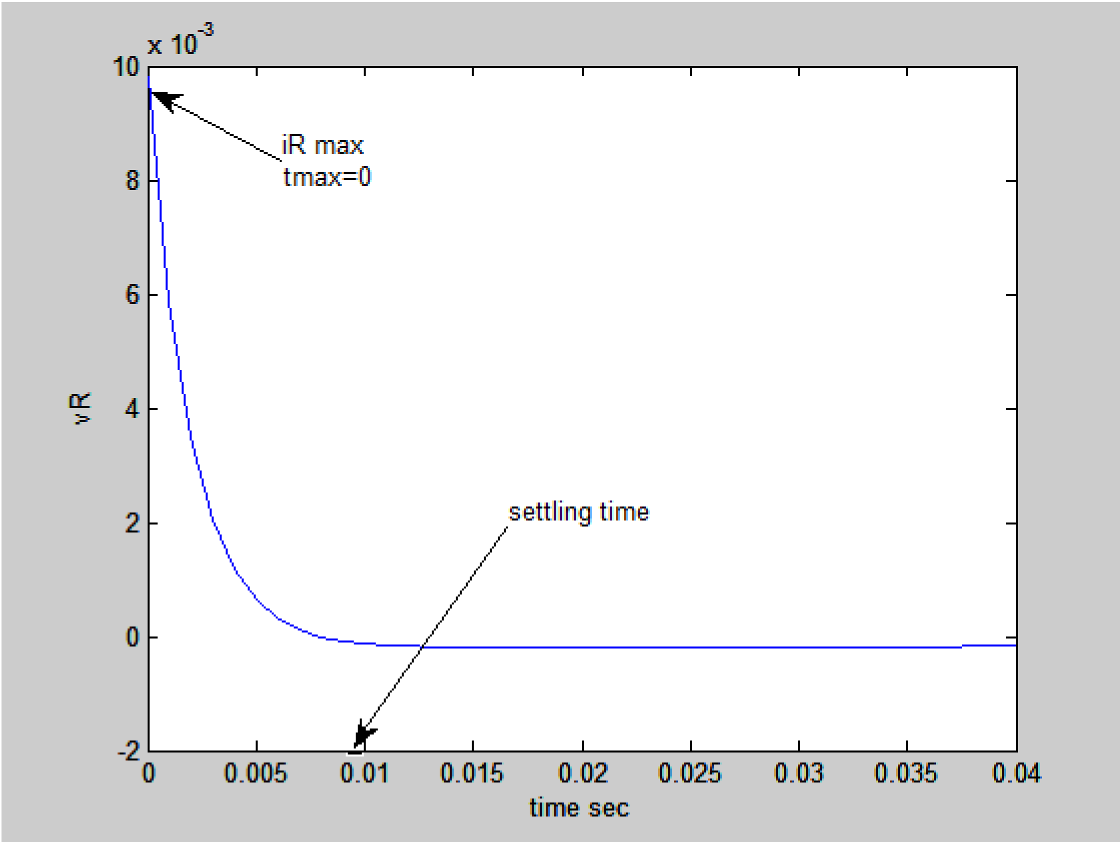
$$v_R(t) = -2.04\times10^{-4}e^{-4t} + 0.01e^{-495.97t}$$

$$|v_{R\max}| \Rightarrow \frac{dv_R}{dt} = 8.16\times10^{-4}e^{-4t} + 4.9597e^{-495.97t} = 0 \Rightarrow t_{\max} = 0$$

(b) $|v_{R\max}| = 9.796\times10^{-3}V$

$$1\%|v_{R\max}| = 9.796\times10^{-5}V$$

$$t_s = 9.33\times10^{-3}s$$



16.

(a) at $t = 0^-$

$i_R(0^-) = 0, i_L(0^-) = 5\mu A$
$v_C(0^-) = 0$

at $t = 0^+ \Rightarrow v_C(0^+) = 0V$

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 0.2\times 4\times 10^{-3}} = 625s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(1\times 10^{-3}\right)\left(4\times 10^{-3}\right)}} = 500 rad/s$$

$$\alpha > \omega_0 \Rightarrow overdamped$$

$$v_C(t) = A_1 e^{s_1 t} + Ae^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -625 + \sqrt{(625)^2 - (500)^2}$$

$$s_1 = -250$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0{}^2} = -625 - \sqrt{(625)^2 - (500)^2}$$

$$s_2 = -1000$$

$$v_R(t) = A_1 e^{-250t} + A_2 e^{-1000t}$$

$$v_C(0^-) = v_C(0^+) = 0V$$

$$v_C(0) = 0V$$

$$A_1 + A_2 = 0$$

$$\frac{dv_C}{dt} = -250A_1 e^{-250t} - 1000e^{-1000t}$$

$$i_C = C\frac{dv_C}{dt}$$

$$i_C(0^+) = -i_L(0^+) - i_R(0^+)$$

$$i_R(0^+) = \frac{v_C(0^+)}{R} = 0$$

$$i_L(0^+) = i_L(0^-) = 5\mu A \Rightarrow i_C(0) = -i_L(0^+) = -5\mu A$$

$$-5\times 10^{-6} = 4\times 10^{-3}(-250A_1 - 1000A_2)$$

$$A_1 = -1.67\times 10^{-6}, A_2 = 1.67\times 10^{-6}$$

$$v(t) = -1.67\times 10^{-6} e^{-250t} + 1.67\times 10^{-6} e^{-1000t}$$

(b)

$$i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$v_C(t) = A_1 e^{-250t} + A_2 e^{-1000t}$$
$$i_L(0^-) = i_L(0^+) = 5\mu A \Rightarrow i_L(0) = 5\mu A$$
$$5\mu A = A_1 + A_2$$
$$\frac{di_L}{dt} = -250A_1 e^{-250t} - 1000A_2 e^{-1000t}$$
$$v_L(t) = L\frac{di_L}{dt}$$
$$v_L(0^+) = v_C(0^+) = 0V$$
$$250A_1 + 1000A_2 = 0$$
$$A_1 = 6.67\times10^{-6}, A_2 = -1.67\times10^{-6}$$
$$\therefore i_L(t) = 6.67\times10^{-6} e^{-250t} - 1.67\times10^{-6} e^{-1000t}$$
$$\frac{di_L}{dt} = -250\times6.67\times10^{-6} e^{-250t_m} - 1000\times1.67\times10^{-6} e^{-1000t_m} = 0$$
$$t_m = 0$$
$$i_L(t_m) = 5\times10^{-6} A$$

(c)

$$i_{L\max} = 5\times10^{-6} A$$
$$1\% i_{L_{\max}} = 0.05\times10^{-6} A$$
$$0.05\times10^{-6} = 6.67\times10^{-6} e^{-250t_s} - 1.67\times10^{-6} e^{-1000t_s}$$
$$0.05\times10^{-6} = 6.67\times10^{-6} e^{-250t_s}$$
$$0.0075 = e^{-250t_s}$$
$$\ln(0.0075) = -250t_s$$
$$t_s = 19.57\times10^{-3} s$$

17.

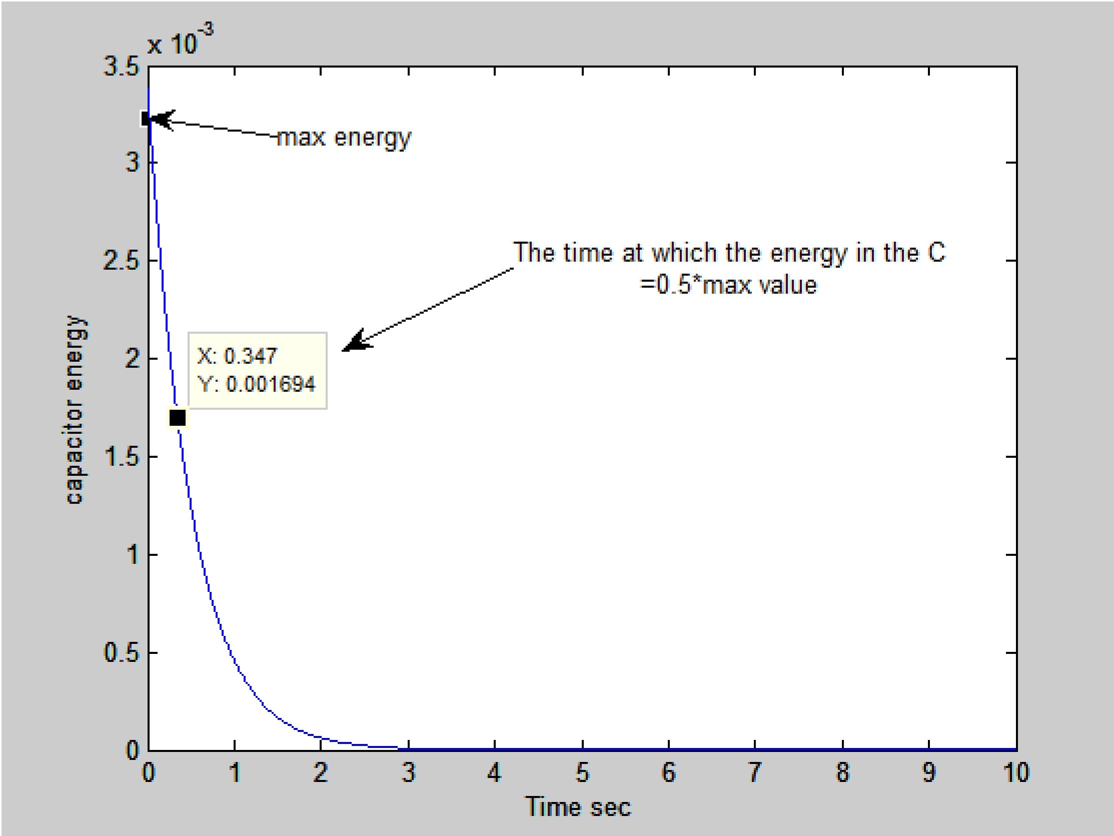
$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 14\times 360\times 10^{-6}} = 99.2$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times 360\times 10^{-6}}} = 52.7 rad/\sec$$
$$\alpha > \omega_0 \Rightarrow overdamped$$
$$v(t) = A_1 e^{s_1 t} + A e^{s_2 t}$$
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -99.2 + \sqrt{(99.2)^2 - (52.7)^2}$$
$$s_1 = -15.16 s^{-1}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0{}^2} = -99.2 - \sqrt{(99.2)^2 - (52.7)^2}$$
$$s_2 = -183.24 s^{-1}$$
$$v(t) = A_1 e^{-15.16t} + A_2 e^{183.24t}$$
$$t = 0^-$$
$$i_L(0^-) = -i_R(0^-) = -310 mA$$
$$v_C(0^-) = v_R(0^-) = (14)(310\times 10^{-3}) = 4.34V$$
$$t = 0^+$$
$$i_L(0^+) =_L (0^-) = -310 mA$$
$$v_C(0^+) = v_C(0^-) = 4.34V$$
$$v_C(t) = A_1 e^{-15.16t} + A_2 e^{-183.24t} \Rightarrow v_C(0) = 4.34 = A_1 + A_2$$
$$\frac{dv_C}{dt} = -15.16 A_1 e^{-15.16t} - 183.24 A_2 e^{-183.24t}$$
$$i_C = C\frac{dv_C}{dt}$$
$$i_C(0^+) = -i_L(0^+) - i_R(0^+)$$
$$i_R(0^+) = \frac{v_C(0^+)}{R} = \frac{4.34}{14} = 0.31A, i_C(0^+) = -i_L(0^+) - i_R(0^+) = 0$$
$$\frac{dv_C}{dt}\Big|_{t=0} = -15.16 A_1 - 183.24 A_2 = 0 \Rightarrow A_1 = 4.73, A_2 = \text{-}0.391$$
$$\therefore v_C(t) = 4.73 e^{-15.16t} - 0.391 e^{-183.24t}$$
$$i_L = \frac{1}{L}\int_0^t v dt - i_L(0) = \int_0^t [4.73 e^{-15.16t} - 0.391 e^{-183.24t}] dt + 0.31 = -0.312 e^{-15.16t} + 0.002 e^{-183.24t} A$$

18.

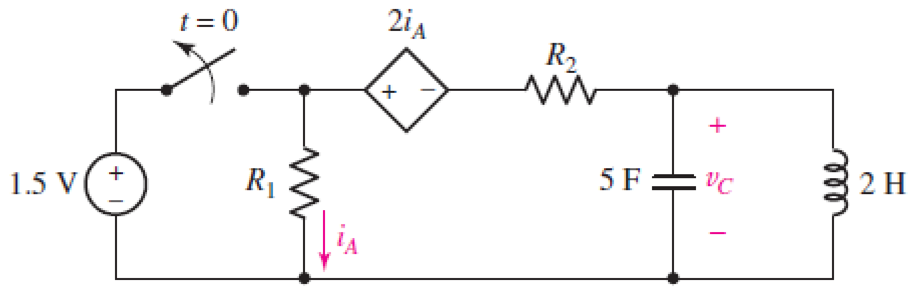
$$\alpha=\frac{1}{2RC}=\frac{1}{2\times1\times360\times10^{-6}}=1388.9s^{-1}$$
$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{1\times360\times10^{-6}}}=52.7rad/\sec$$
$$\alpha>\omega_0\Rightarrow overdamped$$
$$v_C(t)=A_1e^{s_1t}+Ae^{s_2t}$$
$$s_1=-\alpha+\sqrt{\alpha^2-\omega_0{}^2}=-1388.9+\sqrt{(1388.9)^2-(52.7)^2}$$
$$s_1=-1s^{-1}$$
$$s_2=-\alpha-\sqrt{\alpha^2-\omega_0{}^2}=-1388.9-\sqrt{(1388.9)^2-(52.7)^2}$$
$$s_2=-2776.8s^{-1}$$
$$v(t)=A_1e^{-t}+A_2e^{-27768t}$$
$$i_L(0^-)=-i_R(0^-)=-310mA$$
$$v_C(0^+)=v_C(0^-)=4.34V$$
$$v_C(t)=A_1e^{-t}+A_2e^{-27768t}\Rightarrow v_C(0)=4.34=A_1+A_2$$
$$i_C=C\frac{dv_C}{dt}$$
$$i_C(0^+)=-i_L(0^+)-i_R(0^+)=0\Rightarrow\frac{dv_C}{dt}=-A_1-2776.8A_2=0$$
$$A_1=4.34,A_2=-0.002$$
$$v_C(t)=4.34e^{-t}-0.002e^{-27768t}$$
$$w_C=\frac{1}{2}Cv_C{}^2=\frac{1}{2}\times360\times10^{-6}\left[4.34e^{-t}-0.002e^{-27768t}\right]^2=180\times10^{-6}\left[4.34e^{-t}-0.002e^{-27768t}\right]^2$$

(b)

$$w_C=\frac{1}{2}Cv_C{}^2=\frac{1}{2}\times360\times10^{-6}\left[4.34e^{-t}-0.002e^{-27768t}\right]^2=180\times10^{-6}\left[4.34e^{-t}-0.002e^{-27768t}\right]^2$$
$$w_{C\max}=3.387\times10^{-3}J\Rightarrow t_{\max}=0\sec$$
$$\frac{1}{2}w_{C\max}=1.694\times10^{-3}J$$
$$\therefore t\Big|_{\frac{1}{2}w_{C\max}}=0.347\sec$$


x 10^{-3}
3.5
3
2.5
2
1.5
1
0.5
0
capacitor energy
max energy
The time at which the energy in the C
=0.5*max value
X: 0.347
Y: 0.001694
0 1 2 3 4 5 6 7 8 9 10
Time sec

20. $R_1 = 0.752\Omega, R_2 = 1.268\Omega$



(a) $w_C = \frac{1}{2}Cv^2{}_C$

At $t = 0^-$, $v_C(0^-) = 0$ due to the presence of the inductor.

Performing mesh analysis:

$$-1.5 + R_1 i_A = 0$$
$$i_A = i_1 - i_2 \Rightarrow -1.5 + 0.752 i_1 - 0.752 i_2 = 0$$

Mesh two:

$$-R_1 i_A + 2i_A + R_2 i_2 = 0 \Rightarrow 0.02 i_2 + 1.25 i_1 = 0$$

$$i_1, i_2 \Rightarrow i_1 = 0.0315A, i_2 = -1.967A$$
$$i_A(0^-) = i_1 - i_2 = 2A$$
$$t > 0 \Rightarrow -v_C(0^+) + R_2 i_A(0^+) - 2i_A(0^+) + R_1 i_A(0^+) = 0$$

Solving for $i_A(0^+) = 0$

$$-v_{LC} + R_2 - 2(1) + R_1 = 0 \Rightarrow v_{LC} = 0.02V \Rightarrow R_{Th} = \frac{v_{LC}}{1A} = 0.02\Omega$$
$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 0.02 \times 5} = 5s^{-1}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 2}} = 0.3162 rad / \sec$$

$$\alpha > \omega_0 \Rightarrow overdamped$$

$$v_C(t) = A_1 e^{s_1 t} + A e^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -5 + \sqrt{(5)^2 - (0.3162)^2}$$

$$s_1 = -0.01 s^{-1}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0{}^2} = -5 - \sqrt{(5)^2 - (0.3162)^2}$$

$$s_2 = -9.99 s^{-1}$$

$$v(t) = A_1 e^{-0.01t} + A_2 e^{-9.99t}$$

$$i_L(0^-) = i_2(0^-) = -1.967A$$

$$v_C(0^+) = v_C(0^-) = 0V$$

$$v_C(t) = A_1 e^{-0.01t} + A_2 e^{-9.99t} \Rightarrow v_C(0) = 0 = A_1 + A_2$$

$$i_C = C\frac{dv_C}{dt}, i_R(0^+) = i_A(0^+) = 0$$

$$i_C(0^+) = -i_L(0^+) - i_R(0^+) = 1.967A \Rightarrow \frac{dv_C}{dt} = -0.01A_1 - 9.99A_2 = \frac{1.967}{5} = 0.3934$$

$$A_1 = 0.03934, A_2 = -0.03934$$

$$v_C(t) = 0.03934e^{-0.01t} - 0.03934e^{-9.99t}$$

$$\boxed{w_C = \frac{1}{2}Cv_C{}^2 = \frac{1}{2}\times 5\times\left[0.03934e^{-0.01t} - 0.03934e^{-9.99t}\right]^2 = 2.5\times\left[0.03934e^{-0.01t} - 0.03934e^{-9.99t}\right]^2}$$

(b)

21. L = 8H, C = 2μF

(a) $\alpha = \dfrac{1}{2RC}, \omega_0 = \dfrac{1}{\sqrt{LC}}$

$\alpha = \omega_0 \Rightarrow$ Critical damping

$$\frac{1}{4R^2C^2} = \frac{1}{LC} \Rightarrow LC = 4R^2C^2 \Rightarrow R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{8}{2\times10^{-6}}} = 1000\Omega = 1k\Omega$$

(b) $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2\times1000\times2\times10^{-6}} = 250s^{-1}$

(c) $v = Ri_r$

$$v = e^{-\alpha t}(A_1t + A_2)$$
$$v = e^{-250t}(A_1t + A_2)$$
$$i_r = \frac{v}{R} = \frac{1}{R}e^{-250t}(A_1t + A_2)$$

(d) Show that $i_r = \dfrac{1}{R}e^{-\alpha t}(A_1t + A_2)$ is a solution to

$$\frac{d^2i_r}{dt^2} + 2\alpha\frac{di_r}{dt} + \alpha^2 i_r = 0$$
$$\frac{di_r}{dt} = \frac{1}{R}\left[e^{-\alpha t}(A_1) - \alpha e^{-\alpha t}(A_1t + A_2)\right] = \frac{1}{R}\left[A_1 - \alpha A_1t - \alpha A_2\right]e^{-\alpha t}$$
$$\frac{d^2i_r}{dt^2} = \frac{-\alpha}{R}(2A_1 - \alpha A_1t - \alpha A_2)e^{-\alpha t}$$
$$\frac{d^2i_r}{dt^2} + 2\alpha\frac{di_r}{dt} + \alpha^2 i_r = \frac{-\alpha}{R}(2A_1 - \alpha A_1t - \alpha A_2)e^{-\alpha t} + 2\alpha\left(\frac{1}{R}\left[A_1 - \alpha A_1t - \alpha A_2\right]e^{-\alpha t}\right) + \alpha^2\frac{1}{R}e^{-\alpha t}(A_1t + A_2)$$
$$= \frac{1}{R}\left(-2\alpha A_1 + \alpha^2A_2 + \alpha^2A_1t + 2\alpha A_1 - 2\alpha^2A_1t - 2\alpha^2A_2 + \alpha^2A_1t + \alpha^2A_2\right)e^{-\alpha t} = 0$$

22. $\omega_0 = \alpha$

$$L = 4R^2C$$

$$1H = 1\Omega^2 F$$

$$1\Omega = 1\frac{V}{A}$$

$$1F = 1\frac{A.s}{V}$$

$$1H = \frac{V.s}{A}$$

$$L = 4R^2C \Rightarrow \frac{V.s}{A} = \frac{V^2}{A^2} \times \frac{A.s}{V} = \frac{V.s}{A}$$

$$\boxed{\therefore 1H = 1\Omega^2 F}$$

23. R = 40Ω, L = 51.2μH, C = 8μF

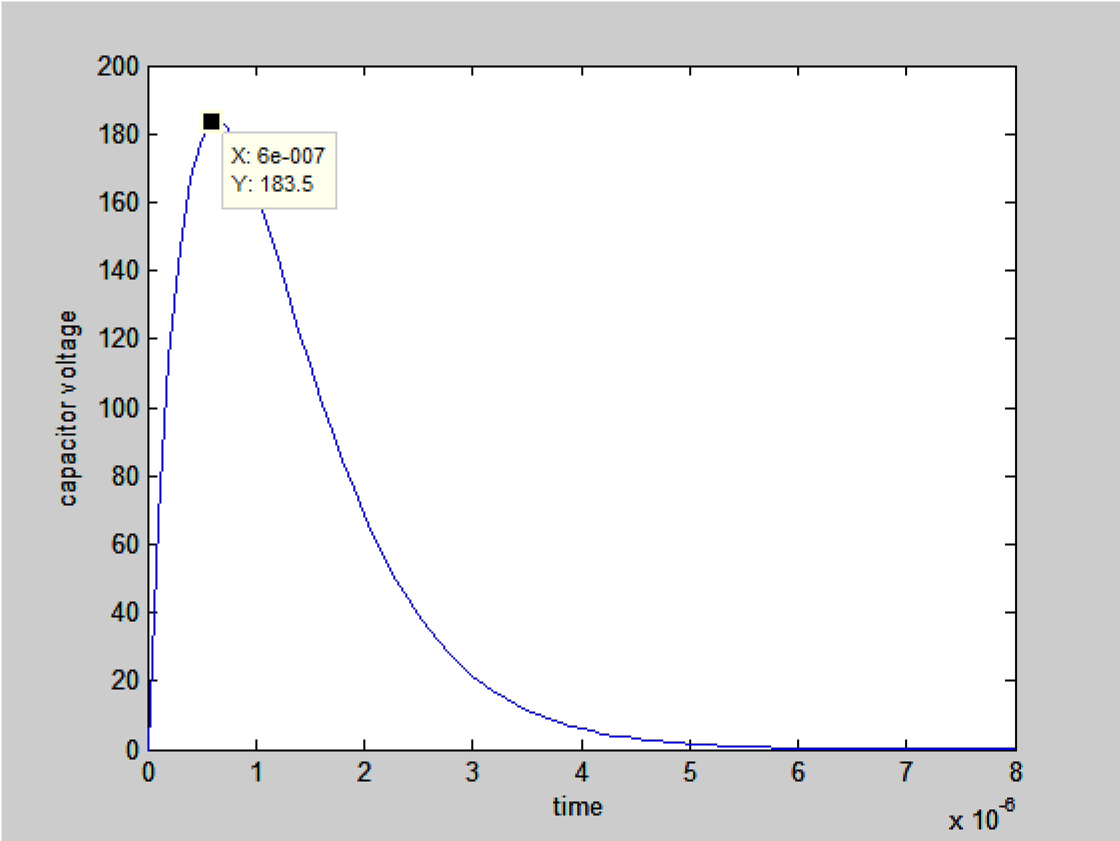
(a) $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2\times 40\times 8\times 10^{-9}} = 1.563\times 10^{6}\, s^{-1}$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{51.2\times 10^{-6}\times 8\times 10^{-9}}} = 1.563\times 10^{6}\, rad/s$$

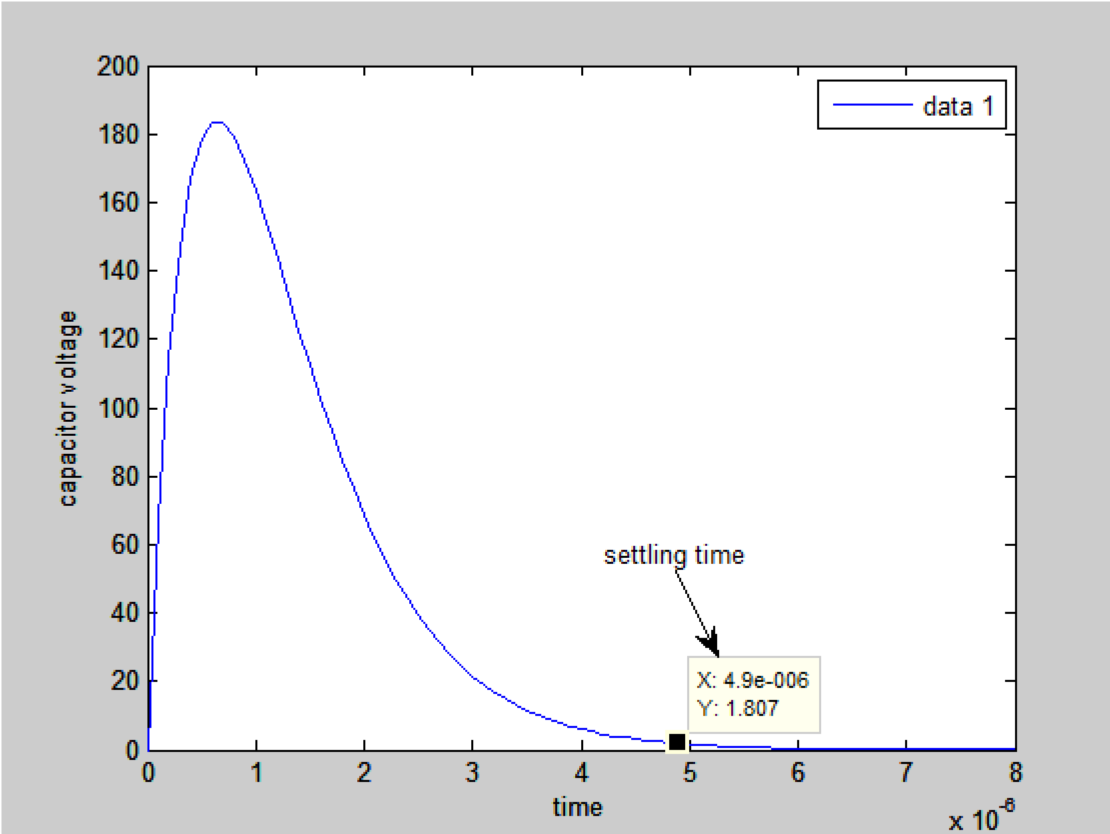
(b) We could never make α exactly equal to ω_0 because in practice it is unusual to obtain components that are closer than 1 percent of their specified values.

(c) L stores 1mJ, C initially discharged

$$w_L = \frac{1}{2}Li^2 \Rightarrow w_L = 1mJ = \frac{1}{2}L{i_L}^2(0)$$
$$i_L(0) = \sqrt{\frac{2\times 1\times 10^{-3}}{51.2\times 10^{-6}}} = 6.25A$$
$$v_C(t) = e^{-\alpha t}(A_1 t + A_2)$$
$$v_C(t) = A_1 e^{-1.563\times 10^{6} t} t + A_2 e^{-1.563\times 10^{6} t}$$
$$v_C(0) = 0$$
$$0 = 0 + A_2 \Rightarrow A_2 = 0$$
$$v_C(t) = A_1 e^{-1.563\times 10^{6} t} t$$
$$\frac{dv_C}{dt} = A_1 e^{-1.563\times 10^{6} t} - A_1 1.563\times 10^{6}\, t e^{-1.563\times 10^{6} t}$$
$$\frac{dv_C}{dt}\Big|_{t=0} = A_1$$
$$\frac{dv_C}{dt}\Big|_{t=0} = \frac{i_C(0)}{C}, i_C(0) = i_L(0) + i_R(0)$$
$$i_R(0) = 0 \Rightarrow i_C(0) = i_L(0)$$
$$\frac{dv_C}{dt}\Big|_{t=0} = \frac{i_L(0)}{C} = \frac{6.25}{8\times 10^{-9}} = 7.813\times 10^{8} V/s$$
$$A_1 = 7.813\times 10^{8} V/s$$
$$v_C(t) = 7.813\times 10^{8} e^{-1.563\times 10^{6} t} t$$
$$v_C(500\times 10^{-9}) = 178.81V$$
$$t_m = 0.64\times 10^{-6}\ \text{sec}$$
$$V_m = 183.5V$$



$$v_{C\max} = 183.5V$$
$$1\% v_{C\max} = 1.835V$$
$$1.835 = 7.813\times10^{8} t_s e^{-1.563\times10^{6} t_s}$$
$$t_s = 4.9\times10^{-6} s$$



200
180
160
140
120
100
80
60
40
20
0
capacitor voltage
0
1
2
3
4
5
6
7
8
time
x 10^-6
data 1
settling time
X: 4.9e-006
Y: 1.807

25. R = 40 Ω, C = 2 pF.

(a) $\alpha = \omega_0 \Rightarrow \dfrac{1}{2RC} = \dfrac{1}{\sqrt{LC}} \Rightarrow L = 4R^2C = (4)(40^2)(2\times10^{-12})$

$L = 1.28\times10^{-8}\,H$

(b) In practice it is unusual to obtain components that are closer than 1% of their specified values.

(c) $i_L(0) = 0, w_C = 10\,pJ$

$$w_C = \frac{1}{2}Cv^2 \Rightarrow 10\times10^{-12} = \frac{1}{2}\times2\times10^{-12}\times v_C(0) \Rightarrow v_C(0) = 10V$$

$$v_R(t) = e^{-\alpha t}(A_1t + A_2) \Rightarrow \alpha = \frac{1}{2RC} = 625\times10^7\,s^{-1}$$

$$v_R(t) = e^{-625\times10^7 t}(A_1t + A_2)$$

$$v_C(0) = 10V = v_R(0)$$

$$10 = 0 + A_2 \Rightarrow A_2 = 10$$

$$v_R(t) = e^{-625\times10^7 t}(A_1t + 10)$$

$$\frac{dv_R}{dt} = e^{-625\times10^7 t}(A_1 - 625\times10^7 A_1t - 6.25\times10^{10}) \Rightarrow \frac{dv_R}{dt}\Big|_{t=0^+} = A_1 - 6.25\times10^{10}$$

$$\frac{dv_C}{dt}\Big|_{t=0} = \frac{i_C(0)}{C}, i_C(0) = i_L(0) + i_R(0), i_L(0) = 0 \Rightarrow \frac{i_C(0)}{C} = \frac{0.25}{2\times10^{-12}} = 2.083\times10^{10}V/s$$

$$\frac{dv_R}{dt}\Big|_{t=0^+} = A_1 - 6.25\times10^{10} = 2.083\times10^{10}$$

$$A_1 = 8.333\times10^8$$

$$v_R(t) = e^{-625\times10^7 t}(8.333\times10^8 t + 10)$$

$$P = \frac{v_R{}^2(t)}{R} = \frac{[e^{-625\times10^7 t}(8.333\times10^8 t + 10)]^2}{40} \Rightarrow P(2ns) = 4.7257\times10^{-11}W$$

26. $i_s(t) = 30u(-t)mA$

(a) R_1 so that $v(0^+) = 6V$

At $t = 0^-$, the current source is on, the inductor can be treated as a short circuit, and C as an open circuit. Thus $v(0^-)$ appears across R_1 and is given by

$$v(0^-) = (30mA)R_1$$
$$v(0^+) = v(0^-) = v(0) = 6V$$
$$R_1 = \frac{v(0)}{30\times10^{-3}} = 200\Omega$$

(b) v (2ms)

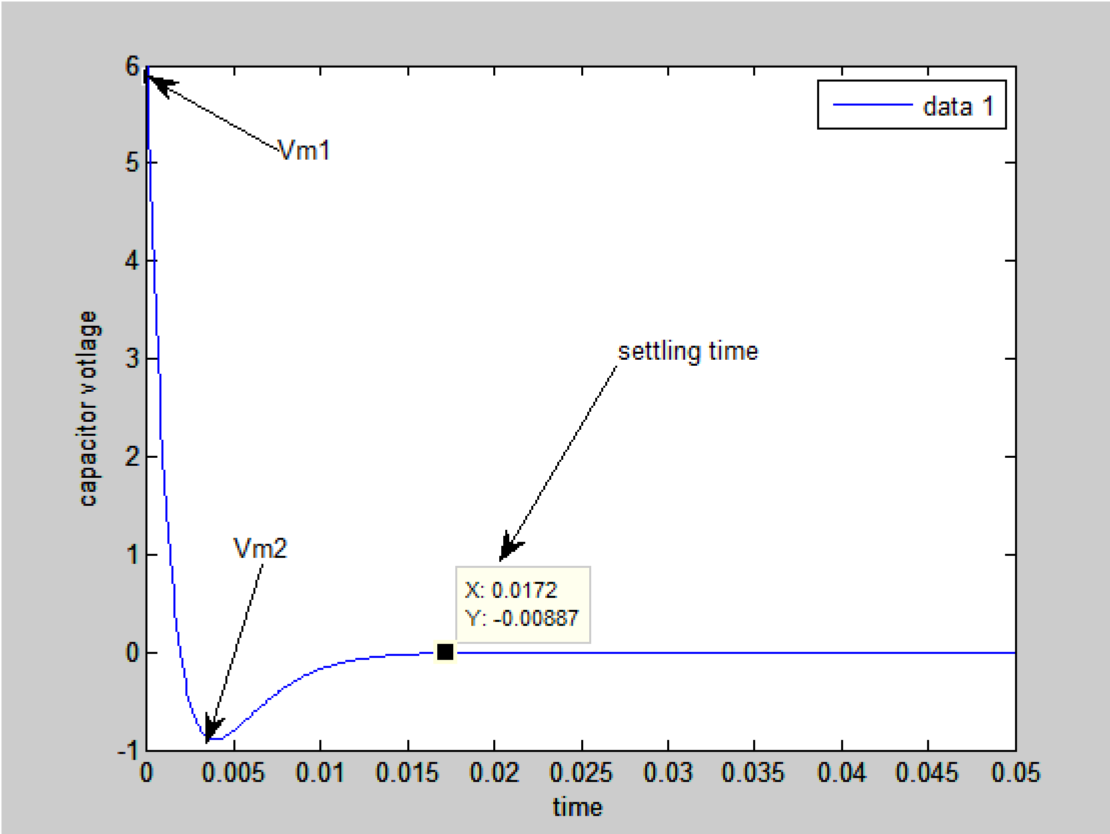
$$v = e^{-\alpha t}(A_1 t + A_2)$$

At $t = 0^+$ the current source has turned itself off and R_1 is shorted. We are left with a parallel RLC circuit comprised of $R = 5\Omega, C = 200\mu F, L = 20mH$

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times5\times200\times10^{-6}} = 500s^{-1}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times10^{-3}\times200\times10^{-6}}} = 500rad/\sec$$
$$\alpha = \omega_0 \Rightarrow critcallyd\,amped$$
$$v(t) = e^{-\alpha t}(A_1 t + A_2)$$
$$v(t) = A_1 te^{-500t} + A_2 e^{-500t}$$
$$t = 0^-$$
$$i_L(0^-) = 30mA$$
$$v_C(0^-) = v_C(0^+) = 6V$$
$$v(0) = 6 = A_2$$
$$v_C(t) = A_1 te^{-500t} + 6e^{-500t}$$
$$\frac{dv_C}{dt} = A_1 e^{-500t} - 500A_1 te^{-500t} - 3000e^{-500t}$$
$$\frac{dv_C}{dt}\Big|_{t=0} = A_1 - 3000$$
$$t = 0^+$$
$$i_C(0^+) = -i_L(0^+) - i_R(0^+), i_R(0^+) = \frac{v_C(0^+)}{R} = \frac{6}{5} = 1.2A$$
$$i_L(0^+) =_L (0^-) = 30mA \Rightarrow i_C(0^+) = -30\times10^{-3} - 1.2 = -1.23A$$
$$i_C = C\frac{dv_C}{dt} \Rightarrow \frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C}$$
$$A_1 - 3000 = \frac{-1.23}{200\times10^{-6}} = -6150 \Rightarrow A_1 = -3150$$
$$v_C(t) = -3150te^{-500t} + 6e^{-500t}$$
$$v(2ms) = -0.11V$$

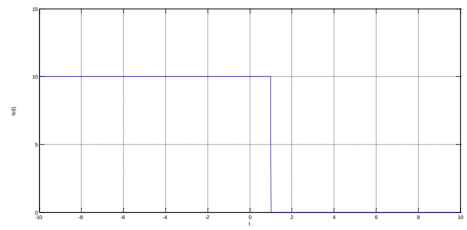
(c) t_s for v_C

$$\frac{dv_C}{dt} = -3150e^{-500t} - 3150(-500)te^{-500t} - 6(500)e^{-500t} = 0$$
$$-3150 + 1.575\times10^6 t_{\min} - 3000 = 0 \Rightarrow t_{mn} = 3.9\times10^{-3}\sec$$
$$v_C(t_{\min}) = V_{mi} = -0.8925V$$
$$v_{C_{\max}} = 6V \Rightarrow t_{\max} = 0$$
$$-3150t_s e^{-500t_s} + 6e^{-500t_s} = 0.00893$$
$$\therefore t_s = 0.0172s$$



data 1
Vm1
Vm2
settling time
X: 0.0172
Y: -0.00887
capacitor votlage
time
6
5
4
3
2
1
0
-1
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05

27. $i_s(t)=10u(1-t)\mu A$ R_1 so that $i_L\left(0^+\right)=2\mu A$



At $t=0^-$, the current source is on, the inductor can be treated as a short circuit, and C as an open circuit. Thus $v(0^-)$ appears across R_1 and is given by

$$v\left(0^-\right)=(10\mu A)(R_1 // 5)=\left(\frac{5R_1}{R_1+5}\right)(10\mu A)$$

$$i_L\left(0^-\right)=i_L\left(0^+\right)=2\mu A=\frac{v\left(0^-\right)}{R_1}$$

$$R_1=20\Omega$$

$$v\left(0^-\right)=40\times10^{-6}V$$

$$\alpha=\frac{1}{2RC}=\frac{1}{2\times5\times200\times10^{-6}}=500s^{-1},\omega_0=\frac{1}{\sqrt{LC}}=500s^{-1}$$

$$i_L(t)=e^{-\alpha t}\left(A_1t+A_2\right)\Rightarrow i_L(t)=e^{-500t}\left(A_1t+A_2\right)$$

$$i_L(0)=2\times10^{-6}A\Rightarrow 2\times10^{-6}=A_2\Rightarrow i_L(t)=e^{-500t}\left(A_1t+2\times10^{-6}\right)$$

$$\frac{di_L}{dt}=e^{-500t}\left(A_1-500A_1t-1\times10^{-3}\right)$$

$$v_C(0^-)=(14)\left(310\times10^{-3}\right)=4.34V$$

$$t=0^+$$

$$v_C(0^-)=v_C(0^+)=40\times10^{-6}V$$

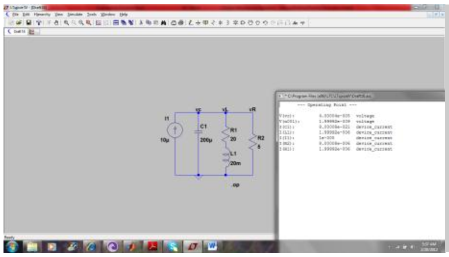
$$v_L(0)=v_C(0)=40\times10^{-6}V=L\frac{di_L}{dt}\Big|_{t-0}$$

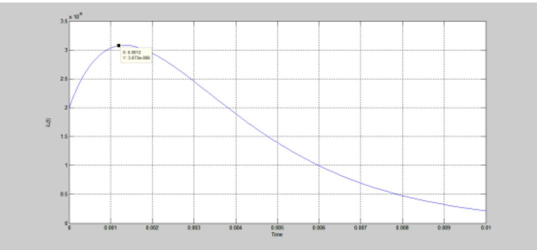
$$(20\times10^{-3})\left(A_1-1\times10^{-3}\right)=40\times10^{-6}\Rightarrow A_1=3\times10^{-3}$$

$$i_L(t)=e^{-500t}\left(3\times10^{-3}t+2\times10^{-6}\right)$$

$$i_L(500ms)=0$$

$$i_L(1.002ms)=3.033\times10^{-6}$$





28.

(a) Critically damping

$$\alpha = \omega_0 \Rightarrow L = 4R^2C \Rightarrow L = (4)(14)^2\left(360\times10^{-6}\right)$$
$$L = 0.282H$$

(b) $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2\times14\times360\times10^{-6}} = 99.21s^{-1}$

$$i_L(t) = e^{-\alpha t}\left(A_1t + A_2\right) \Rightarrow i_L(t) = e^{-99.21t}\left(A_1t + A_2\right)$$
$$t = 0^-$$
$$i_L(0^-) = -310\times10^{-3}A, i_R(0^-) = 310\times10^{-3}A$$
$$v_C(0^-) = (14)\left(310\times10^{-3}\right) = 4.34V$$
$$t = 0^+$$
$$v_C(0^-) = v_C(0^+) = 4.34V$$
$$i_R(0^+) = \frac{v_C(0^+)}{R} = 310\times10^{-3}A, i_L(0^+) = -310\times10^{-3}A$$
$$i_L(0) = -310\times10^{-3} = A_2$$
$$i_L(t) = e^{-99.21t}\left(A_1t - 310\times10^{-3}\right)$$
$$\frac{di_L}{dt} = e^{-99.21t}\left(A_1 - 99.21A_1t + 30.755\right)$$
$$v_L(0) = v_C(0) = 4.34 = L\frac{di_L}{dt}\Big|_{t-0}$$
$$(0.282)\left(A_1 + 30.755\right) = 4.34 \Rightarrow A_1 = -15.365$$
$$i_L(t) = e^{-99.21t}\left(-15.365t - 310\times10^{-3}\right)$$
$$w_L = \frac{1}{2}Li_L^{\ 2} = \frac{1}{2}(0.282)\left[e^{-99.21t}\left(-15.365t - 310\times10^{-3}\right)\right]^2$$
$$w_L(10ms) = 2.04\times10^{-88}J$$

$$v_C(t) = e^{-\alpha t}\left(A_1 t + A_2\right) \Rightarrow v_C(t) = e^{-99.21t}\left(A_1 t + A_2\right)$$
$$t = 0^-$$
$$i_L(0^-) = -310\times10^{-3}\,A, i_R(0^-) = 310\times10^{-3}\,A$$
$$v_C(0^-) = (14)\left(310\times10^{-3}\right) = 4.34V$$
$$v_C(0^-) = v_C(0^+) = 4.34V$$
$$i_R(0^+) = \frac{v_C(0^+)}{R} = 310\times10^{-3}\,A, i_L(0^+) = -310\times10^{-3}\,A$$
$$v_C(0) = 4.34 = A_2$$
$$v_C(t) = e^{-99.21t}\left(A_1 t + 4.34\right)$$
$$\frac{dv_C}{dt} = e^{-99.21t}\left(A_1 - 99.21A_1 t - 430.57\right)$$
$$i_C = C\frac{dv_C}{dt}, i_C(0) = -i_L(0) - i_R(0) = 0$$
$$\left(A_1 - 430.57\right) = 0 \Rightarrow A_1 = 430.57$$
$$v_C(t) = e^{-99.21t}\left(430.57t + 4.34\right)$$
$$w_C = \frac{1}{2}Cv_C{}^2 = \frac{1}{2}(360\times10^{-6})\left[e^{-99.21t}\left(430.57t + 4.34\right)\right]^2$$
$$w_C(10ms) = 1.8499\times10^{-3}\,J$$

29. $R_1 = R_2 = 10\Omega$

(a)

$$\alpha = \omega_0$$

$$\alpha = \frac{1}{2RC}, \omega_0 = \frac{1}{\sqrt{LC}}$$

$$R_{Th} = \frac{v_{LC}}{1A} = 80\Omega$$

$$\alpha = 3125s^{-1}$$

$$\boxed{L = 0.0512H}$$

30. Underdamped

(a)

$$\alpha < \omega_0, \alpha = \frac{1}{2RC}, \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{2RC} < \frac{1}{\sqrt{LC}} \Rightarrow 2RC > \sqrt{LC} \Rightarrow 4R^2C^2 > LC$$

$$R > \frac{1}{2}\sqrt{\frac{L}{C}}$$

(b) $C = 1nF, L = 10mH$

$$R > \frac{1}{2}\sqrt{\frac{L}{C}} \Rightarrow R > \frac{1}{2}\sqrt{\frac{10\times10^{-3}}{1\times10^{-9}}} \Rightarrow R > 1581.14\Omega$$

$$R = 1582\Omega$$

$$\alpha = \frac{1}{2RC} = 3.16055\times10^5\, s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 3.16227\times10^5\, rad/\sec$$

$$\therefore \alpha < \omega_0$$

(c) $\zeta = \dfrac{\alpha}{\omega_0}$

If the damping ratio increased the circuit becomes less underdamped since α will be larger than ω_0

(d)

$$\alpha = \frac{1}{2RC} = 3.16055\times10^5\, s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 3.16227\times10^5\, rad/\sec \Rightarrow \omega_0{}^2 = 1\times10^{11}$$

$$\omega_d = \sqrt{1\times10^{11} - \left(3.16056\times10^5\right)^2} = 10.421\times10^3\, rad/s$$

31. $R = 10k\Omega, L = 72\mu H, C = 18pF$

(a) $\alpha, \omega_0, \omega_d$

$$\alpha = \frac{1}{2RC} = \frac{1}{2\left(10\times10^{3}\right)\left(18\times10^{-12}\right)} = 2.78\times10^{6}s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(72\times10^{-6}\right)\left(18\times10^{-12}\right)}} = 2.78\times10^{7}rad/\sec$$

$$\therefore \alpha < \omega_0 \Rightarrow underdamped$$

$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} = 27.66\times10^{6}rad/s$$

(b) $v_C(t) = v(t) = e^{-\alpha t}(B_1\cos\omega_d t + B_2\sin\omega_d t)$

$$v_C(t) = v(t) = e^{-2.78\times10^6 t}(B_1\cos 27.66\times10^6 t + B_2\sin 27.66\times10^6 t)$$

(c) $w_C = 1nJ$

$$w_C = \frac{1}{2}Cv_C{}^2(0) \Rightarrow v_C(0) = 10.54V$$

$$v(0) = 10.54 = B_1$$

$$v_C(t) = e^{-2.78\times10^6 t}(10.54\cos 27.66\times10^6 t + B_2\sin 27.66\times10^6 t$$

$$\frac{dv_C}{dt}\Big|_{t=0} = -29.3\times10^6 + 27.66\times10^6 B_2$$

$$\frac{dv_C}{dt}\Big|_{t=0} = \frac{i_C}{C} = 0$$

$$-29.3\times10^6 + 27.66\times10^6 B_2 = 0 \Rightarrow B_2 = 1.06$$

$$v_C(t) = e^{-2.78\times10^6 t}(10.54\cos 27.66\times10^6 t + 1.06\sin 27.66\times10^6 t$$

$$v_C(300ns) = -1.551V$$

32. $R = 1.5k\Omega, L = 10mH, C = 1mF$

(a) $\alpha = \frac{1}{2RC} = \frac{1}{2(1.5\times10^{3})(1\times10^{-3})} = 0.33s^{-1}$

$$\omega_0 = \frac{1}{\sqrt{10\times10^{-3}\times1\times10^{-3}}} = 316.23rad/\sec$$
$$\omega_d = \sqrt{\omega_0^{\ 2} - \alpha^2} = 316.23rad/s$$

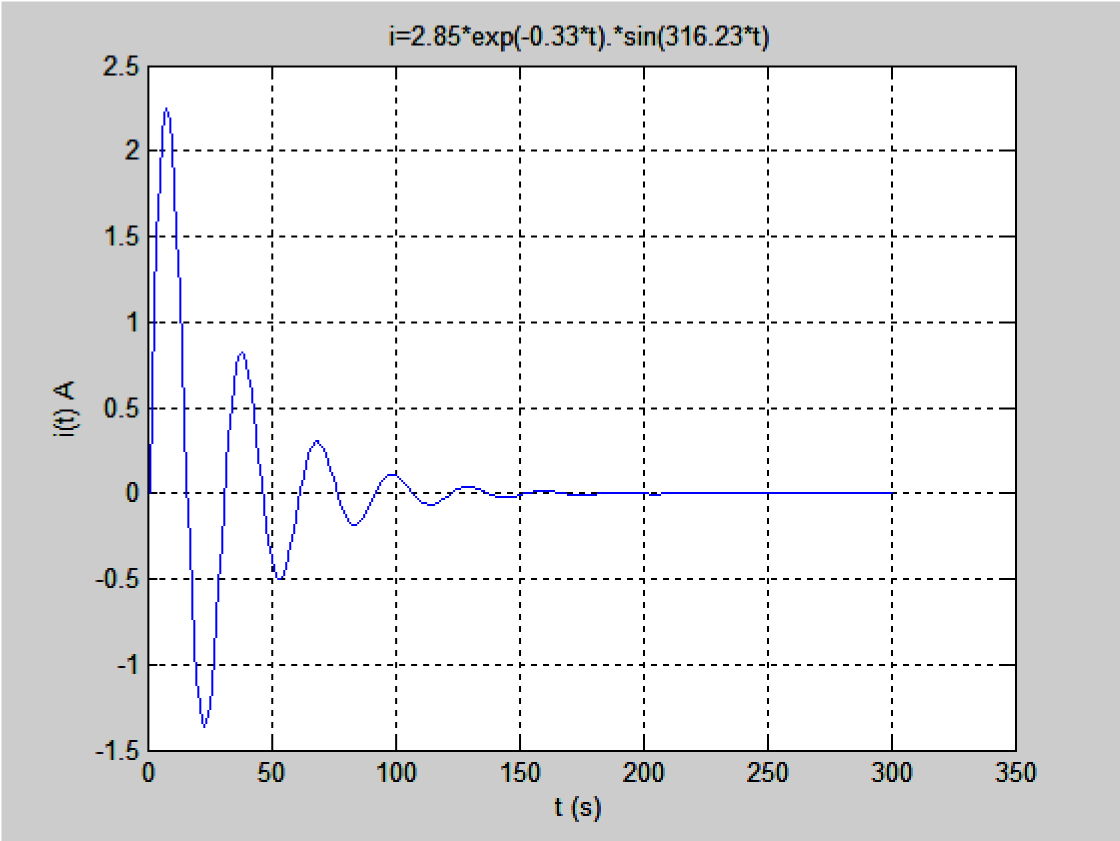
(b) $i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$

$$i(t) = e^{-0.33t}(B_1 \cos 316.23t + B_2 \sin 316.23t)$$

(c) $i_L(0^-) = 0, v_C(0^-) = 9V$

$$i_L(0^-) = 0 = B_1$$
$$i(t) = e^{-0.33t} B_2 \sin 316.23t$$
$$\frac{di(t)}{dt} = 316.23B_2 e^{-0.33t} \cos 316.23t - 0.33B_2 e^{-0.33t} \sin 316.23t$$
$$v_L(t) = L\frac{di}{dt}, v_L(0^+) = v_C(0^+) = 9V$$
$$v_L(0) = (10\times10^{-3})(316.23B_2) = 9 \Rightarrow B_2 = 2.85$$
$$i_L(t) = 2.85e^{-0.33t} \sin 316.23t$$
$$\frac{di_L(t)}{dt} = 2.85\times[316.23e^{-0.33t} \cos 316.23t - 0.33e^{-0.33t} \sin 316.23t] = 0$$
$$t_m = 4.96\times10^{-3}s$$
$$i_{\max} = 2.845A$$

33. $i(t) = 2.85^{-0.33t} \sin 316.23t$ for R = 1.5kΩ



For R = 15kΩ

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times15\times10^{3}\times1\times10^{-3}} = 0.033s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 316.23 rad/\sec$$

$$\alpha < \omega_0 \Rightarrow underdamped$$

$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} = 316.23 rad/s$$

$$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

$$i(t) = e^{-0.033t}(B_1 \cos 316.23t + B_2 \sin 316.23t)$$

$$i_L(0^-) = 0, v\left(0^-\right) = 9V$$

$$i(0) = B_1 = 0$$

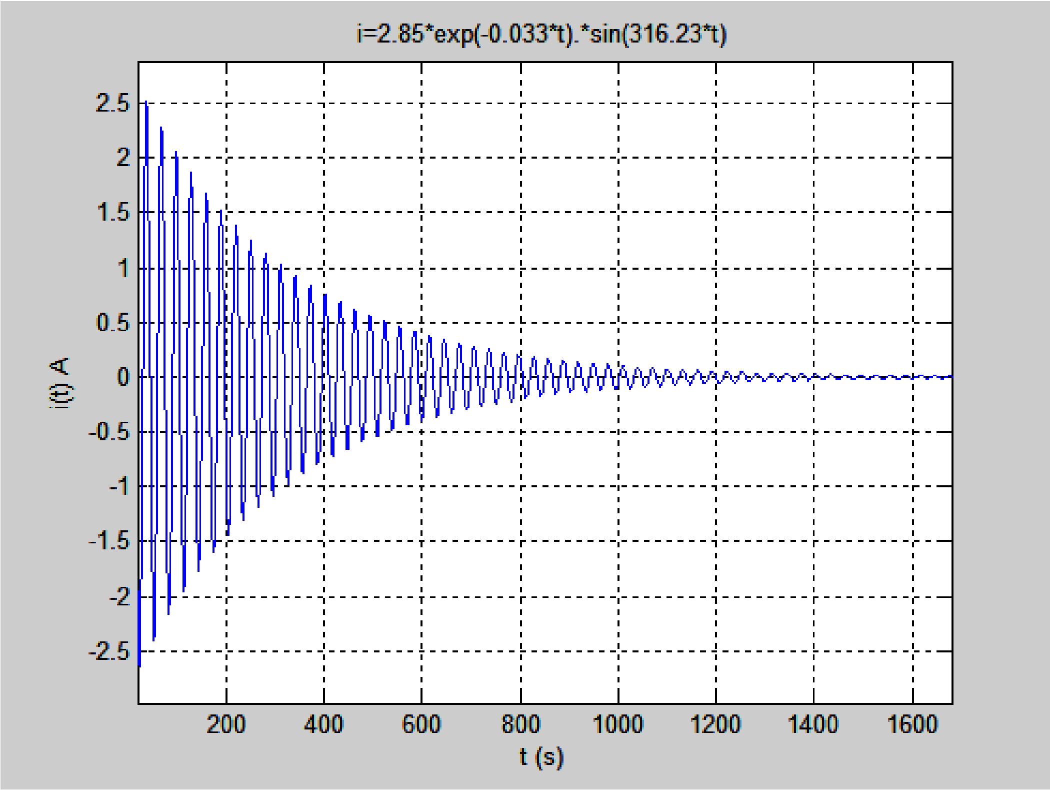
$$i(t) = e^{-0.033t}(B_2 \sin 316.23t)$$

$$\frac{di}{dt}\Big|_{t=0} = B_2 316.23$$

$$\frac{di}{dt}\Big|_{t=0} = \frac{v_L(0)}{L}, v_L(0^+) = v_C(0^+) = 9,$$

$$\Rightarrow B_2 = 2.85$$

$$\therefore i_L(t) = e^{-0.033t}(2.85 \sin 316.23t)$$



For R = 150kΩ

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times150\times10^3\times1\times10^{-3}} = 0.0033s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 316.23rad/\sec$$

$$\alpha < \omega_0 \Rightarrow underdamped$$

$$\omega_d = \sqrt{\omega_0^{\ 2} - \alpha^2} = 316.23rad/s$$

$$i(t) = e^{-\alpha t}(B_1\cos\omega_d t + B_2\sin\omega_d t)$$

$$i(t) = e^{-0.0033t}(B_1\cos 316.23t + B_2\sin 316.23t)$$

$$i_L(0^-) = 0, v(0^-) = 9V$$

$$i(0) = B_1 = 0$$

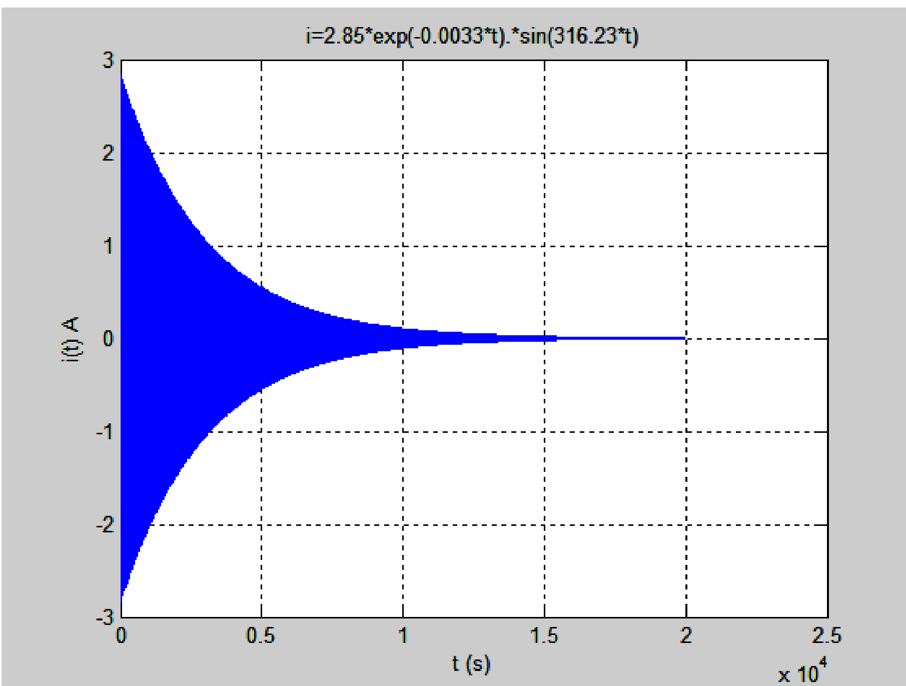
$$i(t) = e^{-0.0033t}(B_2\sin 316.23t)$$

$$\frac{di}{dt}\Big|_{t=0} = B_2 316.23$$

$$\frac{di}{dt}\Big|_{t=0} = \frac{v_L(0)}{L}, v_L(0^+) = v_C(0^+) = 9,$$

$$\Rightarrow B_2 = 2.85$$

$$\therefore i_L(t) = e^{-0.0033t}(2.85\sin 316.23t)$$



34. $v(t), t > 0$

(a) R = 2kΩ, L = 10mH, C = 1mF

$$\alpha = \frac{1}{2RC} = \frac{1}{2\left(2\times10^{3}\right)\left(1\times10^{-3}\right)} = 0.25s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(10\times10^{-3}\right)\left(1\times10^{-3}\right)}} = 316.23 rad/\sec$$

$$\therefore \alpha < \omega_0 \Rightarrow underdamped$$

$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} = 316.23 rad/s$$

$$v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

$$v(t) = e^{-0.25t}(B_1 \cos 316.23t + B_2 \sin 316.23t)$$

$$i_L(0^-) = 0, v\left(0^-\right) = 9V$$

$$v(0) = B_1 = 9$$

$$v(t) = e^{-0.25t}(9\cos 316.23t + B_2 \sin 316.23t)$$

$$\frac{dv}{dt}\Big|_{t=0} = 9\times -0.25 + B_2 316.23$$

$$\frac{dv}{dt}\Big|_{t=0} = \frac{i_C(0)}{C}, i_L(0^-) = 0,$$

$$\frac{dv}{dt}\Big|_{t=0} = 0 \Rightarrow B_2 = 0.0071$$

$$\boxed{\therefore v(t) = e^{-0.25t}(9\cos 316.23t + 0.0071\sin 316.23t)}$$

(b) R = 2Ω

$$\alpha = \frac{1}{2RC} = \frac{1}{2(2)\left(1\times10^{-3}\right)} = 250s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(10\times10^{-3}\right)\left(1\times10^{-3}\right)}} = 316.23 rad/\sec$$

$$\therefore \alpha < \omega_0 \Rightarrow underdamped$$

$$\omega_d = \sqrt{\omega_0^{\ 2} - \alpha^2} = 193.65 rad/s$$

$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

$$v(t) = e^{-250t}(B_1 \cos 193.65t + B_2 \sin 193.65t)$$

$$i_L(0^-) = 0, v\left(0^-\right) = 9V$$

$$v(0) = B_1 = 9$$

$$v(t) = e^{-.250t}(9\cos 193.65t + B_2 \sin 193.65t)$$

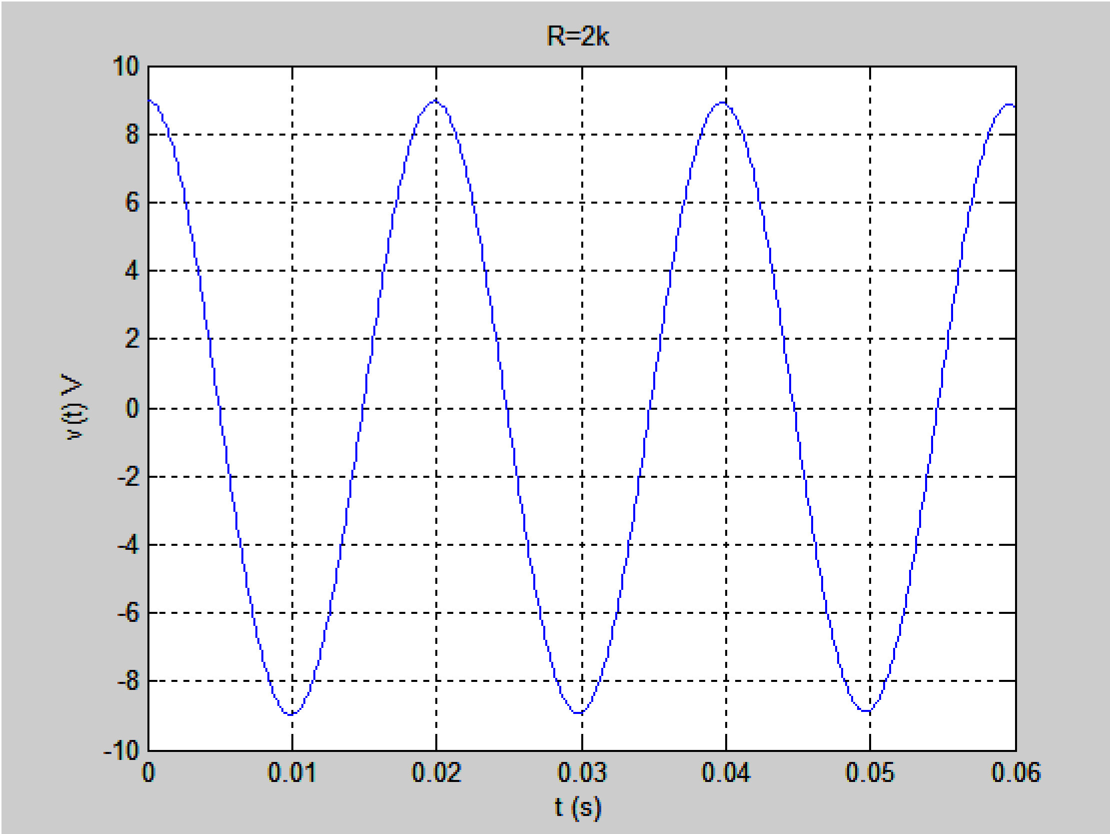
$$\frac{dv}{dt}\Big|_{t=0} = 9\times -250 + B_2 193.65$$

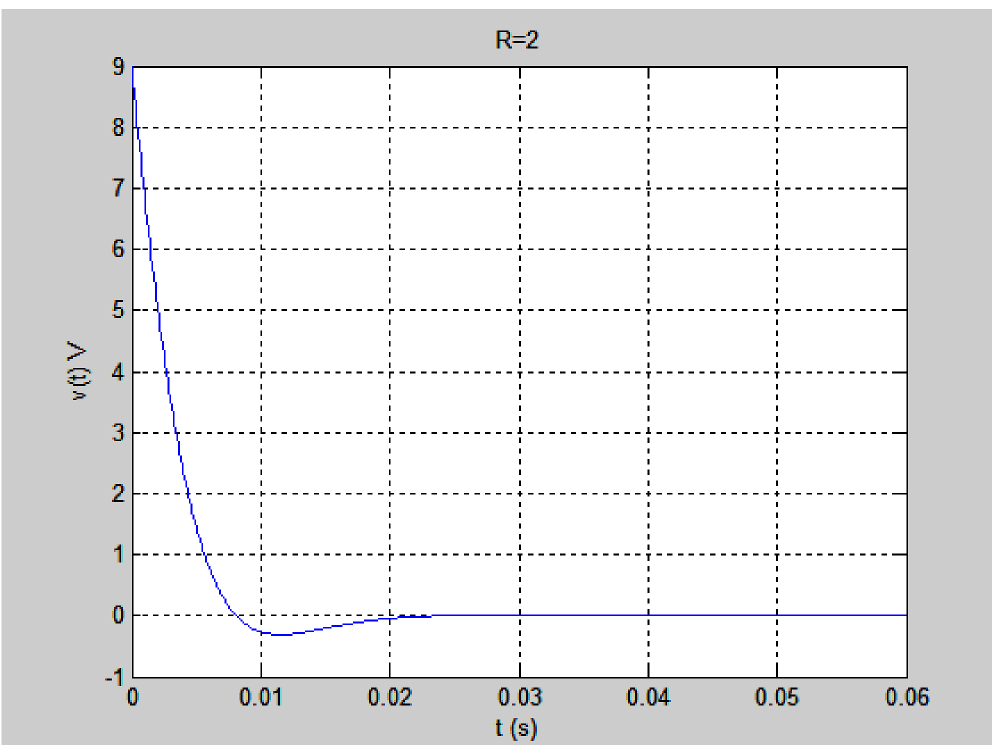
$$\frac{dv}{dt}\Big|_{t=0} = \frac{i_C(0)}{C}, i_L(0^-) = 0,$$

$$\frac{dv}{dt}\Big|_{t=0} = 0 \Rightarrow B_2 = 11.62$$

$$\therefore v(t) = e^{-250t}(9\cos 193.65t + 11.62\sin 193.65t)$$

(c)


R=2k
10
8
6
4
2
0
-2
-4
-6
-8
-10
v(t) V
0
0.01
0.02
0.03
0.04
0.05
0.06
t (s)


R=2
9
8
7
6
5
4
3
2
1
0
-1
v(t) V
0
0.01
0.02
0.03
0.04
0.05
0.06
t (s)

35. at t=0 both the 3 A source and the 2 Ω are removed leaving the 2.5μF, 20 mH, and 50Ω resistor in parallel.

(a) $i_C(0^-) = 0$

(b) $i_L(0^-) = \dfrac{v_C(0^-)}{2} = 2..88A$

(c) $i_R(0^-) = \dfrac{5.76}{50} = 0.115A$

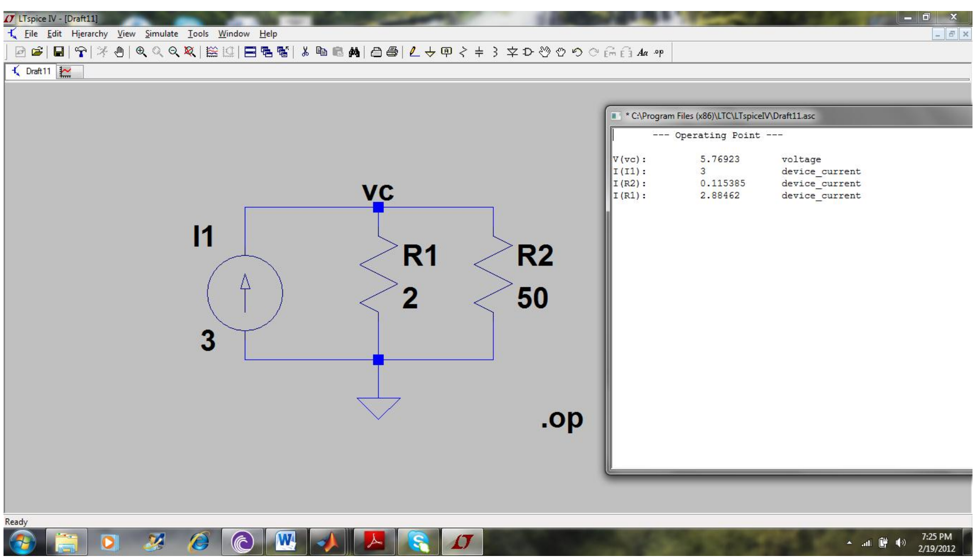
(d) $v_C(0^-) = (2\Omega // 50\Omega)(3A) = 1.92 \times 3 = 5.76V$

(e) $i_C(0^+) = -i_L(0^+) - i_R(0^+) = -2.88 - 0.115 = -2.995A$

(f) $i_L(0^+) = i_L(0^-) = 2.88A$

(g) $i_R(0^+) = \dfrac{v_C(0^+)}{R} = \dfrac{5.76}{50} = 0.115A$

(h) $v_C(0^+) = v_C(0^-) = 5.76V$



36. $v_L(t)$ for t > 0

$$\alpha = \frac{1}{2RC} = \frac{1}{2(50)\left(2.5\times10^{-6}\right)} = 4000s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(20\times10^{-3}\right)\left(2.5\times10^{-6}\right)}} = 20\times10^{6}\, rad/\sec$$

$$\therefore \alpha < \omega_0 \Rightarrow underdamped$$

$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} = 2000 rad/s$$

$$v_L(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$

$$v_L(t) = e^{-4000t}(B_1 \cos 2000t + B_2 \sin 2000t)$$

$$i_L(0^+) = 2.88A, v_L\left(0^+\right) = v_C\left(0^+\right) = 5.76V$$

$$v_L(0) = B_1 = 5.76V$$

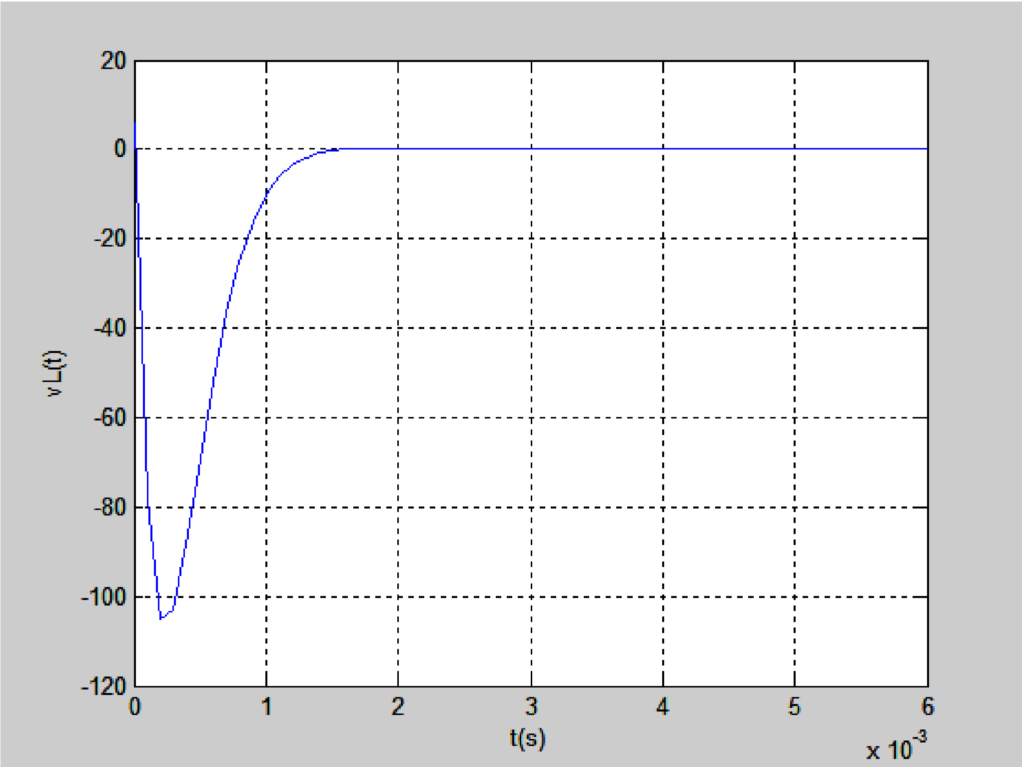
$$v_L(t) = e^{-4000t}(5.76 \cos 2000t + B_2 \sin 2000t)$$

$$\frac{dv_L}{dt}\Big|_{t=0} = 23.04\times10^3 + 2000B_2$$

$$\frac{dv}{dt}\Big|_{t=0} = \frac{i_C(0^+)}{C} = \frac{-2.995}{2.5\times10^{-6}}$$

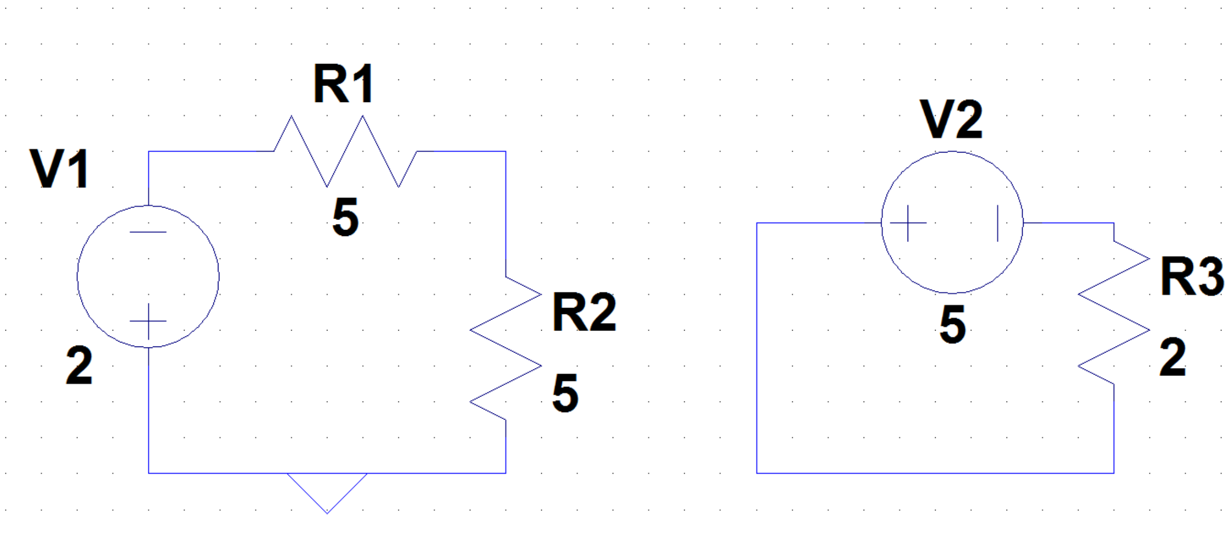
$$\Rightarrow B_2 = -615.02$$

$$\therefore v_L(t) = e^{-4000t}(5.76 \cos 2000t - 615.02 \sin 2000t)$$



37.

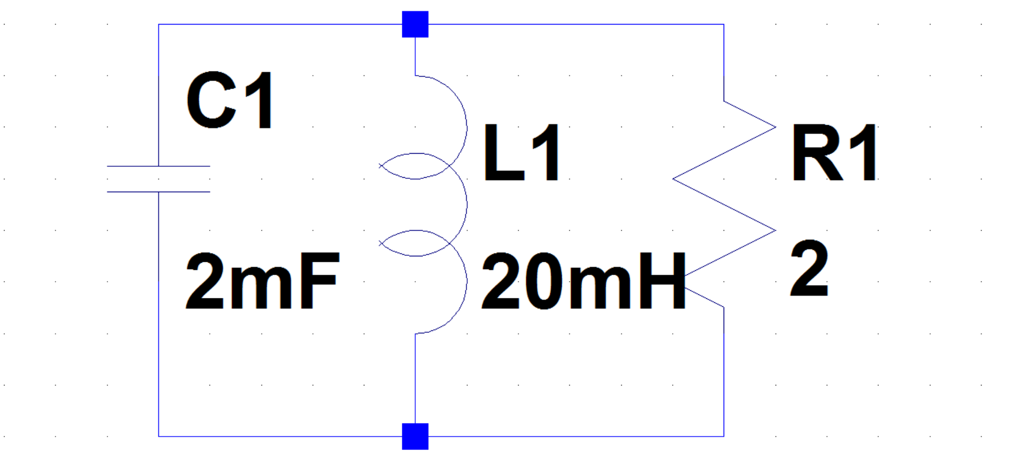
At $t = 0^-$ we will have the following circuit:



$v_C(0^-) = -2 \times \frac{5}{10} = -1V$

$i_L(0^-) = \frac{5}{2} = 2.5A$

At $t = 0^+$ we will have the following circuit:



$$\alpha = \frac{1}{2RC} = \frac{1}{2(2)\left(2\times10^{-3}\right)} = 125s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(20\times10^{-3}\right)}\left(2\times10^{-3}\right)} = 158 rad/\sec$$

$$\therefore \alpha < \omega_0 \Rightarrow underdamped$$

$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} = 96.64 rad/s$$

$$v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

$$v(t) = e^{-125t}(B_1 \cos 96.64t + B_2 \sin 96.64t)$$

$$i_L(0^-) = 2.5A, v\left(0^-\right) = -1V$$

$$v(0) = B_1 = -1$$

$$v(t) = e^{-125t}(-\cos 96.64t + B_2 \sin 96.64t)$$

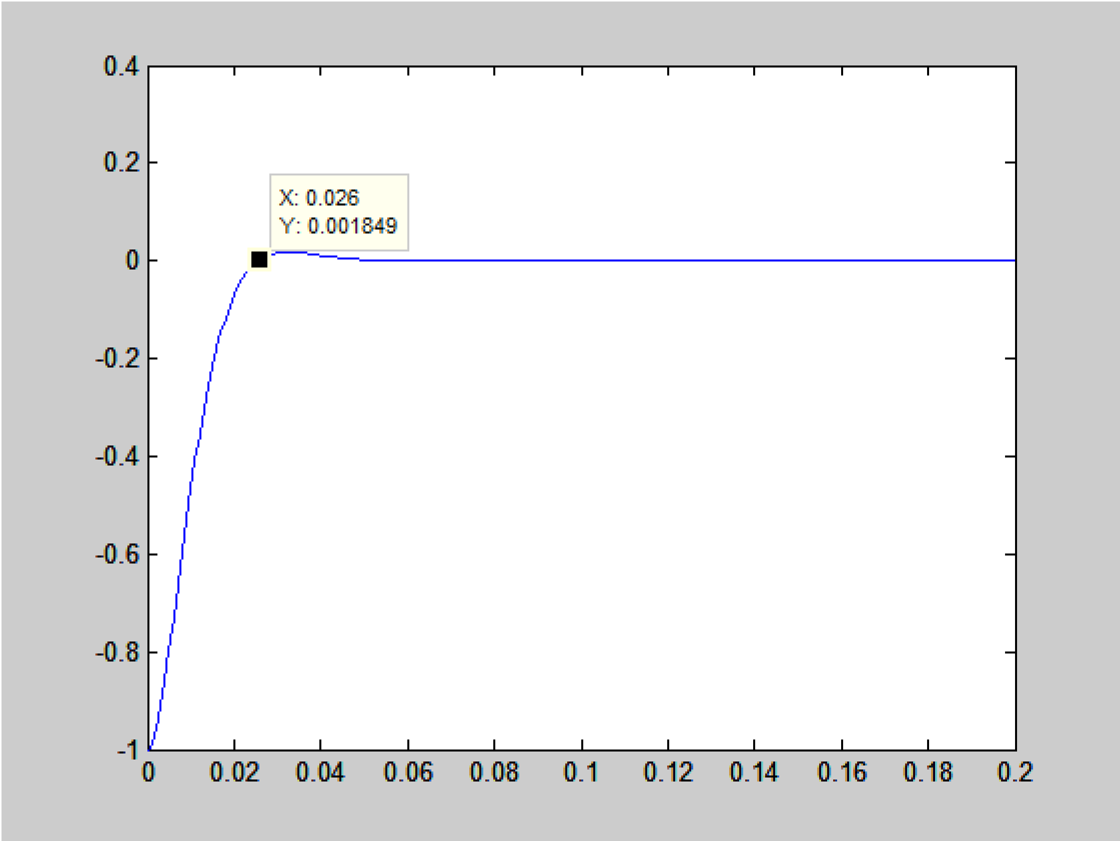
$$\frac{dv}{dt}\Big|_{t=0} = 125 + B_2 96.64$$

$$\frac{dv}{dt}\Big|_{t=0} = \frac{i_C(0)}{C}, i_L(0^-) = 2.5A, i_R(0^+) = -0.5A \Rightarrow i_C(0^+) = -2.5 - (-0.5) = -2A$$

$$\frac{dv}{dt}\Big|_{t=0} = \Rightarrow B_2 = -1.2935$$

$$\therefore v(t) = e^{-125t}(-\cos 96.64t - 1.2935 \sin 96.64t)$$

$$v(t) = 0 \Rightarrow \boxed{t = 0.0257\sec}$$



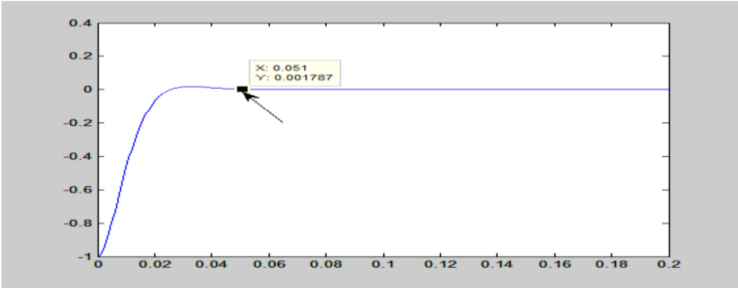
(b)

$$\frac{dv_C}{dt} = 0$$

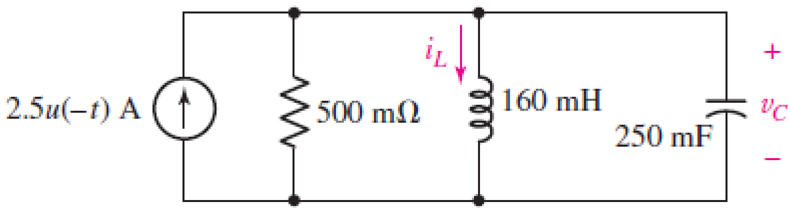
$$v_{C_{\max}} = 0.0171V \Rightarrow t_{\max} = 0.033\sec$$

$$0.1 \times v_{C_{\max}} = 0.00171$$

$$\therefore t_s = 0.051s$$



39. For $t < 0$ s, we see from the circuit below that the capacitor and the resistor are shorted by the presence of the inductor. Hence, $i_L(0^-) = 2.5$A and $v_C(0^-) = 0$ V.



When the 2.5-A source turns off at $t = 0$ s, we are left with a parallel RLC circuit.

(a) $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(500\times10^{-3})(250\times10^{-3})} = 4s^{-1}$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(160\times10^{-3})(250\times10^{-3})}} = 5rad/\sec$$

$$\therefore \alpha < \omega_0 \Rightarrow underdamped$$

$$\omega_d = \sqrt{\omega_0^{\,2} - \alpha^2} = 3rad/s$$

(b)

$$i_L(t) = e^{-\alpha t}(B_1\cos\omega_d t + B_2\sin\omega_d t)$$
$$i_L(t) = e^{-4t}(B_1\cos 3t + B_2\sin 3t)$$
$$i_L(0^-) = i_L(0^+) = 2.5$$
$$i_L(0) = B_1 = 2.5$$
$$i_L(t) = e^{-4t}(2.5\cos 3t + B_2\sin 3t) \Rightarrow \frac{di_L}{dt} = e^{-4t}(-10\cos 3t - 7.5\sin 3t - 4B_2\sin 3t + 3B_2\cos 3t)$$
$$\left.\frac{di_L}{dt}\right|_{t=0^+} = -10 + 3B_2$$
$$\left.\frac{di_L}{dt}\right|_{t=0} = \frac{v_L(0)}{L}, v_L(0^+) = v_C(0^+) = 0,$$
$$\Rightarrow B_2 = 3.33$$
$$\boxed{\therefore i_L(t) = e^{-4t}(2.5\cos 3t + 3.33\sin 3t)}$$

(c) $\boxed{w_L = \dfrac{1}{2}Li_L^{\,2} = \dfrac{1}{2}(160\times10^{-3})\left[e^{-4t}(2.5\cos 3t + 3.33\sin 3t)\right]^2}$

40. $R = 500M\Omega$

(a) $\alpha = \frac{1}{2RC} = \frac{1}{2(500\times10^{6})(250\times10^{-3})} = 4\times10^{-9}\, s^{-1}$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(160\times10^{-3})(250\times10^{-3})}} = 5 rad/\sec$

$\therefore \alpha < \omega_0 \Rightarrow underdamped$

$\omega_d = \sqrt{\omega_0^{\,2} - \alpha^2} = 5 rad/s$

(b)

$i_L(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$

$i_L(t) = e^{-5t}(B_1 \cos 5t + B_2 \sin 5t)$

$i_L(0^-) = i_L(0^+) = 2.5$

$i_L(0) = B_1 = 2.5$

$i_L(t) = e^{-5t}(2.5\cos 5t + B_2 \sin 5t) \Rightarrow \frac{di_L}{dt} = e^{-5t}(-12.5\cos 5t - 12.5\sin 5t - 5B_2 \sin 5t + 5B_2 \cos 5t)$

$\frac{di_L}{dt}\Big|_{t=0^+} = -12.5 + 5B_2$

$\frac{di_L}{dt}\Big|_{t=0} = \frac{v_L(0)}{L}, v_L(0^+) = v_C(0^+) = 0,$

$\Rightarrow B_2 = \frac{12.5}{5} = 2.5$

$\therefore i_L(t) = e^{-5t}(2.5\cos 5t + 2.5\sin 5t)$

(c)

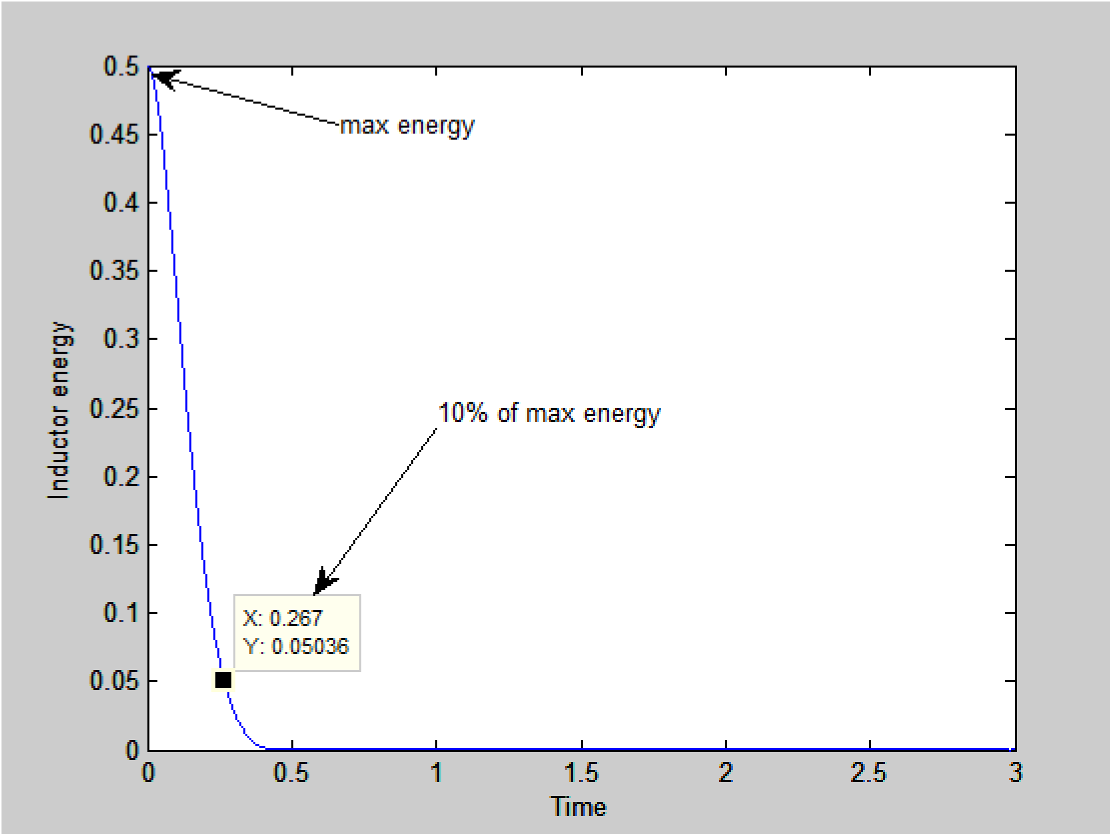
$i_L(t) = e^{-5t}(2.5\cos 5t + 2.5\sin 5t)$

$w_L = \frac{1}{2}Li_L^{\,2} = \frac{1}{2}\times160\times10^{-3}\left[e^{-5t}(2.5\cos 5t + 2.5\sin 5t)\right]^2$

$w_{L\max} = 0.5 \Rightarrow t_{\max} = 0$

$10\% w_{L\max} = 0.05$

$\therefore t\Big|_{10\% w_{L\max}} = 0.267\sec$



max energy
10% of max energy
X: 0.267
Y: 0.05036
Inductor energy
Time
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.5
1
1.5
2
2.5
3

41. $C = 160mF, L = 250mH$

(a) Critically damped response

$$\alpha = \frac{R}{2L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(250\times10^{-3})(160\times10^{-3})}} = 5rad/\sec$$

$$\alpha = \omega_0 \Rightarrow \frac{R}{2L} = \omega_0$$

$$R = 2L\omega_0 = 5\times2\times250\times10^{-3} = 2.5\Omega$$

(b) just barely underdamped

$$\alpha < \omega_0 \Rightarrow R < 2.5\Omega$$

(c) parallel RLC

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5rad/\sec$$

$$\alpha = \frac{1}{2RC}$$

$\alpha = \omega_0$ Critically damped

$$R = \frac{1}{2C\omega_0}$$

$$\therefore R = 0.625\Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5rad/\sec$$

$$\alpha = \frac{1}{2RC}$$

$\alpha < \omega_0$ Underdamped

$$R > \frac{1}{2C\omega_0}$$

$$\boxed{\therefore R > 0.625\Omega}$$

42. $R = 2\Omega, C = 1mF, L = 2mH$

$v_C(0^-) = 1V, i_L(0^-) = 0$

$$\alpha = \frac{R}{2L} = 500s^{-1}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3})(1\times10^{-3})}} = 707.11rad/\sec$$
$$\alpha < \omega_0 \Rightarrow underdamped$$
$$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$
$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} = 500rad/s$$
$$i(t) = e^{-500t}(B_1 \cos 500t + B_2 \sin 500t)$$
$$i(0) = 0 \Rightarrow B_1 = 0$$
$$i(t) = e^{-500t}(B_2 \sin 500t)$$
$$\left.\frac{di}{dt}\right|_{t=0} = 500B_2 = \frac{v_L(0)}{L} = \frac{v_C(0) - Ri(0)}{L} = \frac{1}{2\times10^{-3}}$$
$$B_2 = 1$$
$$i(t) = e^{-500t}\sin 500t$$
$$\boxed{\begin{aligned} i(1ms) &= e^{-500\times10^{-3}}\sin 500\times10^{-3} = 0.291A \\ i(2ms) &= e^{-500\times2\times10^{-3}}\sin 500\times2\times10^{-3} = 0.3096A \\ i(3ms) &= e^{-500\times3\times10^{-3}}\sin 500\times3\times10^{-3} = 0.223A \end{aligned}}$$

43. $R = 2\Omega, C = 1mF, L = 2mH$

$$R_{new} = 2//2 = 1\Omega$$
$$\alpha = \frac{R}{2L} = 250s^{-1}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3})(1\times10^{-3})}} = 707 rad/\sec$$
$$\alpha < \omega_0 \Rightarrow underdamped$$
$$v_C(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$
$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} = 661 rad/s$$
$$v_C(t) = e^{-250t}(B_1 \cos 661t + B_2 \sin 661t)$$
$$v(0) = 1 \Rightarrow B_1 = 1$$
$$v_C(t) = e^{-250t}(\cos 661t + B_2 \sin 661t)$$
$$\left.\frac{dv_C}{dt}\right|_{t=0} = -250 + 661B_2 = \frac{i_C(0)}{C} = 0$$
$$B_2 = 0.378$$
$$v_C(t) = e^{-250t}(\cos 661t + 0.378 \sin 661t)V, t > 0$$
$$\boxed{v_C(4ms) = \text{-}0.2569V}$$

44. $R = 2\Omega, C = 1mF, L = 2mH$

$v_C(0^-) = 2V, i_L(0^-) = 1mA$

$$\alpha = \frac{R}{2L} = 500s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3})\left(1\times10^{-3}\right)}} = 707.11 rad/\sec$$

$$\alpha < \omega_0 \Rightarrow underdamped$$

$$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

$$\omega_d = \sqrt{\omega_0^{\ 2} - \alpha^2} = 500 rad/s$$

$$i(t) = e^{-500t}(B_1 \cos 500t + B_2 \sin 500t)$$

$$i(0^-) = 1mA \Rightarrow B_1 = 1\times10^{-3}$$

$$i(t) = e^{-500t}(1\times10^{-3} \cos 500t + B_2 \sin 500t)$$

$$\left.\frac{di}{dt}\right|_{t=0} = -0.5 + 500B_2 = \frac{v_L(0)}{L} = \frac{v_C(0) - Ri(0)}{L} = \frac{2-(2)\left(1\times10^{-3}\right)}{2\times10^{-3}} = 999$$

$$B_2 = 1.999$$

$$\boxed{i(t) = e^{-500t}(1\times10^{-3} \cos 500t + 1.999 \sin 500t) A, t > 0}$$

45. $R = 1k\Omega, C = 2mF, L = 1mH$

$v_C(0^-) = -4V, i_L(0^-) = 0$

(a)

$$\alpha = \frac{R}{2L} = 5\times10^5\, s^{-1}$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times10^{-3})(1\times10^{-3})}} = 707.11 rad/\sec$$
$$\alpha > \omega_0 \Rightarrow overdamped$$
$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t > 0$$
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -0.5$$
$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0{}^2} = -999999.5$$
$$i(t) = A_1 e^{-0.5t} + A_2 e^{-9999995t}$$
$$i_L(0) = -2.99\times10^{-4} A = i_L(0^+)$$
$$i(0) = A_1 + A_2 = 0$$
$$\frac{di}{dt} = -0.5A_1 e^{-0.5t} - 999999.5A_2 e^{-9999995t}$$
$$\left.\frac{di}{dt}\right|_{t=0} = -0.5A_1 - 999999.5A_2 = \frac{v_L(0)}{L} = \frac{v_C(0) - Ri(0)}{L} = \frac{-4-(0)(1\times10^3)}{1\times10^{-3}} = -4\times10^3 A/s$$
$$A_1 = -0.004, A_2 = 0.0004$$
$$i(t) = -0.004e^{-0.5t} + 0.004e^{-9999995t} A, t > 0$$

46. $R = 140\Omega, C = 0.5F, L = 12H$

(a) $\alpha = \dfrac{R}{2L} = \dfrac{140}{2\times 0.5} = 140s^{-1}$

(b) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(12)(0.5)}} = 0.41 rad/\sec$

(c) $i(0^-) = i(0^+) = 0.5A$

(d)

$$\alpha > \omega_0 \Rightarrow overdamped$$

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t > 0$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -6\times 10^{-4}$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0{}^2} = -279.99$$

$$i(t) = A_1 e^{-6\times 10^{-4} t} + A_2 e^{-279.99t}$$

$$i(0) = A_1 + A_2 = 0.5$$

$$\frac{di}{dt} = -6\times 10^{-4} A_1 e^{-6\times 10^{-4} t} - 279.99 A_2 e^{-279.99t}$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = -6\times 10^{-4} A_1 - 279.99 A_2 = \frac{v_L(0)}{L} = \frac{v_C(0) - Ri(0)}{L} = \frac{70 - (140)(0.5)}{12} = 0$$

(e)

$$A_1 = 0.499, A_2 = 1.07\times 10^{-6}$$

$$i(t) = 0.499 e^{-6\times 10^{-4} t} + 1.07\times 10^{-6} e^{-279.99t} A, t > 0$$

$$\therefore i(6s) = 0.497A$$

50. $R=1\Omega, C=20mF, L=10\mu H$

(a) $\boxed{\alpha=\frac{R}{2L}=\frac{1}{2\times10\times10^{-6}}=5\times10^{4}s^{-1}}$

$$\boxed{\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(10\times10^{-6})\left(20\times10^{-3}\right)}}=2236.1rad/\sec}$$

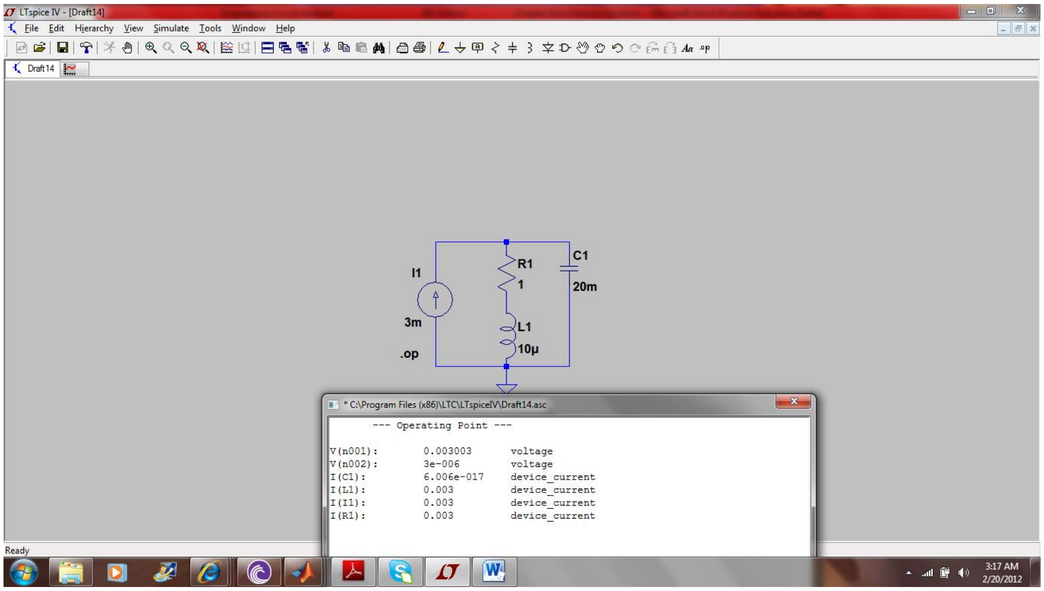
(b) $i_s=3u(-t)+2u(t)mA$

At $t=0^{-}\Rightarrow i_s=3mA$

$$\boxed{\begin{aligned}&v_R(0^{-})=(1)(3\times10^{-3})=3\times10^{-3}=3mV\\&i_L(0^{-})=3mA\\&v_C(0^{-})=v_R(0^{-})=3mV\end{aligned}}$$

At $t=0^{+}\Rightarrow i_s=2mA$

$$\boxed{\begin{aligned}&v_R(0^{+})=(1)(3\times10^{-3})=3\times10^{-3}=3mV\\&i_L(0^{+})=i_L(0^{-})=3mA\\&v_C(0^{+})=v_C(0^{-})=3mV\\&i_L(\infty)=2mA\\&v_C(\infty)=\left(2mA\right)\left(1\right)=2mV\end{aligned}}$$


LTspice IV - [Draft14]
File Edit Hierarchy View Simulate Tools Window Help
Draft14
I1
3m
.op
R1
1
L1
10µ
C1
20m
* C:\Program Files (x86)\LTC\LTspiceIV\Draft14.asc
--- Operating Point ---
V(n001): 0.003003 voltage
V(n002): 3e-006 voltage
I(C1): 6.006e-017 device_current
I(L1): 0.003 device_current
I(I1): 0.003 device_current
I(R1): 0.003 device_current
Ready
3:17 AM
2/20/2012

51.

At $t = 0^-$ only the right-hand current source is active.

$$i_C(0^-) = 0A, i_R(0^-) = -10mA, i_L(0^-) = 10mA$$
$$v_L(0^-) = 0, v_C(0^-) = (10mA)(20\times10^3) = 200V, v_R(0^-) = (-10mA)(20\times10^3) = -200V$$

During the interval from $t = 0^-$ to $t = 0^+$, the left-hand current source becomes active.

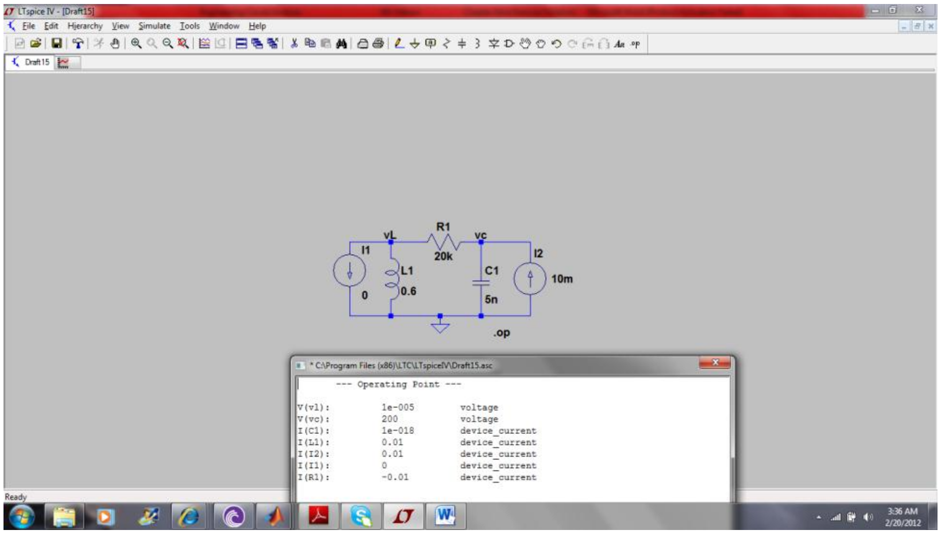
$$i_L(0^+) = i_L(0^-) = 10mA, i_R(0^+) = -10mA - 15mA = -25mA, i_C(0^+) = 10\times10^{-3} + i_R(0^+) = -15mA,$$
$$v_C(0^+) = 200V, v_R(0^+) = (-25mA)(20\times10^3) = -500V, v_L(0^+) = v_R(0^+) + v_C(0^+) = -500 + 200 = -300V$$
$$v_L = L\frac{di_L}{dt} \Rightarrow \frac{di_L}{dt}\Big|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{-300}{0.6} = -500A/s$$
$$\frac{dv_C}{dt}\Big|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{-15\times10^{-3}}{5\times10^{-9}} = -3\times10^6 V/s$$

At the left hand node:

$$-15\times10^{-3} - i_L - i_R = 0, t > 0$$
$$0 - \frac{di_L}{dt} - \frac{di_R}{dt} = 0 \Rightarrow \frac{di_R}{dt}\Big|_{t=0^+} = -\frac{di_L}{dt}\Big|_{t=0^+} = 500A/s$$

At the right hand node:

$$10\times10^{-3} - i_C + i_R = 0, t > 0$$
$$0 - \frac{di_C}{dt} + \frac{di_R}{dt} = 0 \Rightarrow \frac{di_C}{dt}\Big|_{t=0^+} = -\frac{di_R}{dt}\Big|_{t=0^+} = 500A/s$$
$$v_R = Ri_R \Rightarrow \frac{dv_R}{dt}\Big|_{t=0^+} = R\frac{di_R}{dt}\Big|_{t=0^+} = 20\times10^3\times500 = 10\times10^6 V/s$$
$$-v_L + v_R + v_C = 0 \Rightarrow v_L = v_R + v_C$$
$$\frac{dv_L}{dt}\Big|_{t=0^+} = \frac{dv_R}{dt}\Big|_{t=0^+} + \frac{dv_C}{dt}\Big|_{t=0^+} = 10\times10^6 - 3\times10^6 = 7\times10^6 V/s$$



52. $v_s(t) = -8 + 2u(t)V$

(a) $v_C(0^+)$

At $t = 0^-$

$$i_R(0^-) = \frac{-8}{15} = -0.533A$$

$$v_R(0^-) = (15)(-0.53) = -7.995V$$

$$v_R(0^-) = v_C(0^-) = 8V$$

$$\boxed{v_C(0^+) = 8V}$$

(b)

$$i_L(0^-) = i_R = \frac{8}{15} = 0.533A$$

$$\boxed{i_L(0^+) = 0.533A}$$

53. $v_s = 1 - 2u(t)V$

At $t = 0^- \Rightarrow v_s = 1$

(a)

$$i_L(0^-) = i_R(0^-) = \frac{v_s}{R} = \frac{1}{500 \times 10^{-3}} = 2A$$

$$\boxed{i_L(0^+) = i_L(0^-) = 2A}$$

(b)

$$v_C(0^-) = v_R(0^-) = Ri_R = 1V$$

$$\boxed{v_C(0^+) = v_C(0^-) = 1V}$$

(c)

$$v_s = 1 - 2 = -1V$$

$$i_L(\infty) = \frac{-1}{500 \times 10^{-3}}$$

$$\boxed{i_L(\infty) = -2A}$$

(d)

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 500 \times 10^{-3} \times 5 \times 10^{-3}} = 200 s^{-1}$$

$$\alpha^2 = 40000$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 10^{-3} \times 6 \times 10^{-3}}} = 182.57 rad/s$$

$$\omega_0{}^2 = 33333.33$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0{}^2} = -200 + \sqrt{40000 - 33333.33} = -118.34 s^{-1}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -281.65$$

$$\alpha > \omega_0 \Rightarrow overdamped$$

$$v_C(0^+) = v_C(0^-) = 1V$$

$$i_L(0^+) = i_L(0^-) = 2A$$

$$v_C(t) = -1 + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$v_C(t) = -1 + A_1 e^{-0.66t} + A_2 e^{-39.34t}$$

$$v_C(0) = 1 = -1 + A_1 + A_2 \Rightarrow A_1 + A_2 = 2$$

$$\left.\frac{dv_C}{dt}\right|_{t=0^+} = -118.34 A_1 - 281.65 A_2 = 0$$

60. $v(0)=0, i(0)=1mA$

$\alpha=0$ this is a series RLC with R=0, or a parallel RLC with R=∞

$$\omega_0^2=\frac{1}{LC}=\frac{1}{10\times10^{-12}\times2\times10^{-9}}=5\times10^{19}$$

$$\omega_d=\sqrt{\omega_0^2-\alpha^2}=7.071\times10^9\,rad/\sec$$

The response form is as follow:

$$v(t)=A\cos\omega_d t+B\sin\omega_d t$$

$$v(0^-)=v(0^+)=0$$

$$\therefore v(t)=B\sin\omega_d t$$

$$\frac{dv}{dt}=\omega_d B\cos\omega_d t$$

$$i_C=C\frac{dv}{dt}\Rightarrow i_C(0^+)=CB\omega_d=-i_L(0^+)=-i_L(0^-)=-1\times10^{-3}$$

$$B=\frac{-1\times10^{-3}}{2\times10^{-9}\times7.071\times10^9}=-7.0711\times10^{-5}$$

$$v(t)=-7.0711\times10^{-5}\sin 7.071\times10^9 t$$

In designing the op amp stage, we first write the differential equation:

$$\frac{1}{10\times10^{-12}}\int_0^t v\,dt'+10^{-3}+2\times10^{-9}\frac{dv}{dt}=0,(i_C+i_L=0)$$

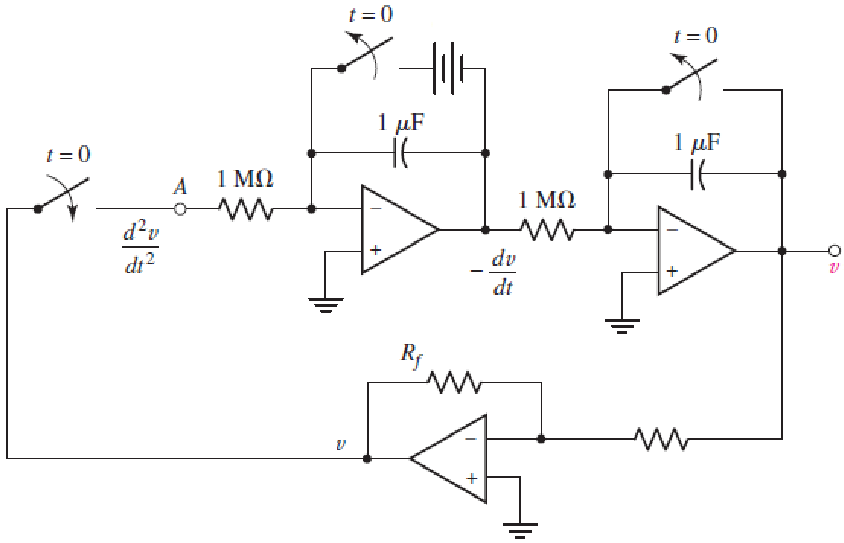
Take the derivative of both sides:

$$\frac{1}{10\times10^{-12}}v+2\times10^{-9}\frac{d^2v}{dt^2}=0$$

$$\frac{d^2v}{dt^2}=\frac{-1}{2\times10^{-9}\times10\times10^{-12}}v=\frac{-1}{20\times10^{-21}}v$$

$$\left.\frac{dv}{dt}\right|_{t=0^+}=\left(7.0711\times10^{-5}\right)\left(7.071\times10^9\right)=4.99\times10^5$$

One possible solution is:



t = 0
t = 0
t = 0
1 μF
1 μF
A
1 MΩ
1 MΩ
d²v/dt²
− dv/dt
v
Rf
v

61. $v(0)=0, i(0)=1mA$

$\alpha=0$ this is a series RLC with R=0, or a parallel RLC with R=∞

$$\omega_0^2=\frac{1}{LC}=\frac{1}{10\times10^{-12}\times2\times10^{-9}}=5\times10^{19}$$

$$\omega_d=\sqrt{\omega_0^2-\alpha^2}=7.071\times10^9\,rad/\sec$$

The response form is as follow:

$$i(t)=A\cos\omega_d t+B\sin\omega_d t$$

$$i(0^-)=i(0^+)=1mA=A=1\times10^{-3}$$

$$\therefore i(t)=A\cos\omega_d t+B\sin\omega_d t$$

$$\frac{di}{dt}=-A\omega_d\sin\omega_d t+\omega_d B\cos\omega_d t$$

$$v_L=L\frac{di}{dt}=10\times10^{-12}\frac{di}{dt}$$

$$v_L(0^+)=v_C(0^+)=v_C(0^-)=0=LB\omega_d$$

$$B=0$$

$$i(t)=10^{-3}\cos7.071\times10^9 t$$

In designing the op amp stage, we first write the differential equation:

$$\frac{1}{10\times10^{-12}}\int_0^t vdt'+10^{-3}+2\times10^{-9}\frac{dv}{dt}=0,(i_C+i_L=0)$$

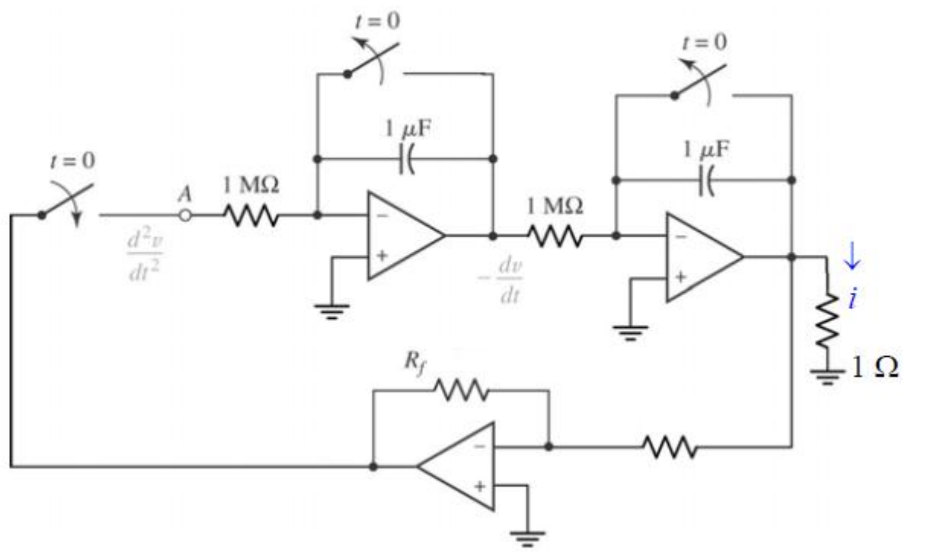
Take the derivative of both sides:

$$\frac{1}{10\times10^{-12}}v+2\times10^{-9}\frac{d^2v}{dt^2}=0$$

$$\frac{d^2v}{dt^2}=\frac{-1}{2\times10^{-9}\times10\times10^{-12}}v=\frac{-1}{20\times10^{-21}}v$$

$$\left.\frac{di}{dt}\right|_{t=0^+}=0$$

One possible solution is:



t = 0
t = 0
t = 0
1 μF
1 μF
A
1 MΩ
1 MΩ
$\frac{d^2v}{dt^2}$
$-\frac{dv}{dt}$
i
1 Ω
R_f

62. $v(0)=0, i(0)=1mA, L=20H//10PH, C=5\mu F$

$\alpha=0$ this is a series RLC with R=0, or a parallel RLC with R=∞

$$L_{eq}=10\times10^{-12}//20=1\times10^{-11}H$$

$$\omega_0^2=\frac{1}{LC}=\frac{1}{1\times10^{-11}\times2\times10^{-9}}=2\times10^{16}$$

$$\omega_d=\sqrt{\omega_0^2-\alpha^2}=1.41\times10^8 rad/\sec$$

The response form is as follow:

$$i(t)=A\cos\omega_d t+B\sin\omega_d t$$

$$i(0^-)=i(0^+)=1mA=A=1\times10^{-3}$$

$$\therefore i(t)=A\cos\omega_d t+B\sin\omega_d t$$

$$\frac{di}{dt}=-A\omega_d\sin\omega_d t+\omega_d B\cos\omega_d t$$

$$v_L=L\frac{di}{dt}=1\times10^{-11}\frac{di}{dt}$$

$$v_L(0^+)=v_C(0^+)=v_C(0^-)=0=LB\omega_d$$

$$B=0$$

$$i(t)=10^{-3}\cos1.41\times10^8 t$$

In designing the op amp stage, we first write the differential equation:

$$\frac{1}{10\times10^{-12}}\int_0^t vdt'+10^{-3}+\frac{1}{20}\int_0^t vdt'+2\times10^{-9}\frac{dv}{dt}=0,(i_C+i_{L_1}+i_{L_2}=0)$$

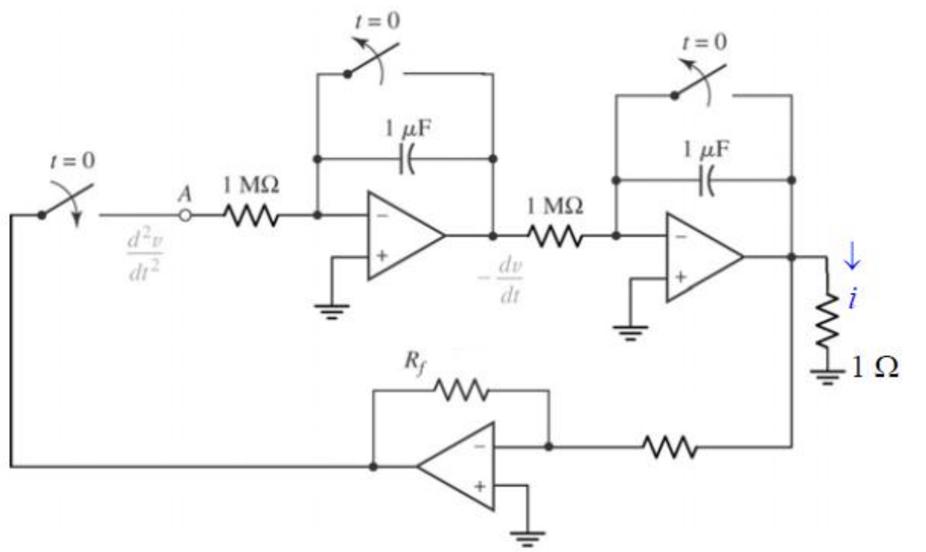
Take the derivative of both sides:

$$\frac{1}{10\times10^{-12}}v+\frac{1}{20}v+2\times10^{-9}\frac{d^2v}{dt^2}=0$$

$$\frac{d^2v}{dt^2}=-25\times10^6 v$$

$$\frac{di}{dt}\bigg|_{t=0^+}=0$$

One possible solution is:



t = 0
t = 0
t = 0
1 μF
1 μF
A
1 MΩ
1 MΩ
i
1 Ω
R_f

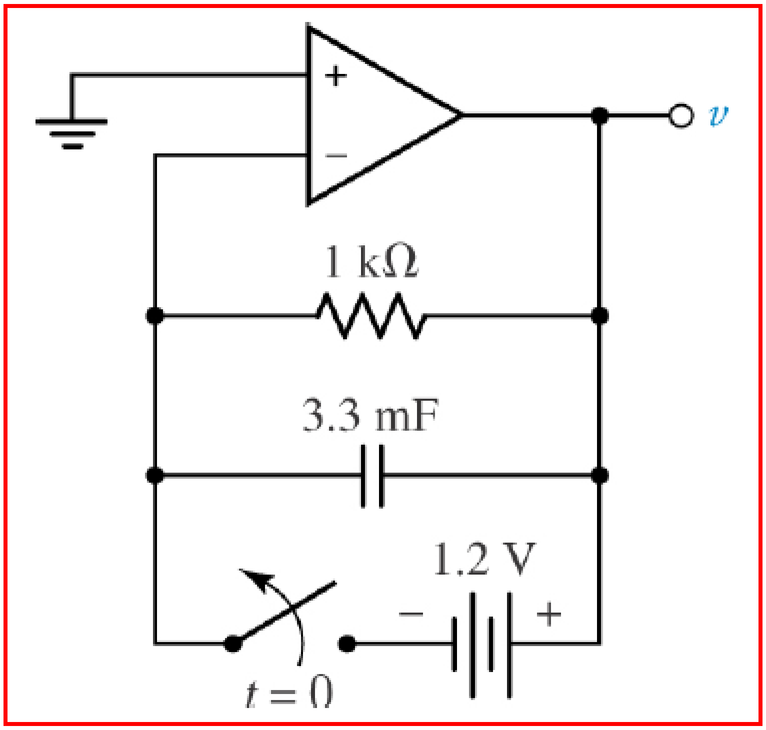
63. RC circuit

$R = 1k\Omega, C = 3.3mF$

$v_C(0^-) = v_C(0^+) = 1.2V$

(a) $\boxed{\frac{v}{1000} + 3.3\times10^{-3}\frac{dv}{dt} = 0}$

(b) One possible solution is



64. RL circuit

$R = 20\Omega, L = 5H$

$i_L(0^-) = i_L(0^+) = 2A$

(a)

$v_R = v_L$

$20(-i_L) = 5\frac{di_L}{dt}$

$\boxed{\frac{di_L}{dt} = -4i_L}$

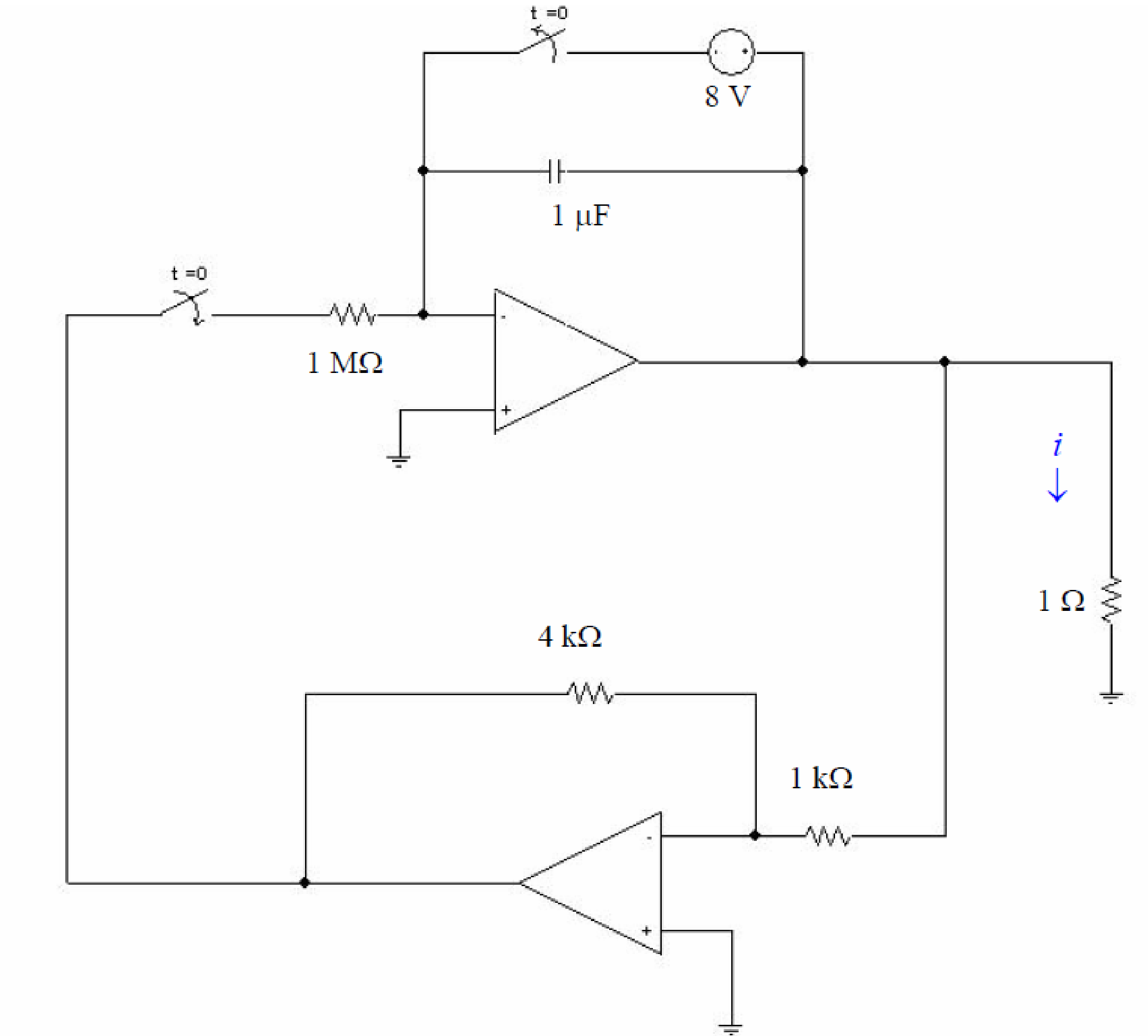
(b)

$i_L(t) = Ae^{-t/\tau}, \tau = L/R = 0.25$

$i_L(0^-) = 2A$

$i_L(t) = 2e^{-4t}$

$\frac{di_L}{dt}\Big|_{t=0^+} = -8$


t =0
8 V
1 µF
t =0
1 MΩ
i
1 Ω
4 kΩ
1 kΩ

1. (a) $\boxed{-0.7822;\ -0.544;1.681}$

(b) $\boxed{4\cos 2t\text{: } 4;\ -1.665;\ -3.960}$

$\boxed{4\sin(2t+90^{o})\text{: } 4;\ -1.665;\ -3.960}$

(c) $\boxed{3.2\cos(6t+15^{o})\text{: } 3.091;\ 1.012;\ 2.429}$

$\boxed{3.2\cos(6t+105^{o})\text{: } 3.091;\ 1.012;\ 2.429}$

2. (a)

$$5\sin 300t = \boxed{5\cos(300t - 90°)}$$

$$1.95\sin(\pi t - 92°) = \boxed{1.95\cos(\pi t - 182°)}$$

$$2.7\sin(50t + 5°) - 10\cos 50t$$

$$= 2.7\sin 50t\cos 5° + 2.7\cos 50t\sin 5° - 10\cos 50t$$

$$= 2.6897\sin 50t - 9.7647\cos 50t = \boxed{10.13\cos(50t + 15.4°)}$$

(b)

$$66\cos(9t - 10°) = \boxed{66\sin(9t + 80°)}$$

$$4.15\cos 10t = \boxed{4.15\sin(10t + 90°)}$$

$$10\cos(100t - 9°) + 10\sin(100t + 19°)$$

$$= 11.0195\sin 100t + 13.1325\cos 100t$$

$$= 17.14\cos(100t - 40°)$$

$$= \boxed{17.14\sin(100t + 50°)}$$

3. (a) v_1 leads i_1 by $\boxed{-45^o}$

(b) v_1 leads by -45 + 80 = $\boxed{35^o}$

(c) v_1 leads by -45 + 40 = $\boxed{-5^o}$

(d) $5\sin(10t - 19^o) = 5\cos(10t - 19^o)$ therefore v_1 leads by -45 + 109 = $\boxed{64^o}$

4. $v_1 = 34\cos(10t + 125°)$

(a) $i_1 = 5\cos 10t; v_1$ lags i_1 by $\boxed{-235°}\left(-360° + 125°\right)$

(b) $i_1 = 5\cos\left(10t - 80°\right); v_1$ lags i_1 by $\boxed{-155°}\left(-235° + 80°\right)$

(c) $i_1 = 5\cos\left(10t - 40°\right); v_1$ lags i_1 by $\boxed{-195°}\left(-235° + 40°\right)$

(d) $i_1 = 5\cos\left(10t + 40°\right); v_1$ lags i_1 by $\boxed{-275°}\left(-235° - 40°\right)$

(e) $i_1 = 5\sin\left(10t - 19°\right) = 5\cos(10t - 109°); v_1$ lags i_1 by $\boxed{-126°}\left(-235° + 109°\right)$

5. (a) $\boxed{\text{cos4t leads sin4t}}$; $\boxed{\text{sin4t lags cos4t}}$

(b) the first is $\boxed{\text{lagging by } 80^o}$

(c) the second is $\boxed{\text{lagging by } 80^o}$

(d) the second is $\boxed{\text{lagging by } 88^o}$

(e) $\boxed{\text{Neither term lags}}$

6. (a) $\cos 3t - 7\sin 3t = 0$

$7.07\cos(3t + 1.4289) = 0$

$3t + 1.4289 = 1.5708$

$3t = 0.1419$

$t = \boxed{0.0473 \text{ s}}$

Also, $3t = 0.1419 + \pi$

$t = \boxed{1.0945 \text{ s}}$

and, $3t = 0.1419 + 2\pi$

$t = \boxed{2.1417 \text{ s}}$

(b) $\cos\left(10t + 45°\right) = 0$

$10t + 0.7854 = 1.5708$

$10t = 0.7854 \Rightarrow t = \boxed{0.0785 \text{ s}}$

Also, $10t = 0.7854 + \pi \Rightarrow t = \boxed{0.3927 \text{ s}}$

and, $10t = 0.7854 + 2\pi \Rightarrow t = \boxed{0.7069 \text{ s}}$

(c) $\cos 5t - \sin 5t = 0$

$5t + 0.7854 = 1.5708$

$5t = 0.7854 \Rightarrow t = \boxed{0.1571 \text{ s}}$

Also, $5t = 0.7854 + \pi \Rightarrow t = \boxed{0.7854 \text{ s}}$

and, $5t = 0.7854 + 2\pi \Rightarrow t = \boxed{1.4137 \text{ s}}$

(d) $\cos 2t + \sin 2t - \cos 5t + \sin 5t = 0$

$1.4142\cos(1.5t - 0.7854) = 0$

$1.5t - 0.7854 = 1.5708$

$1.5t = 2.3562 \Rightarrow t = \boxed{1.5708 \text{ s}}$

Also, $1.5t = 2.3562 + \pi \Rightarrow t = \boxed{3.6652 \text{ s}}$

and, $1.5t = 2.3562 + 2\pi \Rightarrow t = \boxed{5.7596 \text{ s}}$

7. (a) t = $\boxed{0}$; t = $\boxed{550 \text{ ms}}$; t = $\boxed{0}$; t = $\boxed{126 \text{ ms}}$

8. (a) $v(t) = 2t; 0 \le t < 0.5s$

$\therefore v(0.25s) = \boxed{0.5\text{ V}}$

(b) Using the first term of the Fourier series only,

$$v(t) = \frac{8}{\pi^2}\sin \pi t$$

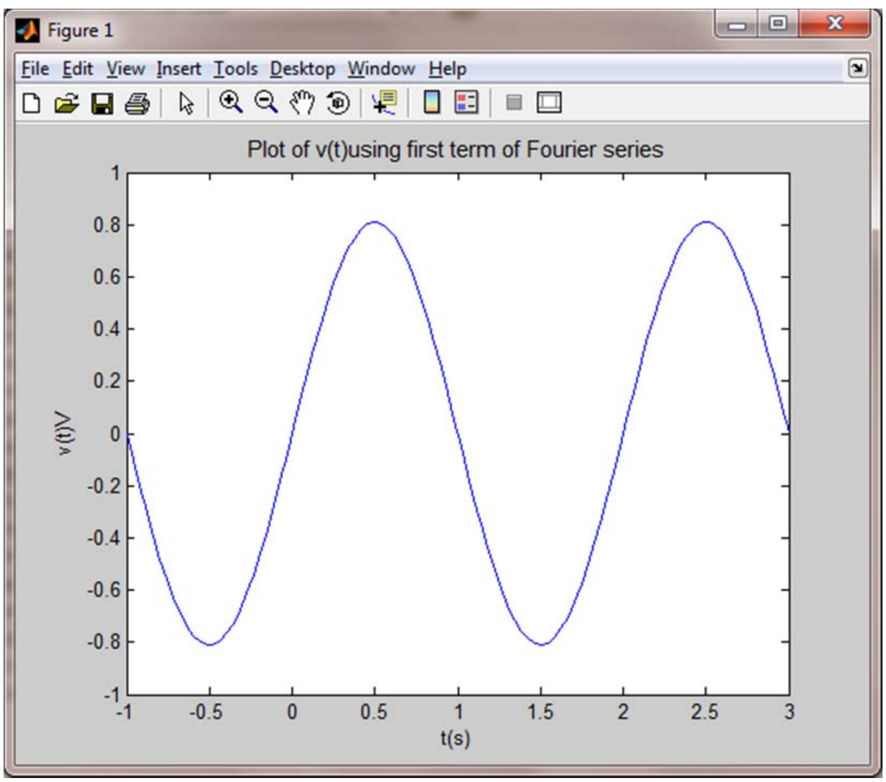
$$v(0.25s) = \frac{8}{\pi^2}\sin 45° = \boxed{0.5732\text{ V}}$$

(c) Using the first three terms of the Fourier series,

$$v(t) = \frac{8}{\pi^2}\sin \pi t - \frac{8}{3^2\pi^2}\sin 3\pi t + \frac{8}{5^2\pi^2}\sin 5\pi t$$

$$v(0.25s) = \boxed{0.4866\text{ V}}$$

(d)

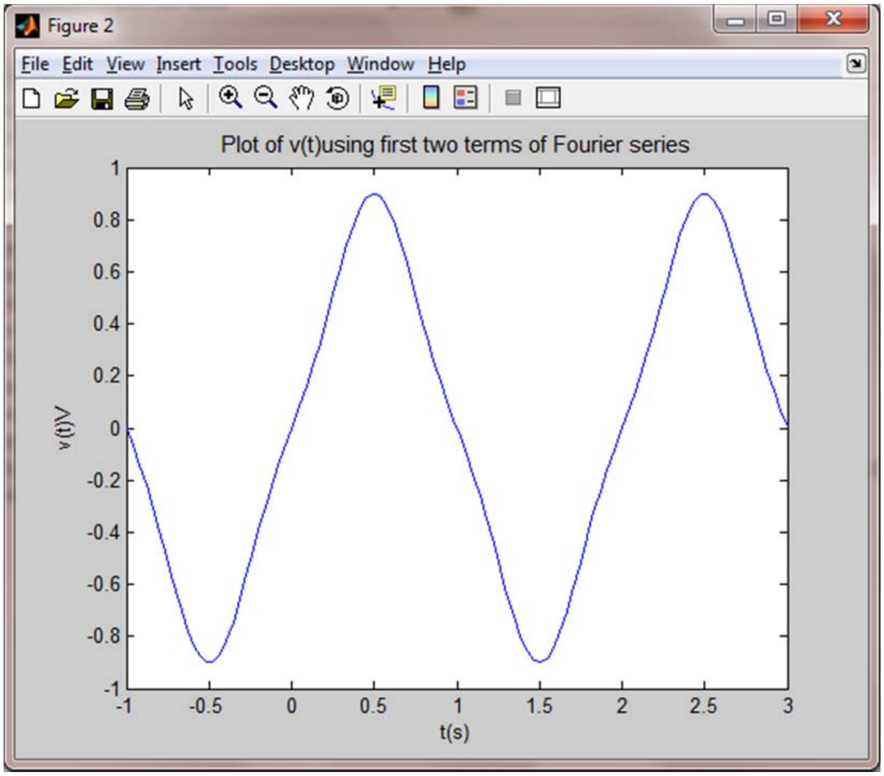



```
t=linspace(-1,3);
v = 8/(pi^2)*sin(pi*t);
figure(1);
plot(t,v);
xlabel('t(s)')
```

ylabel('v(t)V')

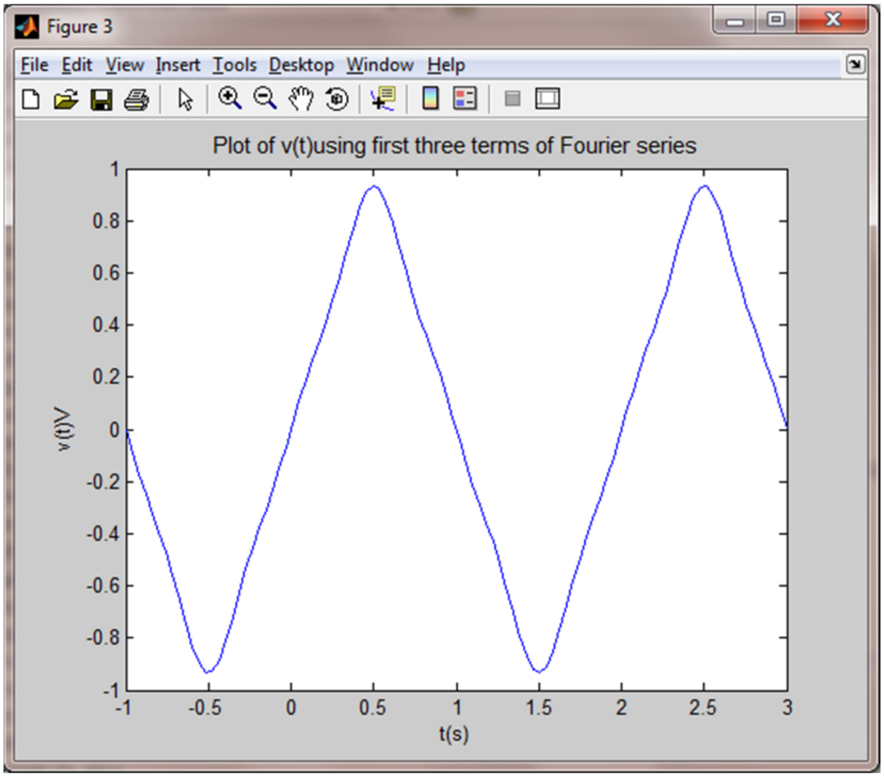
title('Plot of v(t)using first term of Fourier series','FontSize',11)

(e)




```
t=linspace(-1,3);
v = 8/(pi^2)*(sin(pi*t)-1/(3^2)*sin(3*pi*t));
figure(2);
plot(t,v);
xlabel('t(s)')
ylabel('v(t)V')
title('Plot of v(t)using first two terms of Fourier series','FontSize',11)
```

(f)




```
t=linspace(-1,3);
v = 8/(pi^2)*(sin(pi*t)-1/(3^2)*sin(3*pi*t)+ 1/(5^2)*sin(5*pi*t));
figure(3);
plot(t,v);
xlabel('t(s)')
ylabel('v(t)V')
title('Plot of v(t)using first three terms of Fourier series','FontSize',11)
```

9. (b)

V_{rms}	V_m
110 V	156 V
115 V	163 V
120 V	170 V

10. In this problem, when we apply Thevenin's theorem with the inductor as the load, we get,

$$v_{oc} = v_s \cdot \frac{1}{1+10} = \left(4.53\cos\left(0.333\times10^{-3}t+30°\right)\right)\frac{1}{11} = 0.4118\cos\left(0.333\times10^{-3}t+30°\right)\text{ V}$$

$$R_{th} = \frac{1\times10}{1+10} = \frac{10}{11} = 0.909\ \Omega$$

Now for a series RL circuit with $L = 3mH, R_{th} = 0.909\Omega$ and a source voltage of

$0.4118\cos\left(0.333\times10^{-3}t+30°\right)$ V, we get,

$$i_L(t) = \frac{V_m}{\sqrt{R_{th}^{\ 2}+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}+30°\right)$$

$$= \frac{0.4118}{\sqrt{(0.909)^2+\left(0.333\times10^{-3}\times3\times10^{-3}\right)^2}}\cos\left(0.333\times10^{-3}t-\tan^{-1}\frac{\left(0.333\times10^{-3}\times3\times10^{-3}\right)}{0.909}+30°\right)$$

$$= 0.453\cos\left(0.333\times10^{-3}t+30°\right)A$$

$$\therefore i_L(t=0) = 0.453\cos(30°) = \boxed{392.3\text{ mA}}$$

Now,

$$v_L(t) = L\frac{di_L}{dt} = -3\times10^{-3}\times0.333\times10^{-3}\times0.453\sin\left(0.333\times10^{-3}t+30°\right)$$

$$= 0.4526\cos\left(0.333\times10^{-3}t+120°\right)\ \mu\text{V}$$

$$v_L(t=0) = -0.2262\ \mu\text{V}$$

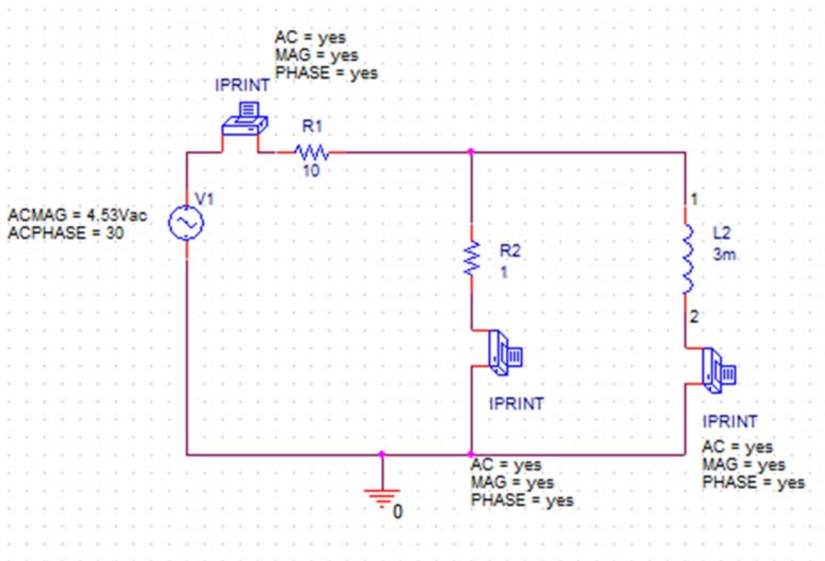
$$i_R(t=0) = \frac{v_L(t=0)}{R} = \boxed{-0.2262\ \mu\text{A}}$$

$$i_s(t=0) = i_R + i_L = \boxed{392.3\text{ mA}}$$

(b) $\boxed{v_L(t) = 0.4526\cos\left(0.333\times10^{-3}t+120°\right)\ \mu\text{V}}$

Pspice Verification:

This has been verified using the phasor in Pspice.




```
 FREQ        IM(V_PRINT1)IP(V_PRINT1)

   5.300E-05    4.530E-01    3.000E+01
**** 06/14/12 15:54:20 ******* PSpice Lite (April 2011) ******* ID# 10813 ****

 ** Profile: "SCHEMATIC1-prob10_10"  [ C:\ORCAD\ORCAD_16.5_LITE\examples\prob10_1

 ****     AC ANALYSIS                      TEMPERATURE =   27.000 DEG C

******************************************************************************

 FREQ        IM(V_PRINT2)IP(V_PRINT2)

   5.300E-05    4.526E-07    1.200E+02
```

11. $\boxed{8.84\cos(100t - 0.785) \text{ A}}$

12. In this problem, when we apply Thevenin's theorem with the inductor as the load, we get,

$$v_{oc} = 25\cos 100t \times \frac{1\|(1+2)}{(1+2)} \times 2 = 12.5\cos 100t \text{ V}$$

$$R_{th} = (1+1)\Omega \| 2\Omega = 1\Omega$$

Now for a series RL circuit with $L = 10mH, R_{th} = 1\Omega$ and a source voltage of $12.5\cos 100t$ V, we get,

$$i_L(t) = \frac{V_m}{\sqrt{R_{th}^{\ 2} + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$= \frac{12.5}{\sqrt{1^2 + \left(100\times 10\times 10^{-3}\right)^2}} \cos\left(100t - \tan^{-1}\frac{\left(100\times 10\times 10^{-3}\right)}{1}\right)$$

$$= 8.84\cos\left(100t - 45°\right) \text{ A}$$

Now,

$$v_L(t) = L\frac{di_L}{dt} = -8.84\times 10\times 10^{-3}\times 100\sin\left(100t - 45°\right)$$

$$= 8.84\cos\left(100t + 45°\right) \text{ V}$$

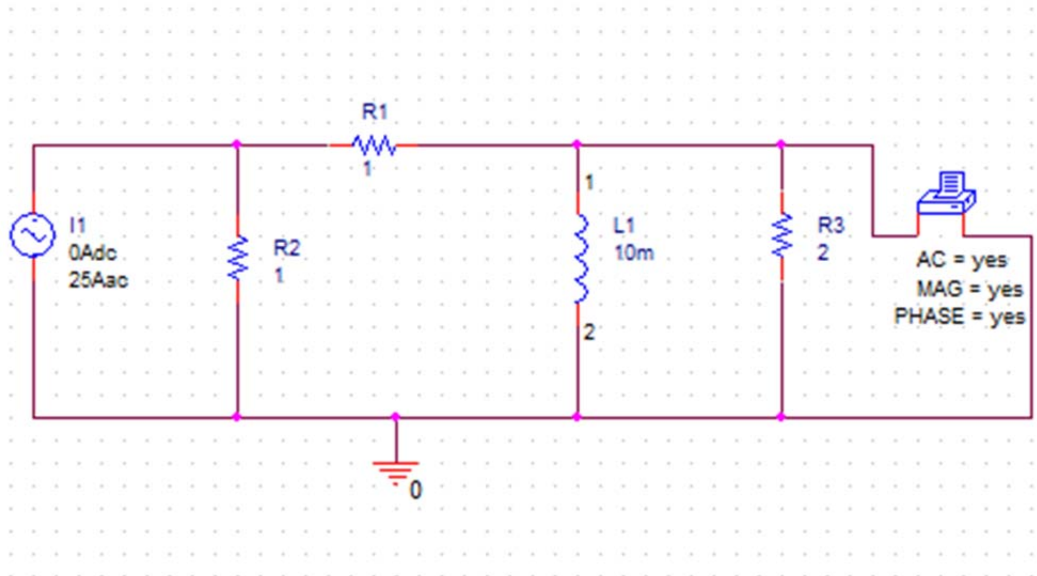
Voltage across the 2Ω resistor is equal to $v_L(t)$.

Power dissipated in 2Ω resistor is given by,

$$p_R(t) = \frac{v_R^{\ 2}}{R} = \frac{v_L^{\ 2}}{R} = \boxed{39.07\cos^2\left(100t + 45°\right) \text{ W}}$$

Pspice Verification:

This has been verified using the phasor in Pspice.



FREQ	VM(N00183,0)	VP(N00183,0)
1.592E+01	8.839E+00	4.500E+01

13. $\boxed{1.92\cos(40t - 0.876)\ \text{V}}$

14. Let ‘i’ be the current flowing in the circuit in the clockwise direction. Then, on applying KVL, we get,

$$15i + v_C = 3\cos 40t$$

Substituting, $i = i_C = C\dfrac{dv_C}{dt}$ on the above KVL equation, we get,

$$30\times 10^{-3}\frac{dv_C}{dt} + v_C = 3\cos 40t$$

Let us choose to express the response as,

$$v_C(t) = A\cos(40t+\theta)$$

$$\frac{dv_C}{dt} = -40A\sin(40t+\theta)$$

On rewriting the KVL equation, we get,

$$-1.2A\sin(40t+\theta) + A\cos(40t+\theta) = 3\cos 40t$$

$$\sqrt{A^2+(-1.2A)^2}\cos\left((40t+\theta)+\tan^{-1}\left(\frac{1.2A}{A}\right)\right) = 3\cos 40t$$

On equating the terms, we get,

$$A = 1.92$$

$$\theta = -\tan^{-1}(1.2) = -50.19°$$

$$\therefore v_C(t) = 1.92\cos(40t - 50.19°)\text{ V}$$

Energy stored in a capacitor is given by,

$$w_C = \frac{1}{2}Cv_C{}^2(t)$$

At t=10 ms,

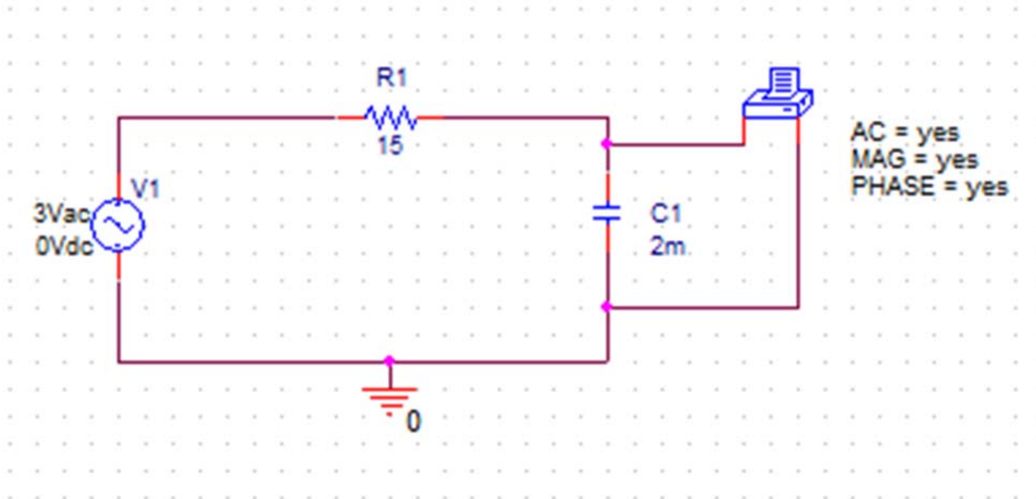
$$w_C(10\text{ms}) = \frac{1}{2}\times 2\times 10^{-3}\times 1.71^2 = \boxed{2.92\text{ mJ}}$$

At t=40 ms,

$$w_C(40\text{ms}) = \frac{1}{2}\times 2\times 10^{-3}\times 1.45^2 = \boxed{2.1\text{ mJ}}$$

Pspice Verification:

Phasor method is used to verify the solution.



FREQ	VM(N00132,0)	VP(N00132,0)
6.366E+00	1.921E+00	-5.019E+01

15. $\boxed{7.02\cos(6t - 0.359)\text{ A}}$

16. (a) $50\angle -75° = 50\cos(-75°) + j50\sin(-75°) = \boxed{12.94 - j48.29}$

(b)

$19e^{j30°} = 19\cos(30°) + j19\sin(30°) = \boxed{16.45 + j9.5}$

$2.5\angle -30° + 0.5\angle 45° = 2.5\cos(-30°) + j2.5\sin(-30°) + 0.5\cos(45°) + j0.5\sin(45°)$

$= \boxed{2.52 - j0.89}$

(c) $(2 + j2)(2 - j2) = 8 = \boxed{8\angle 0°}$

(d) $(2 + j2)(5\angle 22°) = (2.82\angle 45°)(5\angle 22°) = \boxed{14.14\angle 67°}$

17. (a) $\boxed{2.88\angle 11.5^{o}}$

(b) $\boxed{1\angle 90^{o}}$

(c) $\boxed{1\angle 0^{o}}$

(d) $\boxed{2.82 + j0.574}$

(e) $\boxed{2.87 - j4.90}$

18. (a) $4(8-j8)=\boxed{45.25\angle -45°}$

(b) $4\angle 5° - 2\angle 15° = (3.98 + j0.35) - (1.93 + j0.52) = 2.05 - j0.17 = \boxed{2.05\angle -4.74°}$

(c) $(2+j9) - 5\angle 0° = -3 + j9 = \boxed{9.5\angle 108.44°}$

(d) $\dfrac{-j}{10+j5} - 3\angle 40° + 2 = \dfrac{-j}{10+j5} - (2.3 + j1.93) + 2 = -0.34 - j2.01 = \boxed{2.03\angle 260.4°}$

(e) $(10+j5)(10-j5)(3\angle 40°) + 2 = 225\angle 40° + 2 = 174.36 + j144.63 = \boxed{226.54\angle 39.68°}$

19. (a) $\boxed{7.79 + j4.50}$

(b) $\boxed{6.74 - j0.023}$

(c) $\boxed{7.67 + j87.5}$

(d) $\boxed{2.15 + j2.50}$

(e) $\boxed{2.89 + j241}$

20. (a) $\dfrac{2+j3}{1+8\angle 90^\circ}-4=\dfrac{2+j3}{1+j8}-4=-3.6-j0.2=\boxed{3.6\angle 183.18^\circ}$

(b) $\left(\dfrac{10\angle 25^\circ}{5\angle -10^\circ}+\dfrac{3\angle 15^\circ}{3-j5}\right)j2=\left(2\angle 35^\circ+0.52\angle -44.04^\circ\right)\left(2\angle 90^\circ\right)=-1.58+j4.02$

$=\boxed{4.32\angle 111.46^\circ}$

(c) $\left[\dfrac{(1-j)(1+j)+1\angle 0^\circ}{-j}\right]\left(3\angle -90^\circ\right)+\dfrac{j}{5\angle -45^\circ}$

$=9\angle 0^\circ+0.2\angle 135^\circ=8.86+j0.14=\boxed{8.86\angle 0.91^\circ}$

21. $\boxed{88.7\sin(20t - 27.5^{o})\ \text{mA}}$; $\boxed{2.31\sin(20t + 62.5^{o})\ \text{V}}$

22. Let i_L in the complex form be $i_L = Ae^{j(35t+\theta^\circ)}$ A.

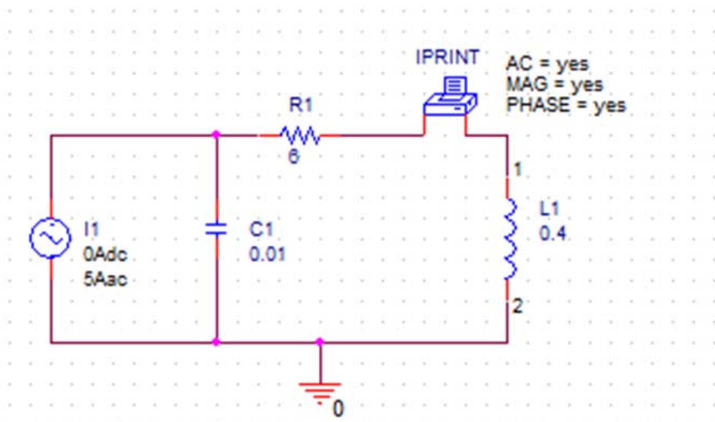
Given, $i_s = 5\sin(35t-10°) = 5e^{j(35t-100^\circ)}$ A

$$v_L = L\frac{di_L}{dt} = 0.4\frac{d}{dt}\left(Ae^{j(35t+\theta^\circ)}\right) = j14Ae^{j(35t+\theta^\circ)}$$
$$v_R = i_L R = 6Ae^{j(35t+\theta^\circ)}$$
$$v_S = v_C = v_R + v_L = A(6+j14)e^{j(35t+\theta^\circ)}$$
$$i_C = C\frac{dv_C}{dt} = 0.01\frac{d}{dt}\left(A(6+j14)e^{j(35t+\theta^\circ)}\right) = A\left(-4.9+j2.1\right)e^{j(35t+\theta^\circ)}$$

Applying KCL, we get,

$$i_s = i_C + i_L$$
$$5e^{j(35t-100^\circ)} = 4.43Ae^{j(35t+\theta^\circ+151.69^\circ)}$$
$$\Rightarrow A = 1.129 \text{ and } \theta = -251.69° = 108.31°$$
$$\therefore i_L = 1.129e^{j(35t+108.31^\circ)} = \boxed{1.129\cos(35t+108.31°) \text{ A}}$$

Pspice Verification:




```
FREQ          IM(V_PRINT1)IP(V_PRINT1)

 5.570E+00    1.129E+00    1.083E+02
```

23.

$$\boxed{\left[(62.5)^2 + (1.25\omega)^2\right]^{1/2} \cos\left(\omega t + 31.3^\circ + \tan^{-1}\frac{1.25\omega}{62.5}\right) \text{ mA}}$$

24. This problem can be easily solved by performing a source transformation which results in a circuit with voltage source, resistance and inductance.

Given, $$i_s = 5e^{j10t}$$ $$v_s = 10e^{j10t}$$

The steady-state expression for $i_L(t)$ can be found as:

$$i_L(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$= \frac{10}{\sqrt{2^2 + 10^2 \times 0.4^2}} \cos\left(10t - \tan^{-1}\left(\frac{10 \times 0.4}{2}\right)\right)$$

$$= \boxed{2.24\cos(10t - 63.44^\circ)\ \text{A}}$$

25. (a) $\boxed{75.9\angle 0^{o}}$

(b) $\boxed{5\angle -42^{o}}$

(c) $\boxed{1\angle 104^{o}}$

(d) $\boxed{8.04\angle -78.4^{o}}$

26. (a) $11\sin 100t = 11\cos(100t - 90°) = \boxed{11\angle -90°}$

(b) $11\cos 100t = \boxed{11\angle 0°}$

(c) $11\cos(100t - 90°) = \boxed{11\angle -90°}$

(d) $3\cos 100t - 3\sin 100t = 3\angle 0° - 3\angle -90° = 3 + j3 = \boxed{4.24\angle 45°}$

27. (a) $\boxed{9\cos(2\pi\times10^3 t + 65^\circ)\ \text{V}}$

(b) $\boxed{500\cos(2\pi\times10^3 t + 6^\circ)\ \text{mA}}$

(c) $\boxed{14.7\cos(2\pi\times10^3 t + 4^\circ)\ \text{V}}$

28. (a) $\dfrac{2-j}{5\angle 45^\circ}\text{ V} = \dfrac{2.24\angle -26.56^\circ}{5\angle 45^\circ} = \boxed{0.45\angle -71.56^\circ\text{ V}}$

(b) $\dfrac{6\angle 20^\circ}{1000} - j\text{ V} = 0.00564 - j1 = \boxed{1\angle -89.67^\circ\text{ V}}$

(c) $(j)(52.5\angle -90^\circ)\text{V} = \boxed{52.5\angle 0^\circ\text{ V}}$

29. (a) $\boxed{0; 11}$

(b) $\boxed{-11; 0}$

(c) $\boxed{0; 11}$

(d) $\boxed{-3; -3}$

30. (a) $v(t) = 9\cos(100\pi t + 65°)$ V

At t = 10 ms: $v(t) = 9\cos 245° = \boxed{-3.8 \text{ V}}$

At t = 25 ms: $v(t) = 9\cos 515° = \boxed{-8.16 \text{ V}}$

(b) $v(t) = 2\cos(100\pi t + 31°)$ V

At t = 10 ms: $v(t) = 2\cos 211° = \boxed{-1.71 \text{ V}}$

At t = 25 ms: $v(t) = 2\cos 481° = \boxed{-1.03 \text{ V}}$

(c) $v(t) = 22\cos(100\pi t + 14°) - 8\cos(100\pi t + 33°)$ V

At t = 10 ms: $v(t) = 22\cos 194° - 8\cos 213° = \boxed{-14.64 \text{ V}}$

At t = 25 ms: $v(t) = 22\cos 464° - 8\cos 483° = \boxed{-0.97 \text{ V}}$

31. (a) $\boxed{2\angle 0^{o}}$

(b) $\boxed{400\angle -90^{o}\ \text{mV}}$

(c) $\boxed{10\angle 90^{o}\ \mu\text{V}}$

32. (a) Phasor current through the resistor:

Using ohm's law, we get:

$$\mathbf{V}_R = \mathbf{I}R$$

$$\mathbf{I} = \frac{\mathbf{V}_R}{R} = \boxed{1\angle 30^\circ \text{ A}}$$

(b) At ω=1 rad/s, the voltage across the capacitor-inductor combination is 0 as their equivalent impedance is 0.

$$Z_{eq} = -j + j = 0$$

$$\therefore \frac{\mathbf{V}_R}{\mathbf{V}_{C-L}} = \boxed{\infty}$$

(c) At ω = 2 rad/s,

$$Z_{eq} = -j0.5 + j2 = j1.5$$

$$\mathbf{V}_{C-L} = (1\angle 30^\circ)(1.5\angle 90^\circ) = 1.5\angle 120^\circ \text{ V}$$

$$\therefore \frac{\mathbf{V}_R}{\mathbf{V}_{C-L}} = \frac{1\angle 30^\circ}{1.5\angle 120^\circ} = \boxed{0.67\angle -90^\circ}$$

33. (a) $\boxed{20\angle 0^\circ \text{ mV}}$

(b) $\boxed{31.8\angle -90^\circ\ \mu\text{V}}$

(c) $\boxed{3.14\angle 90^\circ \text{ V}}$

(d) $\boxed{20\angle -0.1^\circ \text{ V}}$

(e) $\boxed{3.14\angle 89.6^\circ}$

(f) $\boxed{\text{20 mV; 0; 0; 20 mV; 21.9 mV}}$

34. (a) Given:

$$\mathbf{I}_{10} = 2\angle 42° \text{ mA}$$

$$\mathbf{V} = 40\angle 132° \text{ mV}$$

$$\omega = 1000 \text{ rad / s}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}_{10}} = \frac{40\angle 132°}{2\angle 42°} = 20\angle 90° \ \Omega$$

The phase angle of 90degrees shows that it is an $\boxed{\text{inductor.}}$

(b) $\omega = 1000$ rad/s

$$Z_L = j\omega L = j20\,\Omega$$

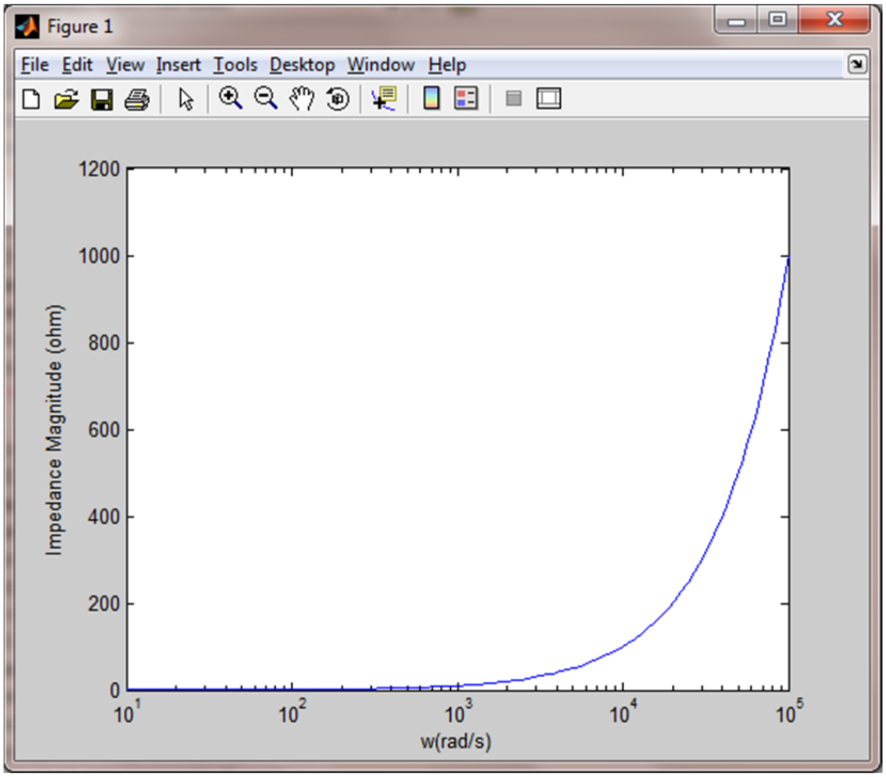
$$\Rightarrow L = \boxed{20\text{ mH}}$$

35. (a) $\boxed{2.5\ \Omega}$; (b) $\boxed{50\angle 35^{o}}$; $\boxed{100\angle 35^{o}}$

36. (a) The equivalent impedance of a 1Ω resistor in series with a 10mH inductor as a function of ω is given by,

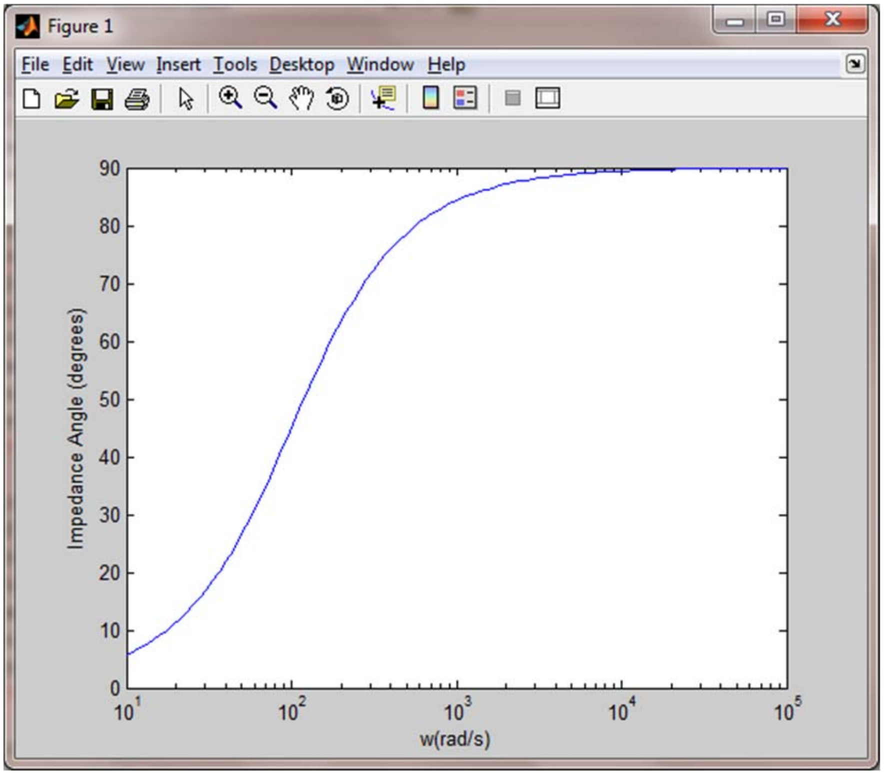
$$\mathbf{Z}_{eq} = R + j\omega L = \boxed{1 + j\omega 0.01\ \ \Omega}$$

(b)




```
w = logspace(1,5,100);
Z = 1+i*w*0.01;
mag = abs(Z);
semilogx(w, mag);
xlabel('w(rad/s)');
ylabel('Impedance Magnitude (ohm)');
```

(c)




```
w = logspace(1,5,100);
Z = 1+i*w*0.01;
theta = angle(Z);
theta_degrees = angledim(theta,'radians','degrees');
semilogx(w, theta_degrees);
xlabel('w(rad/s)');
ylabel('Impedance Angle (degrees)');
```

37. (a) $\boxed{1001\angle -2.9^{o}\ \Omega}$

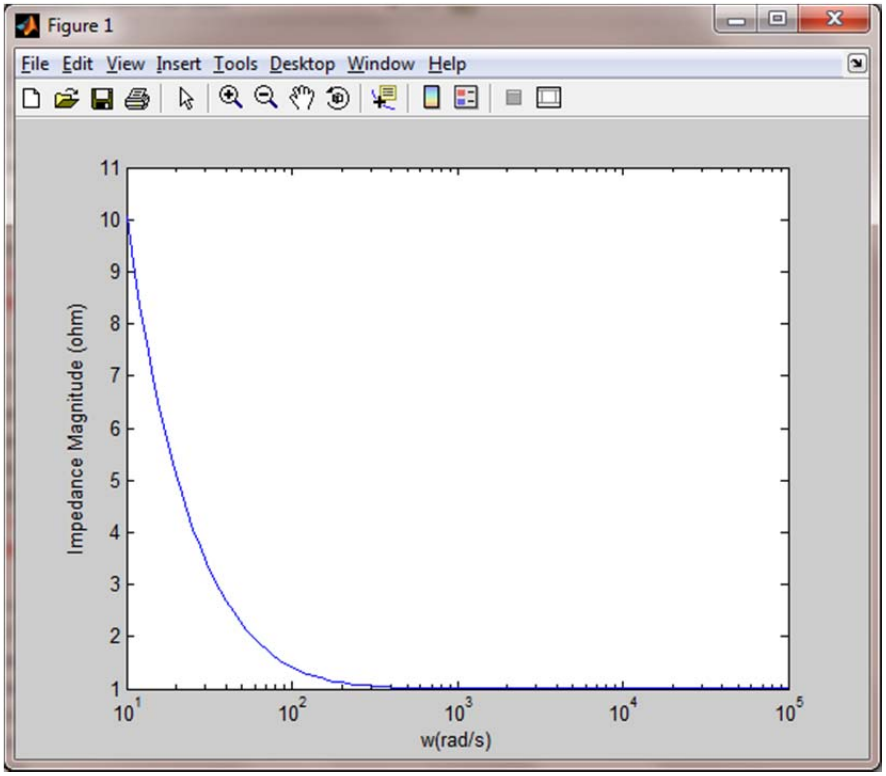
(b) $\boxed{20\angle 90^{o}\ \Omega}$

(c) $\boxed{20\angle 88.8^{o}\ \Omega}$

38. (a) The equivalent impedance of a 1Ω resistor in series with a 10mF capacitor as a function of ω is given by,

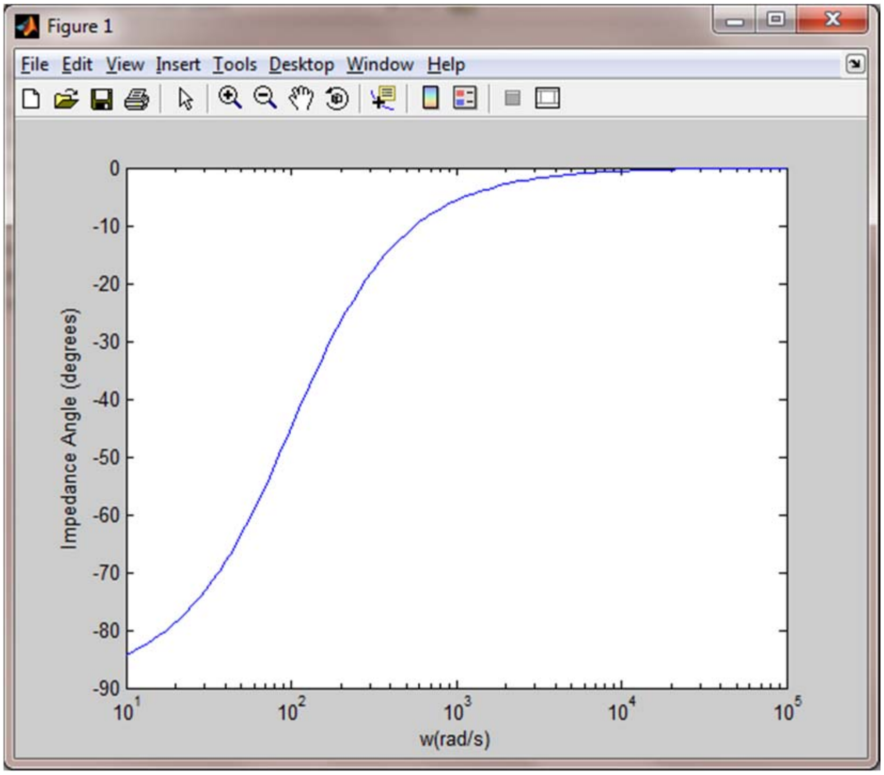
$$\mathbf{Z}_{eq} = R - \frac{j}{\omega C} = \boxed{1 - \frac{j100}{\omega}\ \Omega}$$

(b)




```
w = logspace(1,5,100);
Z = 1-i*100*w.^-1;
mag = abs(Z);
semilogx(w, mag);
xlabel('w(rad/s)');
ylabel('Impedance Magnitude (ohm)');
```

(c)




```
w = logspace(1,5,100);
Z = 1-i*100*w.^-1;
theta = angle(Z);
theta_degrees = angledim(theta,'radians','degrees');
semilogx(w, theta_degrees);
xlabel('w(rad/s)');
ylabel('Impedance Angle (degrees)');
```

39. (a) $\boxed{31.2\angle -38.7^\circ \text{ mS}}$

(b) $\boxed{64.0\angle -51.3^\circ \text{ mS}}$

(c) $\boxed{20\angle 89.9^\circ \text{ S}}$

(d) $\boxed{1\angle -89.9^\circ \text{ mS}}$

(e) $\boxed{1000\angle 89.9^\circ \text{ S}}$

40. Looking into the open terminals we see that the parallel combination of 20 mH and 55 Ω is in series with the series combination of 10 mF and 20Ω , this combination is in parallel with 25 Ω.

(a) ω = 1 rad/s

$$\mathbf{Z}_L = j\omega L = j0.02\,\Omega$$

$$\mathbf{Z}_C = -\frac{j}{\omega C} = -j100\,\Omega$$

$$\mathbf{Z}_{eq} = \frac{\left(\dfrac{55\times j0.02}{55+j0.02}+20-j100\right)\times 25}{\left(\dfrac{55\times j0.02}{55+j0.02}+20-j100\right)+25} = 22.66 - j5.19 = \boxed{23.24\angle -12.9^\circ\,\Omega}$$

(b) ω = 10 rad/s

$$\mathbf{Z}_L = j\omega L = j0.2\,\Omega$$

$$\mathbf{Z}_C = -\frac{j}{\omega C} = -j10\,\Omega$$

$$\mathbf{Z}_{eq} = \frac{\left(\dfrac{55\times j0.2}{55+j0.2}+20-j10\right)\times 25}{\left(\dfrac{55\times j0.2}{55+j0.2}+20-j10\right)+25} = 11.74 - j2.88 = \boxed{12.08\angle -13.78^\circ\,\Omega}$$

(c) ω = 100 rad/s

$$\mathbf{Z}_L = j\omega L = j2\,\Omega$$

$$\mathbf{Z}_C = -\frac{j}{\omega C} = -j\,\Omega$$

$$\mathbf{Z}_{eq} = \frac{\left(\dfrac{55\times j2}{55+j2}+20-j\right)\times 25}{\left(\dfrac{55\times j2}{55+j2}+20-j\right)+25} = 11.14 - j0.30 = \boxed{11.14\angle -1.54^\circ\,\Omega}$$

41. $\boxed{11.3\angle -5.3^{\circ}\ \Omega}$

42. (a) 3Ω in series with 2mH

$$\mathbf{Z}_{eq} = 3 + j4 = 5\angle 53.13°\Omega$$

$$\mathbf{V} = \mathbf{IZ} = (3\angle -20°)(5\angle 53.13°) = \boxed{15\angle -33.13°\text{ V}}$$

(b) 3Ω in series with 125μF

$$\mathbf{Z}_{eq} = 3 - j4 = 5\angle -53.13°\Omega$$

$$\mathbf{V} = \mathbf{IZ} = (3\angle -20°)(5\angle -53.13°) = \boxed{15\angle -73.13°\text{ V}}$$

(c) 3Ω, 2mH, and 125μF in series

$$\mathbf{Z}_{eq} = 3 + j4 - j4 = 3\angle 0°\Omega$$

$$\mathbf{V} = \mathbf{IZ} = (3\angle -20°)(3) = \boxed{9\angle -20°\text{ V}}$$

(d) 3Ω, 2mH and 125μF in series but ω = 4 krad/s

$$\mathbf{Z}_{eq} = 3 + j8 - j2 = 6.71\angle 63.44°\Omega$$

$$\mathbf{V} = \mathbf{IZ} = (3\angle -20°)(6.71\angle 63.44°) = \boxed{20.13\angle -43.44°\text{ V}}$$

43. (a) $\boxed{30 - j0.154\ \Omega}$

(b) $\boxed{23.5 + j9.83\ \Omega}$

(c) $\boxed{30 + j0.013\ \Omega}$

(d) $\boxed{30 + j1.3\times10^{-5}\ \Omega}$

(e) $\boxed{30 + 1.3\times10^{-8}\ \Omega}$

44. One method is to use the current divider rule in order to calculate i(t). In the given circuit, there are three parallel branches.

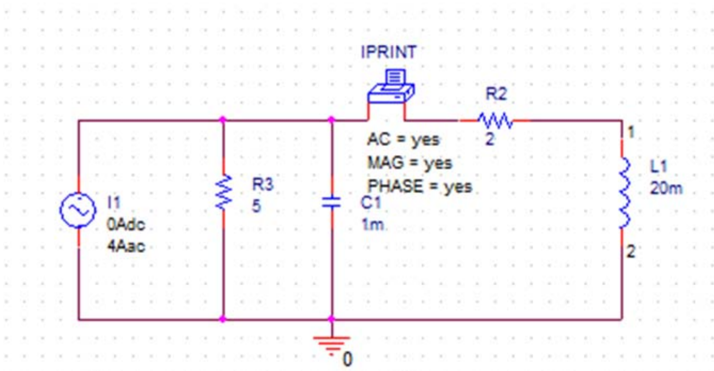
$$\mathbf{Z}_{eq} = \frac{1}{5^{-1} + \left(-j10\right)^{-1} + \left(2 + j2\right)^{-1}} = 2 + j0.67 = 2.11\angle 18.52°\ \Omega$$

$$\mathbf{Z} = \left(2 + j2\right)\Omega = 2.83\angle 45°\Omega$$

$$\mathbf{I} = \mathbf{I}_s \frac{\mathbf{Z}_{eq}}{\mathbf{Z}} = \frac{\left(4\angle -20°\right)\left(2.11\angle 18.52°\right)}{2.83\angle 45°} = 2.98\angle -46.48°\ \text{A}$$

$$\boxed{\therefore i(t) = 2.98\cos(100t - 46.48°)\ \text{A}}$$

PSpice Verification:




```
FREQ          IM(V_PRINT1)IP(V_PRINT1)

 1.592E+01    2.981E+00   -4.656E+01
```

45. (a) *One possible solution*: $\boxed{\text{A 1 }\Omega\text{ resistor in series with 1 H and }10^{-4}\text{ F.}}$

(b) *One possible solution*: $\boxed{\text{A 6.894 }\Omega\text{ resistor in series with 11.2 mH.}}$

(c) *One possible solution*: $\boxed{\text{A 3 }\Omega\text{ resistor in series with 2.5 mF.}}$

46. One out of many possible design solutions:

(a) At $\omega = 10$ rad/s, the equivalent admittance is given as, $\mathbf{Y} = 1$ S. We can construct this using a 1 S conductance (1Ω resistor) in parallel with an inductor L and a capacitor C such that $\omega C - \frac{1}{\omega L} = 0$. Selecting L as 5H arbitrarily yields the value of a capacitor as 2mF.

$\boxed{\text{Thus, one design can be 1Ω resistor in parallel with 5H inductor and 2mF capacitor.}}$

(b) At $\omega = 10$ rad/s, the equivalent admittance is given as,

$\mathbf{Y} = 12\angle -18°\ \text{S} = 11.4127 - j3.7082\ \text{S}$. We can construct this using a 11.4127 S conductance (87.6 mΩ resistor) in parallel with an inductor L such that

$-\frac{j}{\omega L} = -j3.7082\ \text{S}$. This yields the value of the inductor as 26.9 mH.

$\boxed{\text{Thus, one design can be 87.6 mΩ resistor in parallel with 26.9 mH inductor.}}$

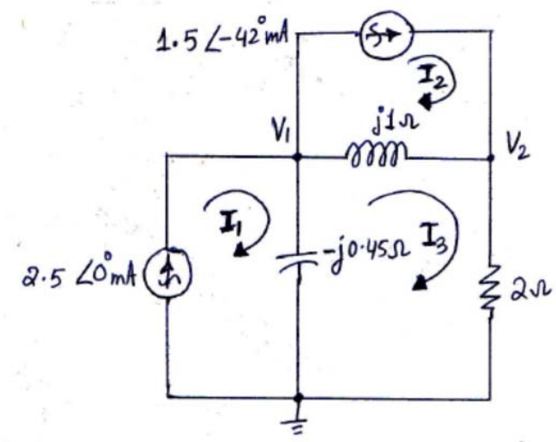
(c) At $\omega = 10$ rad/s, the equivalent admittance is given as, $\mathbf{Y} = 2 + j\ \text{mS}$. We can construct this using a 2 mS conductance (500Ω resistor) in parallel with a capacitor C such that $j\omega C = j0.001\ \text{S}$. This yields the value of the capacitor as 0.1 mF.

$\boxed{\text{Thus, one design can be 500 Ω resistor in parallel with 0.1 mF capacitor.}}$

47. **BOTH SOURCES ARE SUPPOSED TO OPERATE AT 100 rad/s**. Then,

$\boxed{v_1(t) = 2.56\cos(100t + 139.2^\circ)\text{ V}}$; $\boxed{v_2(t) = 4.35\cos(100t + 138.3^\circ)\text{ V}}$.

48. (a)



(b) In mesh 1, we have $\mathbf{I}_1 = 2.5\angle 0°$ mA .

In mesh 2, we have $\mathbf{I}_2 = 1.5\angle -42°$ mA .

In mesh 3, we have,

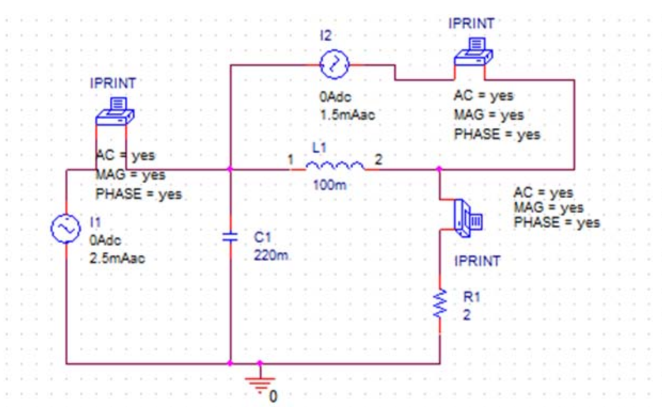
$$(\mathbf{I}_3 - \mathbf{I}_1)\mathbf{Z}_C + (\mathbf{I}_3 - \mathbf{I}_2)\mathbf{Z}_L + 2\mathbf{I}_3 = 0$$

$$\mathbf{I}_3 = \frac{\mathbf{I}_1\mathbf{Z}_C + \mathbf{I}_2\mathbf{Z}_L}{2 + \mathbf{Z}_C + \mathbf{Z}_L} = \frac{2.5\times 10^{-3}\times(-j0.4545) + (1.1147 - j1.0037)\times 10^{-3}\times j}{2 - j0.4545 + j}$$

$$= \frac{1.004\angle -1.23°}{2.073\angle 15.25°} = 0.4843\angle -16.48° \text{ mA}$$

$$\boxed{\begin{aligned} &\therefore i_1(t) = 2.5\cos 10t \text{ mA} \\ &\therefore i_2(t) = 1.5\cos(10t - 42°) \text{ mA} \\ &\therefore i_3(t) = 0.4843\cos(10t - 16.48°) \text{ mA} \end{aligned}}$$

Pspice Verification:



```
FREQ        IM(V_PRINT1)IP(V_PRINT1)

1.592E+00   4.843E-04  -1.649E+01
*** 06/16/12 23:24:09 ******* PSpice Lite (April 2011) ******* ID# 10813 ****
** Profile: "SCHEMATIC1-prob10_48"  [ C:\ORCAD\ORCAD_16.5_LITE\examples\prob1(

****     AC ANALYSIS                     TEMPERATURE =   27.000 DEG C

******************************************************************************

FREQ        IM(V_PRINT2)IP(V_PRINT2)

1.592E+00   2.500E-03   2.120E-11
*** 06/16/12 23:24:09 ******* PSpice Lite (April 2011) ******* ID# 10813 ****
** Profile: "SCHEMATIC1-prob10_48"  [ C:\ORCAD\ORCAD_16.5_LITE\examples\prob1(

****     AC ANALYSIS                     TEMPERATURE =   27.000 DEG C

******************************************************************************

FREQ        IM(V_PRINT3)IP(V_PRINT3)

1.592E+00   1.500E-03  -4.200E+01
```

49. $\boxed{v_1(t) = 928\cos(10t - 86.1^\circ)\ \mu V}$; $\boxed{v_2(t) = 969\cos(10t - 16.5^\circ)\ \mu V}$.

50. In the circuit given by Fig. 10.60, we have,

$$\mathbf{V}_1 - \mathbf{I}_1(j30) = \mathbf{V}_2 + 55(\mathbf{I}_1 - \mathbf{I}_2) \text{ and}$$
$$\mathbf{V}_1 - \mathbf{I}_1(j30) = \mathbf{V}_3 + \mathbf{I}_2(-j20)$$

On simplification. we get,

$$\mathbf{I}_1(55 + j30) - 55\mathbf{I}_2 = -2.2635 + j9.848$$
$$\mathbf{I}_1(j30) - \mathbf{I}_2(j20) = -0.1045 + j9.0665$$

Solving for I_1 and I_2, we get,

$$\mathbf{I}_1 = 0.6247 + j0.3339 = \boxed{0.71\angle 28.12° \text{ A}}$$
$$\mathbf{I}_2 = 0.4838 + j0.4956 = \boxed{0.69\angle 45.69° \text{ A}}$$

51. $\boxed{0.809\angle -4.8^{o}}$

52. Using phasor domain, in mesh 1, we get,

$$2\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) = 2.5\angle 9°$$

$$\Rightarrow \mathbf{I}_1(2 + j10) - \mathbf{I}_2(j10) = 2.4692 + j0.3911 \qquad [1]$$

In mesh 2, we get,

$$j10(\mathbf{I}_2 - \mathbf{I}_1) - j0.3\mathbf{I}_2 + 5\mathbf{I}_1 = 0$$

$$\Rightarrow \mathbf{I}_1(5 - j10) + \mathbf{I}_2(j9.7) = 0 \qquad [2]$$

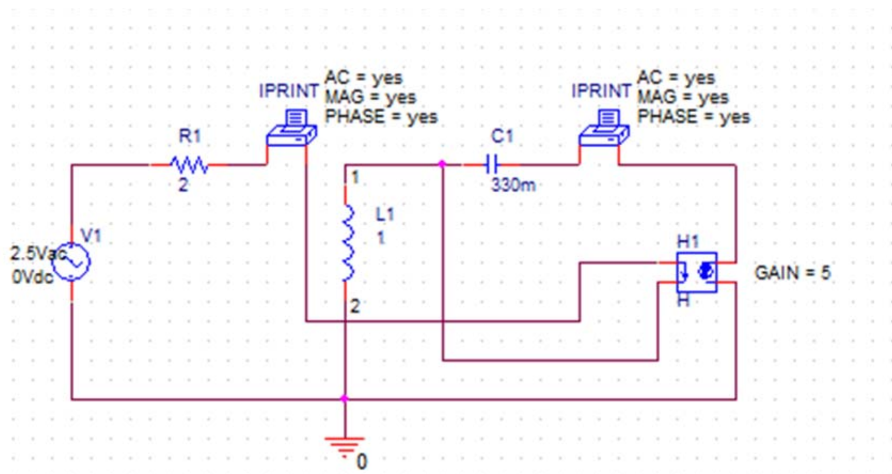
On solving eqns [1] and [2] we get,

$$\mathbf{I}_1 = 0.3421 + j0.0695 = 0.35\angle 11.48° \text{ A}$$

$$\mathbf{I}_2 = 0.3169 + j0.2479 = 0.4\angle 38.04° \text{A}$$

$$\boxed{\begin{aligned}&\therefore i_1(t) = 0.35\cos(10t + 11.48°) \text{ A and}\\ &i_2(t) = 0.4\cos(10t + 38.04°) \text{ A}\end{aligned}}$$

Pspice verification:



```
FREQ          IM(V_PRINT1)IP(V_PRINT1)

 1.592E+00   3.490E-01   1.150E+01
*** 06/17/12 14:13:22 ******* PSpice Lit
** Profile: "SCHEMATIC1-prob10_52"  [ C:

****     AC ANALYSIS

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FREQ          IM(V_PRINT2)IP(V_PRINT2)

 1.592E+00   4.024E-01   3.807E+01
```

53. $\boxed{2.73\angle 152^{o}\ \mathrm{A}}$

54. Using node voltage analysis in phasor domain, we get the nodal equations as,

$$\mathbf{I}_1 + \frac{\mathbf{V}_1}{j2} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j4} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{1 + j3.8} = 0 \qquad [1]$$

$$\mathbf{I}_2 - \frac{\mathbf{V}_2}{2} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j4} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{1 + j3.8} = 0 \qquad [2]$$

$$\mathbf{I}_1 = 15\angle 0° = 15A$$
$$\mathbf{I}_2 = 25\angle 131° = -16.4015 + j18.8677A$$

On simplifying the equations [1] and [2], we get,

$$\mathbf{V}_1(0.0648 - j0.4961) + \mathbf{V}_2(-0.0648 - j0.0039) = -15$$
$$\mathbf{V}_1(0.0648 + j0.0039) + \mathbf{V}_2(-0.5648 - j0.0039) = -16.4015 + j18.8677$$

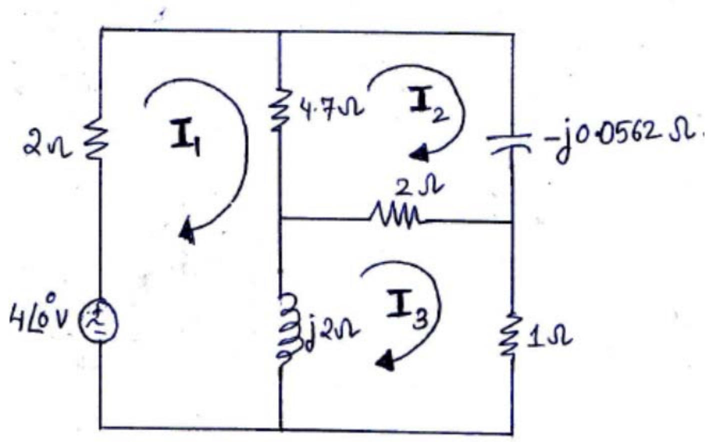
$$\mathbf{V}_2 = -29.5221 + j29.7363 = \boxed{41.9\angle 134.8° \text{ V}}$$

Matlab Verification:

```
>> syms v1 v2;
eqn1 = (15+v1/(2i)+(v1-v2)/(-4i)+(v1-v2)/(1+3.8i));
eqn2 = (-16.4015+18.8677i+(v1-v2)/(1+3.8i)+(v1-v2)/(-4i)-v2/2);
answer=solve(eqn1, eqn2, 'v1', 'v2');
digits(4);
V2 = vpa(answer.v2)
V2 = -29.52+29.74*i
```

55. $\boxed{1.14\cos(20t + 12^{o})\ \text{V}}$

56.



Using phasor domain, in mesh 1, we get,

$$2\mathbf{I}_1 + 4.7(\mathbf{I}_1 - \mathbf{I}_2) + j2(\mathbf{I}_1 - \mathbf{I}_2) = 4$$

$$\Rightarrow (6.7 + j2)\mathbf{I}_1 - 4.7\mathbf{I}_2 - j2\mathbf{I}_3 = 4 \qquad [1]$$

In mesh 2, we get,

$$4.7(\mathbf{I}_2 - \mathbf{I}_1) - j0.0562\mathbf{I}_2 + 2(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$\Rightarrow -4.7\mathbf{I}_1 + (6.7 - j0.0562)\mathbf{I}_2 - 2\mathbf{I}_3 = 0 \qquad [2]$$

In mesh 3, we get,

$$j2(\mathbf{I}_3 - \mathbf{I}_1) + 2(\mathbf{I}_3 - \mathbf{I}_2) + \mathbf{I}_3 = 0$$

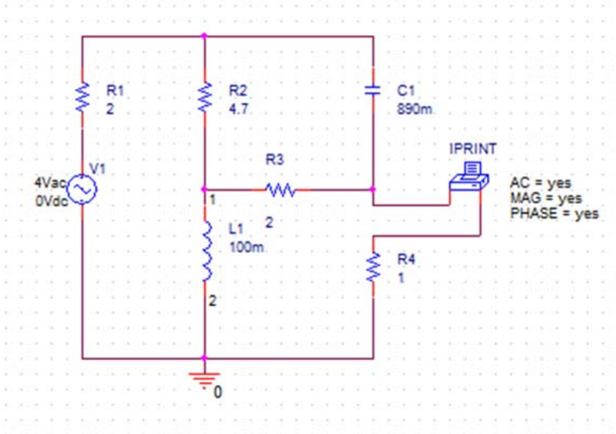
$$\Rightarrow -j2\mathbf{I}_1 - 2\mathbf{I}_2 + (3 + j2)\mathbf{I}_3 = 0 \qquad [3]$$

Here, $\mathbf{I}_x = \mathbf{I}_3$. On solving we get,

$$\mathbf{I}_x = \mathbf{I}_3 = 1.1104 + j0.2394 = 1.136\angle 12.16° \text{ A}$$

$$\boxed{\therefore i_x(t) = 1.136\cos(20t + 12.16°) \text{ A}}$$

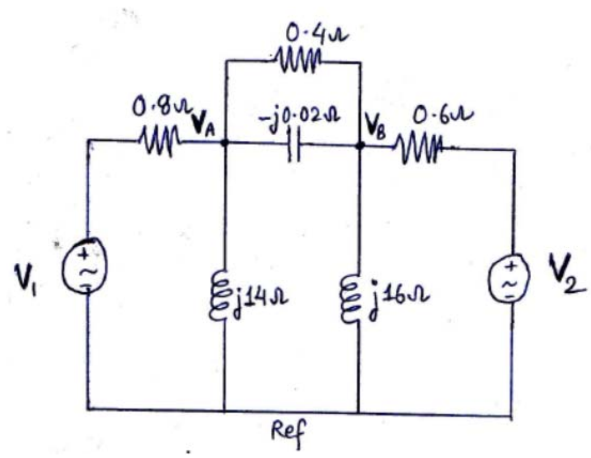
Pspice Verification:



FREQ	IM(V_PRINT1)	IP(V_PRINT1)
3.183E+00	1.136E+00	1.216E+01

57. $\boxed{155\cos(14t + 37^{o})\text{ A}}$; $\boxed{82.2\cos(14t = 101^{o})\text{ A}}$; $\boxed{42.0\cos(14t = 155^{o})\text{ A}}$; $\boxed{71.7\cos(14t + 50^{o})\text{ A}}$.

58.



Using node voltage analysis in phasor domain, we get the nodal equations as,

At node A,

$$\frac{(\mathbf{V}_1-\mathbf{V}_A)}{0.8}-\frac{(\mathbf{V}_A-\mathbf{V}_B)}{0.4}-\frac{(\mathbf{V}_A-\mathbf{V}_B)}{-j0.02}-\frac{\mathbf{V}_A}{j14}=0 \qquad [1]$$

At node B,

$$\frac{(\mathbf{V}_2-\mathbf{V}_B)}{0.6}+\frac{(\mathbf{V}_A-\mathbf{V}_B)}{0.4}+\frac{(\mathbf{V}_A-\mathbf{V}_B)}{-j0.02}-\frac{\mathbf{V}_B}{j16}=0 \qquad [2]$$

Given,

$$\mathbf{V}_1=0.009\angle 0.5^\circ=0.009+j0.000078\text{ V}$$
$$\mathbf{V}_2=0.004\angle 1.5^\circ=0.004+j0.0001\text{ V}$$

On simplifying the nodal equations [1] and [2], we get,

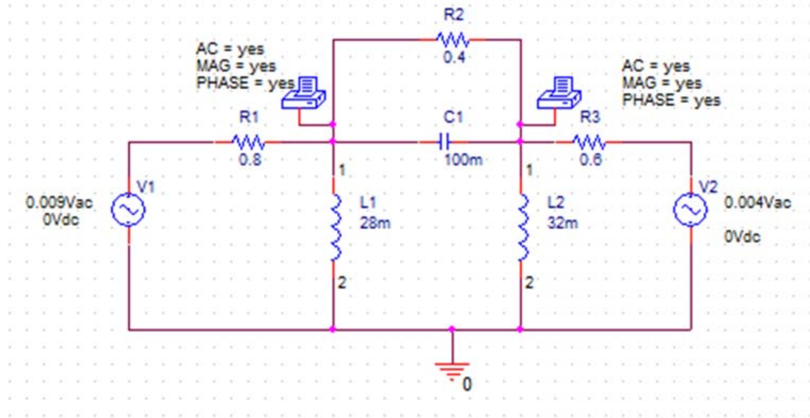
$$\mathbf{V}_A(-3.75-j49.9286)+\mathbf{V}_B(2+j50)=-\frac{0.009+j0.000078}{0.8}$$
$$\mathbf{V}_A(2+j50)+\mathbf{V}_2(-4.1667-j49.9375)=-\frac{0.004+j0.0001}{0.6}$$

and on solving, we get,

$$\mathbf{V}_A=0.00613+j0.00033=0.00613\angle 3.09^\circ\text{ V}$$
$$\mathbf{V}_B=0.00612+j0.00040=0.00613\angle 3.75^\circ\text{ V}$$

$$\boxed{\begin{array}{l}\therefore v_A=0.00613\cos(500t+3.09^\circ)\text{ V and}\\ v_B=0.00613\cos(500t+3.75^\circ)\text{ V}\end{array}}$$

Pspice Verification:




```
FREQ        VM(N00227)  VP(N00227)

 7.957E+01   6.138E-03   3.122E+00
*** 06/18/12 09:52:20 ******* PSpice Li
** Profile: "SCHEMATIC1-prob10_58"  [ C

****     AC ANALYSIS

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FREQ        VM(N00296)  VP(N00296)

 7.957E+01   6.136E-03   3.787E+00
```

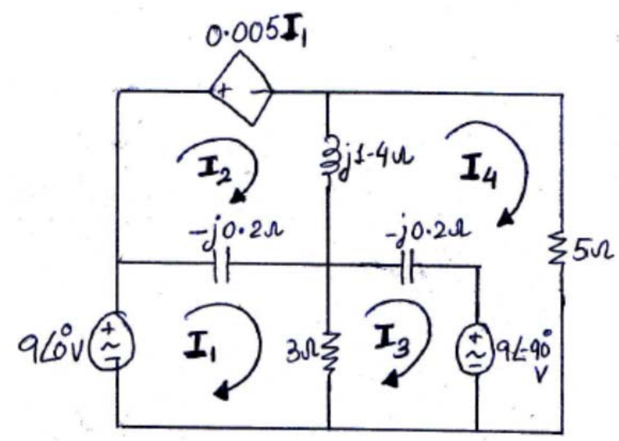
59. (a)

$$\boxed{\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-R_f}{\dfrac{R_f - j/\omega C_1}{A} - \dfrac{j}{\omega C_1}}}$$

(b)

$$\boxed{\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\omega R_f C_1}{\frac{1}{A}\left[\omega R_f C_1 + (R_f C_f - j)\right] + (\omega R_f C_f - j)}}$$

60.



Using phasor domain, in mesh 1, we get,

$$(-j0.2)(\mathbf{I}_1-\mathbf{I}_2)+3(\mathbf{I}_1-\mathbf{I}_3)=9$$

$$\Rightarrow (3-j0.2)\mathbf{I}_1+j0.2\mathbf{I}_2-3\mathbf{I}_3=9 \qquad [1]$$

In mesh 2, we get,

$$0.005\mathbf{I}_1+j1.4(\mathbf{I}_2-\mathbf{I}_3)-j0.2(\mathbf{I}_2-\mathbf{I}_1)=0$$

$$\Rightarrow (0.005+j0.2)\mathbf{I}_1+j1.2\mathbf{I}_2-j1.4\mathbf{I}_4=0 \qquad [2]$$

In mesh 3, we get,

$$-j0.2(\mathbf{I}_3-\mathbf{I}_4)+3(\mathbf{I}_3-\mathbf{I}_1)+\mathbf{I}_3-j9=0$$

$$\Rightarrow -3\mathbf{I}_1+(3-j0.2)\mathbf{I}_3+j0.2\mathbf{I}_4=j9 \qquad [3]$$

In mesh 4, we get,

$$5\mathbf{I}_4-j0.2(\mathbf{I}_4-\mathbf{I}_3)+j1.4(\mathbf{I}_4-\mathbf{I}_2)=-j9$$

$$\Rightarrow -j1.4\mathbf{I}_2+j0.2\mathbf{I}_3+\mathbf{I}_4(5+j1.2)=-j9 \qquad [4]$$

On solving we get,

$$\mathbf{I}_1=-18.33+j20=27.13\angle 132.5^\circ \text{ A}$$

$$\mathbf{I}_2=5.092-j3.432=6.14\angle -33.98^\circ \text{ A}$$

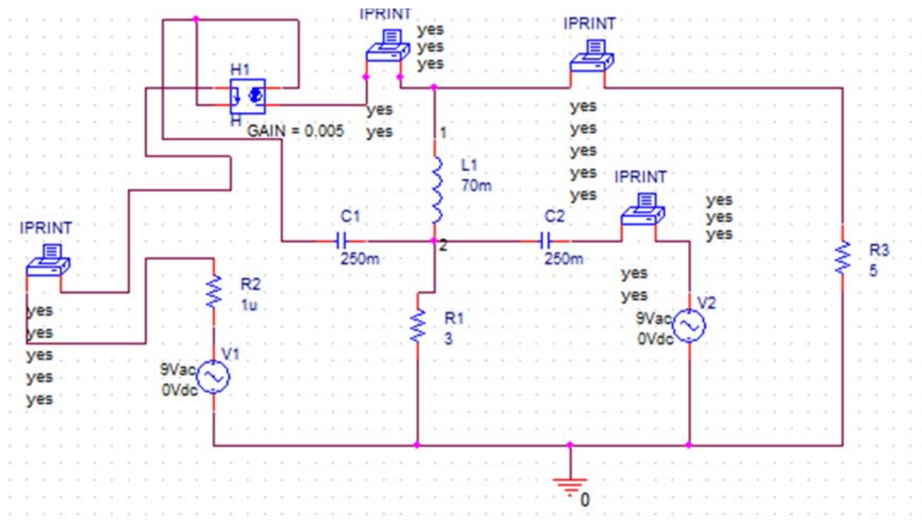
$$\mathbf{I}_3=-19.76+j21.55=29.24\angle 132.5^\circ \text{ A}$$

$$\mathbf{I}_4=1.818-j0.02=1.82\angle -0.63^\circ \text{ A}$$

Therefore,

$$\boxed{\begin{array}{l} i_1(t) = 27.13\cos(20t + 132.5°)\ \text{A} \\ i_2(t) = 6.14\cos(20t - 33.98°)\ \text{A} \\ i_3(t) = 29.24\cos(20t + 132.5°)\ \text{A} \\ i_4(t) = 1.82\cos(20t - 0.63°)\ \text{A} \end{array}}$$

Pspice Verification:




```
 FREQ        IM(V_PRINT1)IP(V_PRINT1)IR(V_PRINT1)II(V_PRINT1)

  3.183E+00   2.712E+01   1.325E+02  -1.833E+01   1.999E+01
*** 06/21/12 13:48:06 ******* PSpice Lite (April 2011) ******* I
** Profile: "SCHEMATIC1-prob10_60_old"  [ C:\ORCAD\ORCAD_16.5_LI

****     AC ANALYSIS                      TEMPERATURE =   27.000

******************************************************************

 FREQ        IM(V_PRINT3)IP(V_PRINT3)IR(V_PRINT3)II(V_PRINT3)

  3.183E+00   3.052E+01   1.350E+02  -2.158E+01   2.157E+01
```

```
 FREQ        IM(V_PRINT4)IP(V_PRINT4)IR(V_PRINT4)II(V_PRINT4)

  3.183E+00   1.818E+00  -6.301E-01   1.818E+00  -2.000E-02
*** 06/21/12 13:48:06 ******* PSpice Lite (April 2011) *******
** Profile: "SCHEMATIC1-prob10_60_old"  [ C:\ORCAD\ORCAD_16.5_

****     AC ANALYSIS                     TEMPERATURE =   27.0

*************************************************************

 FREQ        IM(V_PRINT5)IP(V_PRINT5)IR(V_PRINT5)II(V_PRINT5)

  3.183E+00   6.141E+00  -3.398E+01   5.092E+00  -3.432E+00
```

61. Left hand source contributions: $\boxed{5.58\angle -91.8^{\circ}\ \text{V};\ 1.29\angle -75.9^{\circ}\ \text{V}}$

Right hand source contributions: $\boxed{1.29\angle -75.9^{\circ}\ \text{V};\ 9.08\angle -115^{\circ}\ \text{V}}$

62. Using node voltage analysis in phasor domain, we get the nodal equations as,

At node 1,

$$\mathbf{I}_1 - \mathbf{I}_2 - \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j5} - \frac{\mathbf{V}_1}{j3} = 0 \qquad [1]$$

At node 2,

$$\mathbf{I}_2 + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j5} - \frac{\mathbf{V}_2}{2} = 0 \qquad [2]$$

$$\mathbf{I}_1 = 33\times10^{-3}\angle 3^\circ \text{ mA}$$
$$\mathbf{I}_2 = 51\times10^{-3}\angle -91^\circ \text{ mA}$$

On simplifying the nodal equations [1] and [2], we get,

$$\mathbf{V}_1(j0.1333) + \mathbf{V}_2(-j0.2) = \mathbf{I}_2 - \mathbf{I}_1$$
$$\mathbf{V}_1(j0.2) + \mathbf{V}_2(-0.5 - j0.2) = -\mathbf{I}_2$$

and on solving these we get,

$$\boxed{\begin{aligned}\mathbf{V}_1 &= -0.4694 - j0.1513 = 493.18\angle -162.14^\circ \text{ mV}\\ \mathbf{V}_2 &= -0.0493 - j0.27 = 274.46\angle -100.34^\circ \text{ mV}\end{aligned}}$$

63. $\boxed{7.995\cos(40t + 2.7^o) + 0.343\cos(30t + 90.1^o) \text{ mV}}$;

$\boxed{7.995\cos(40t + 2.4^o) + 1.67\cos(30t - 180^o) \text{ mV}}$

64. Calculate Thevenin Impedance

$$Z_{thevenin} = j2 + 4\angle 10° = 3.94 + j2.69 = \boxed{4.77\angle 34.32°\ \Omega}$$

Calculate Thevenin voltage:

$$\mathbf{V}_1 = (1.5\angle 24°)(j2) = (1.5\angle 24°)(2\angle 90°) = 3\angle 114°\ \text{V}$$

$$\mathbf{V}_2 = -(2\angle 38°)(4\angle 10°) = -8\angle 48°\ \text{V}$$

$$\mathbf{V}_{TH} = \mathbf{V}_1 - \mathbf{V}_2 = (3\angle 114°) + (8\angle 48°) = 4.13 + j8.68 = \boxed{9.61\angle 64.55°\ \text{V}}$$

Current $\mathbf{I}_1$ through the impedance (2-j2) $\boldsymbol{\Omega}$ is found as:

$$\mathbf{Z}_{total} = 3.94 + j2.69 + 2 - j2 = 5.94 + j0.69 = 5.97\angle 6.626°\ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{TH}}{\mathbf{Z}_{total}} = \frac{9.61\angle 64.55°}{5.97\angle 6.63°} = \boxed{1.6\angle 57.92°\ \text{A}}$$

65. $\boxed{1.56\angle 27.8^{o} \text{ A}}$

66. (a) Thevenin Equivalent

$$\mathbf{Z}_{thevenin} = (12 - j34) \parallel (j10) = \frac{340 + j120}{12 - j24} = 1.67 + j13.33 = \boxed{13.43\angle 82.86°\ \Omega}$$

Using current divider to find the current through j10 branch,

$$\mathbf{I} = \frac{\mathbf{I}_s.\mathbf{Z}_{eq}}{\mathbf{Z}} = \frac{22\angle 30°(340 + j120)}{12 - j24}$$

$$= 29.55\angle 22.87°\ \text{A}$$

$$\mathbf{V}_{TH} = \mathbf{V}_{oc} = (29.55\angle 22.87°)(10\angle 90°) = \boxed{295.46\angle 112.87°\ \text{V}}$$

(b) Norton Equivalent

$$\mathbf{Z}_{norton} = \mathbf{Z}_{thevenin} = 1.67 + j13.33 = \boxed{13.43\angle 82.86°\ \Omega}$$

$$\mathbf{I}_N = \mathbf{I}_{sc} = 22\angle 30°\,A$$

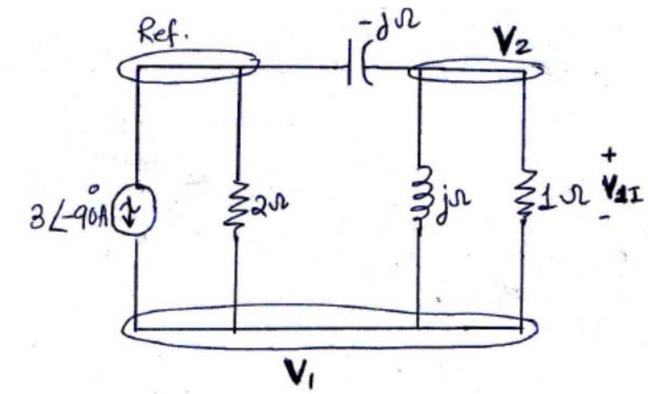
(c) Current flowing from a to b

$$\mathbf{Z}_{total} = 1.67 + j13.33 + 7 - j2 = 8.67 + j11.33 = \boxed{14.26\angle 52.57°\ \Omega}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{TH}}{\mathbf{Z}_{total}} = \frac{295.46\angle 112.87°}{14.26\angle 52.57°} = \boxed{20.72\angle 60.3°\ \text{A}}$$

67. (b) $\boxed{259\angle 84.5^{o}\ \mu V}$

68. Let us first consider the current source only.



Using node voltage analysis in phasor domain, we get the nodal equations as,

At node 1,

$$\mathbf{I}_S + \frac{(\mathbf{V}_2 - \mathbf{V}_1)}{1} = \frac{\mathbf{V}_1}{2} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{j1}$$

$$\Rightarrow \mathbf{V}_1(1.5 - j) + \mathbf{V}_2(-1 + j) = -j3 \qquad [1]$$

At node 2,

$$\frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j1} - \frac{\mathbf{V}_2}{-j} = \frac{(\mathbf{V}_2 - \mathbf{V}_1)}{1}$$

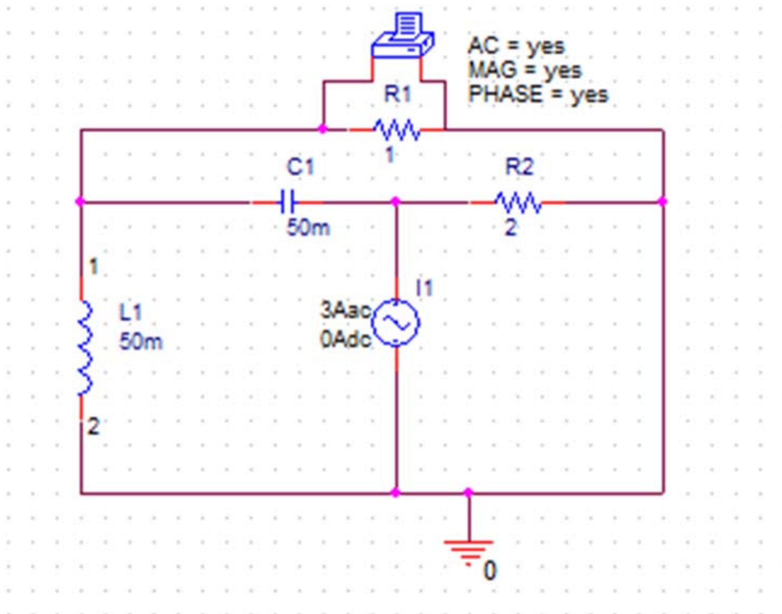
$$\Rightarrow \mathbf{V}_1(1 - j) - \mathbf{V}_2 = 0 \qquad [2]$$

$$\mathbf{I}_S = 3\angle -90° A$$

On solving the nodal equations [1] and [2], we get,

$$\mathbf{V}_{1I} = -0.9231 - j1.3846 \text{ V}$$
$$\mathbf{V}_{2I} = -2.3077 - j0.4615 \text{ V}$$
$$\mathbf{V}_1 = \mathbf{V}_{2I} - \mathbf{V}_{1I} = -1.3846 + j0.9231 = 1.66\angle 146.3° \text{ V}$$
$$\boxed{v_{1I}(t) = 1.66\cos(20t + 146.3°) \text{ V}}$$

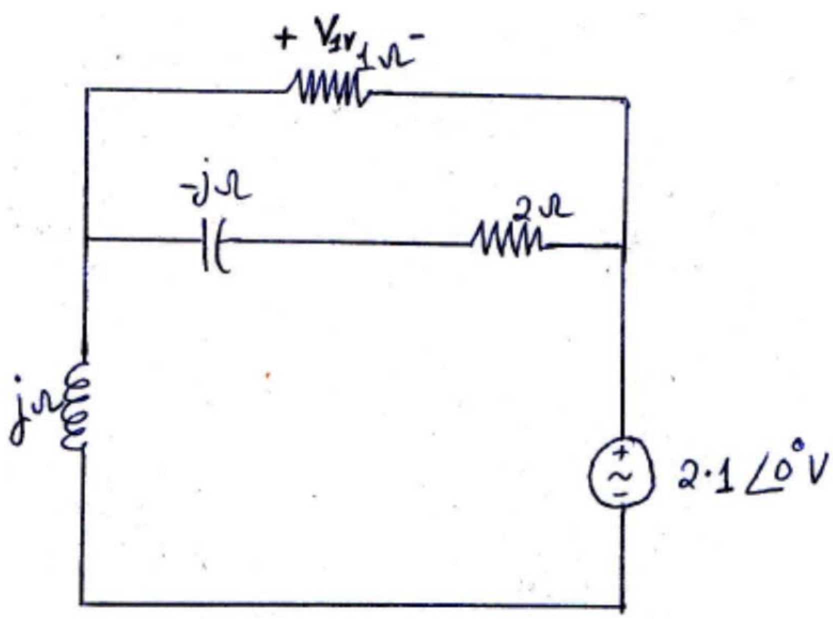
Pspice Verification:




```
FREQ          VM(N00166,0)VP(N00166,0)

 3.183E+00    1.664E+00    1.463E+02
```

Now let us consider the voltage source only.



Then the current flowing in the circuit will be,

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{2.1}{\frac{2-j}{3-j}+j} = 1.1307 - j1.4538 \text{ A}$$

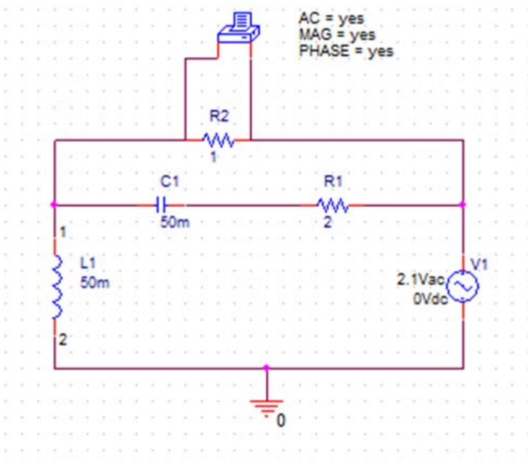
Using current divider to find the current through 1Ω branch,

$$\mathbf{I}_1 = \frac{\mathbf{I}_s.\mathbf{Z}_{eq}}{\mathbf{Z}_R} = (1.1307 - j1.4538)(0.7 - j0.1)$$

$$\mathbf{V}_{1\text{V}} = -\mathbf{I}_1.\mathbf{Z}_R = -0.6461 + j1.1307 = 1.3\angle 119.74°\ \text{V}$$

$$\boxed{v_{1\text{V}}(t) = 1.3\cos(20t + 119.74°)\ \text{V}}$$

Pspice Verification:




```
FREQ          VM(N00211,N00222)VP(N00211,N00222)

 3.183E+00    1.302E+00    1.197E+02
```

69. (a) $\boxed{24\cos^2(20t - 163^\circ)\ \text{W}}$

70. Using phasor analysis, we get the open circuit voltage as,

$\mathbf{V}_{oc} = 1\angle 0° \text{ V}$

For finding the short circuit current through terminal a-b, we can apply KVL,

$1\angle 0° = -j\left(0.25\mathbf{I}_N\left(j2\right)+\mathbf{I}_N\right)+j2\mathbf{I}_N = \left(0.5+j\right)\mathbf{I}_N$

$$\mathbf{I}_N = \frac{1\angle 0°}{\left(0.5+j\right)} = 0.4 - j0.8 = \boxed{0.89\angle -63.43° \text{ A}}$$

Now for finding the equivalent impedance,

$$\mathbf{Z}_N = \frac{\mathbf{V}_{OC}}{\mathbf{I}_N} = 0.5 + j\,\Omega$$

For a parallel combination of a resistor and a capacitor or an inductor,

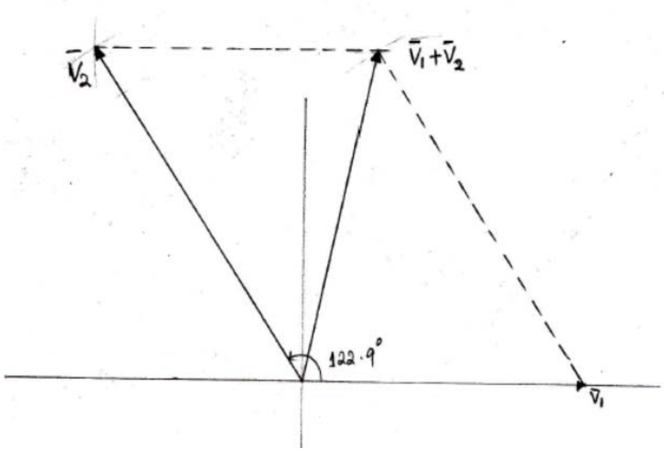
$$\mathbf{Z}_{eq} = \frac{1}{R^{-1}+(jX)^{-1}} = 0.5 + j$$

$\Rightarrow \boxed{R = \frac{1}{0.4} = 2.5\,\Omega}$ and $X = \frac{1}{0.8} = 1.25$ from which at $\omega = 1 \text{ rad/s}$, we get,

the value of $\boxed{L = 1.25\ H}$

71. (b) I_S leads I_R by $\boxed{83^o}$; I_C by $\boxed{-7^o}$; I_x by $\boxed{146^o}$

72. Taking 50 V = 1 inch, from the figure, we get the angle as $\boxed{\pm 122.9^\circ}$. (The figure below shows for the angle +122.9° only.)



Analytical Solution: On solving,

$$|100 + 140\angle\alpha| = 120$$
$$|100 + 140\cos\alpha + j140\sin\alpha| = 120$$
$$\alpha = \pm 122.88^\circ$$

73. (a) $\boxed{\mathbf{V}_R = 51.2\angle -140^{\circ}\text{ V}}$; $\boxed{\mathbf{V}_L = 143\angle 13^{\circ}\text{ V}}$; $\boxed{\mathbf{I}_L = 57\angle -85^{\circ}\text{ A}}$; $\boxed{\mathbf{I}_C = 51.2\angle -50^{\circ}\text{ A}}$; $\boxed{\mathbf{I}_R = 25.6\angle 26^{\circ}\text{ A}}$

74. (a) $\mathbf{V}_S = 120\angle 0°$ V

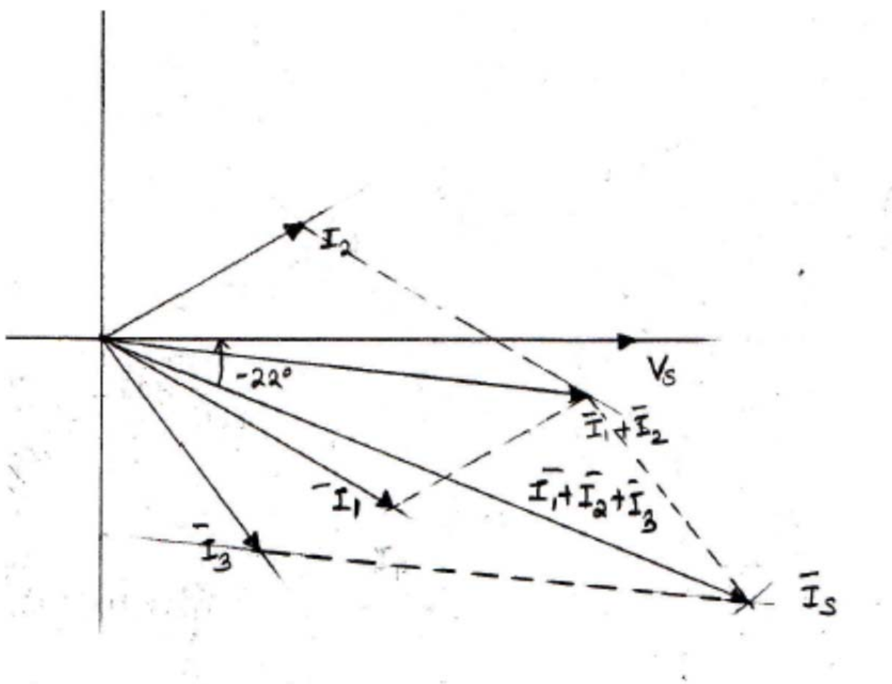
$\mathbf{Z}_1 = 40\angle 30°\ \Omega$

$\mathbf{Z}_2 = 50 - j30 = 58.31\angle -30.96°\ \Omega$

$\mathbf{Z}_3 = 30 + j40 = 50\angle 53.13°\ \Omega$

$$\boxed{\begin{aligned}\mathbf{I}_1 &= \frac{\mathbf{V}_S}{\mathbf{Z}_1} = \frac{120\angle 0°}{40\angle 30°} = 3\angle -30°\text{ A}\\ \mathbf{I}_2 &= \frac{\mathbf{V}_S}{\mathbf{Z}_2} = \frac{120\angle 0°}{58.31\angle -30.96°} = 2.05\angle -30.96°\text{ A}\\ \mathbf{I}_3 &= \frac{\mathbf{V}_S}{\mathbf{Z}_3} = \frac{120\angle 0°}{50\angle 53.13} = 2.4\angle -53.13°\text{ A}\end{aligned}}$$

(b) Here the scale is : 50V = 1 inch and 2A = 1 inch.

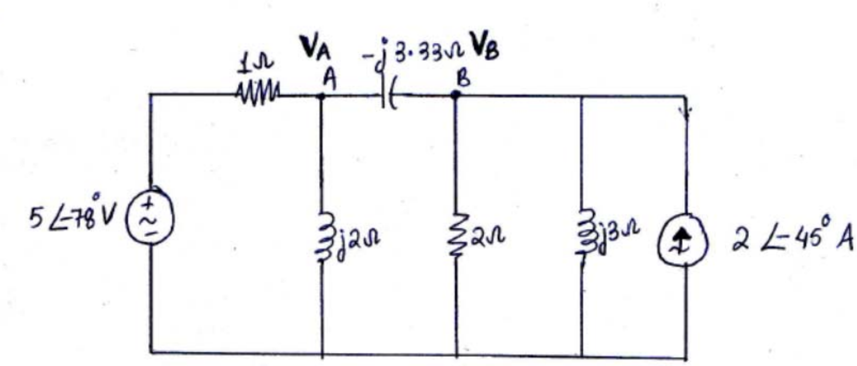


(c) From the graph, we find that,

$\boxed{\mathbf{I}_S = 6.2\angle -22°\text{ A}}$

75. (b) $\boxed{0.333\angle 124^{o}}$

76. (a)



(b) Thevenin Impedance

$$\mathbf{Z}_{TH} = (1 \| j2) + (2 \| j3)\,\Omega$$

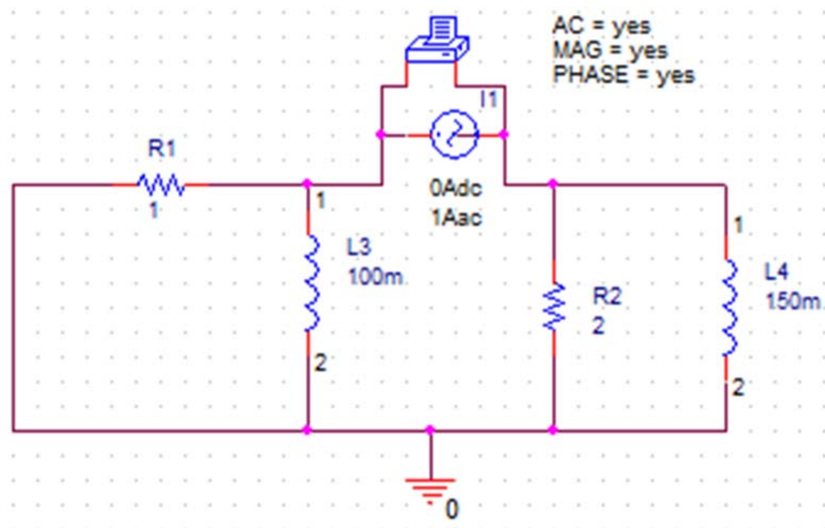
$$= \frac{j2}{1+j2} + \frac{j6}{2+j3}$$

$$= \frac{-18+j10}{-4+j7}$$

$$= \frac{20.6\angle 150.94°}{8.06\angle 119.74°}$$

$$= \boxed{2.55\angle 31.2°\,\Omega}$$

Pspice Verification:



```
FREQ          VM(N00233,N00263)VP(N00233,N00263)

 3.183E+00    2.554E+00    3.120E+01
```

Thevenin voltage:

In order to find the thevenin voltage, after removing the capacitor and on applying node voltage method, we can write the nodal equations as,

At node A,

$$\frac{5\angle -78° - \mathbf{V}_{A'}}{1} = \frac{\mathbf{V}_{A'}}{j2} \qquad [1]$$

At node B,

$$\frac{4\angle -45° - \mathbf{V}_{B'}}{2} = \frac{\mathbf{V}_{B'}}{j3} \qquad [2]$$

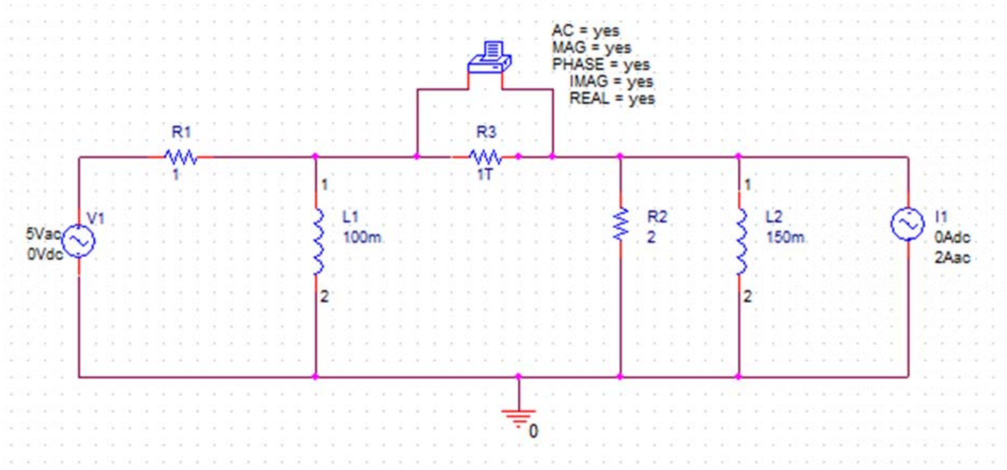
Solving the nodal equations [1] and [2], we get,

$\mathbf{V}_{A'} = 4.472\angle -51.43°$ V and

$\mathbf{V}_{B'} = 3.328\angle -11.31°$ V

$$\mathbf{V}_{TH} = \mathbf{V}_{A'} - \mathbf{V}_{B'}$$
$$= 4.472\angle -51.43° - 3.328\angle -11.31°$$
$$= -0.4752 - j2.8437$$
$$= \boxed{2.88\angle -99.49° \text{ V}}$$

Pspice Verification:



```
FREQ        VM(N00225,N00247)VP(N00225,N00247)VR(N00225,N00247)VI(N00225,N00247)

 3.183E+00   2.884E+00  -9.949E+01  -4.756E-01  -2.844E+00
```

Calculate $v_c(t)$:

$$\mathbf{Z}_c = -j3.33 = 3.33\angle -90°\,\Omega$$
$$\mathbf{Z}_{total} = 2.18 - j2.01 = 2.96\angle -42.67°\,\Omega$$
$$\mathbf{V}_c = \frac{\mathbf{V}_{TH} \times \mathbf{Z}_c}{\mathbf{Z}_{total}}$$
$$= \frac{(2.88\angle -99.49°)(3.33\angle -90°)}{2.96\angle -42.67°}$$
$$= 3.24\angle -146.82° V$$
$$\boxed{v_c(t) = 3.24\cos(20t - 146.82°)\text{ V}}$$

Pspice Verification:

```
FREQ         VM(N00225,N00247)VP(N00225,N00247)

 3.183E+00    3.238E+00   -1.469E+02
```

(c) The current flowing out of the positive terminal of the voltage source is given by $\frac{5\angle -78° - \mathbf{V}_A}{1}$ A. If we apply nodal voltage analysis, we get,

At node A,

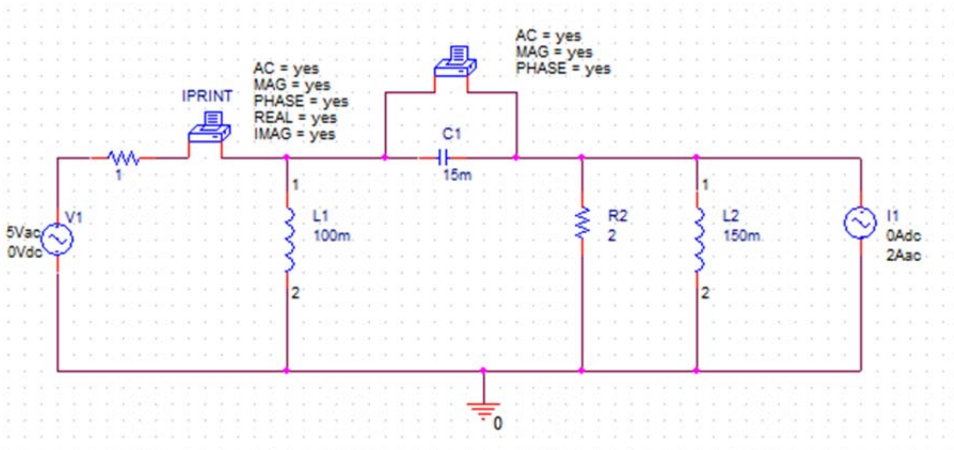
$$\frac{5\angle -78° - \mathbf{V}_A}{1} = \frac{\mathbf{V}_A}{j2} + \frac{\mathbf{V}_C}{-j3.33}$$

From (b), we have, $\mathbf{V}_C = 3.24\angle -146.82°$ V

On solving, we get,

$$\mathbf{V}_A = 2.0348 - j3.057 = 3.67\angle -56.35°\text{ V}$$
$$\mathbf{I} = -0.995 - j1.833 = 2.08\angle -118.49°\text{ A}$$
$$\boxed{i(t) = 2.08\cos(20t - 118.49°)\text{ A}}$$

Pspice Verification:

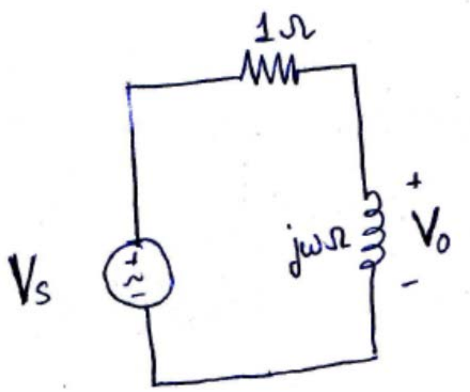


FREQ	IM(V_PRINT2)	IP(V_PRINT2)	IR(V_PRINT2)	II(V_PRINT2)
3.183E+00	2.087E+00	-1.186E+02	-9.984E-01	-1.832E+00

77. If both sources operate at 20 rad/s, $v_c(t) = 510\sin(20t - 124^o)$ mV. However, in the present case,

$\boxed{v_c(t) = 563\sin(20t - 77.3^o) + 594\sin(19t + 140^o) \text{ mV}}$.

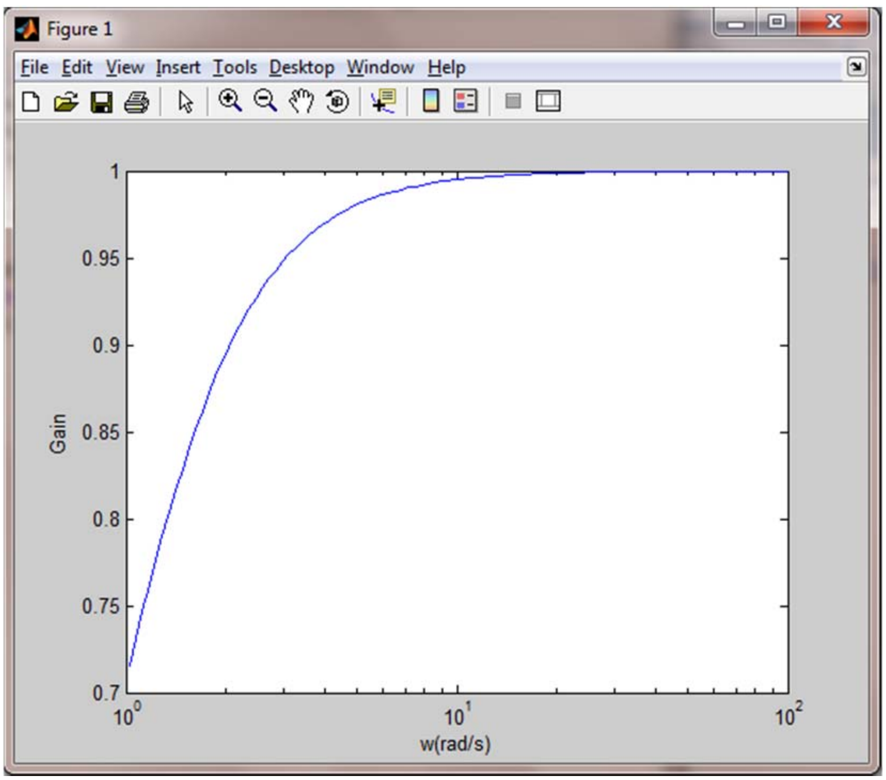
78. (a)



(b) Using the voltage divider rule, we get,

$$\frac{\mathbf{V}_O}{\mathbf{V}_S} = \frac{j\omega}{1+j\omega} = \boxed{\frac{\omega}{\sqrt{1+\omega^2}} \angle\left(90° - \tan^{-1}(\omega)\right)}$$

(c)



(d) From the plot of the gain, we see that the circuit transfers high frequencies more effectively to the output.

79. (b) $\boxed{1\angle -90^{o}}$

(d)

$$\left|\frac{\mathbf{V}_o}{\mathbf{V}_s}\right| = \frac{1}{\sqrt{1+\omega^2}}$$

The circuit transfers low frequencies to the output more effectively, as the “gain” approaches zero as the frequency approaches infinity.

80. One out of many possible design solutions:

Here, the impedance is given as,

$$\mathbf{Z} = \frac{(22 - j7)}{5\angle 8^\circ} = 4.618\angle -25.65^\circ = 4.1629 - j2\,\Omega$$

If $\mathbf{Z} = 4.1629 - j2\,\Omega$ is constructed using a series combination of single resistor, capacitor and an inductor, then, $\mathrm{R} = 4.16\Omega$ and $-j2 = j\omega L - \dfrac{j}{\omega C}$. Selecting L as 200nH arbitrarily yields the value of the capacitor as 0.12pF.

$\boxed{\text{Thus, one design will be } 4.16\Omega \text{ resistor in series with 200nH inductor and 0.12pF capacitor.}}$